



ST STITHIANS GIRLS COLLEGE

AP MATHEMATICS  
GRADE 12

DATE: July 2011

TIME: 3 hours

MARKS: 281

NAME: \_\_\_\_\_ TEACHER: \_\_\_\_\_

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 7 pages, including the front cover. A formula sheet is attached at the end which you may detach.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Unless otherwise stated, round answers to **two** decimal places where necessary. (Note Section B: Statistics is correct to **four** decimal places)
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

## SECTION A: CALCULUS & ALGEBRA

### QUESTION 1

- a) Rewrite  $\frac{(1+3i)^2}{i+1}$  in the form  $a+bi$ , without using a calculator and showing all workings. (6)
- b) Given that  $z_1$  and  $z_2$  are solutions to  $z^2 + 2z + 3 = 0$ , find  $z_1 z_2$ . (9)
- [15]

### QUESTION 2

- a) Determine the following limits:

i)  $\lim_{x \rightarrow 3} \frac{2\sqrt{x} - \sqrt{12}}{3-x}$  Hint: Factorise using difference of 2 squares (8)

ii)  $\lim_{x \rightarrow 0} \frac{5\sin 2x}{\sin 4x}$  (6)

iii)  $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - x}}$  (6)

- b) Prove the identity:  $\frac{1 + \sec 2A}{\cos^2 A} = 2\sec 2A$  (8)

[28]

### QUESTION 3

Find the gradient of the tangent to the curve defined by the equation  $y = x^2 + 2x - 3$  at the point  $(-2; 1)$ .

[10]

### QUESTION 4

Given  $y = x^3 + 2x^2 - 5x + 7$

- a) Write down the first, second and third derivatives of  $y$ . Do not simplify your answers. (6)
- b) Write down a general formula for the  $n$ th derivative of  $y$ . (6)
- c) Hence write down the
- i) 20<sup>th</sup> derivative of  $y$ . (3)
- ii) 21<sup>st</sup> derivative of  $y$ . (2)

[17]

### QUESTION 5

Given the function

- a) Prove that  $f(x) = x^2 - 2x + 1$  has an intercept with the  $x$ -axis between  $x = 1$  and  $x = 2$ . (4)
  - b) Use Newton's Method to determine the intercept with the  $x$ -axis correct to 6 decimal places. (5)
- [9]**

### QUESTION 6

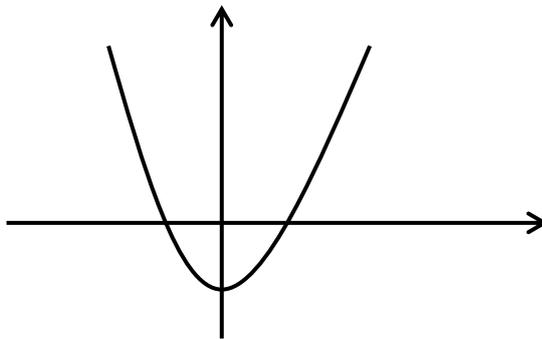
The function  $f(x) = \frac{x^2 - 4}{x^2 + 1}$  is given.

- a) Determine the asymptotes and discontinuities of  $f(x)$ . (8)
  - b) Determine and classify the stationary points of  $f(x)$ . (14)
  - c) Determine the intercepts of  $f(x)$  with the axes. (5)
  - d) Sketch  $f(x)$ , indicating all asymptotes, stationary points and intercepts with the axes. (9)
- [36]**

### QUESTION 7

**You may not just type the question into your calculator. Show appropriate working!**

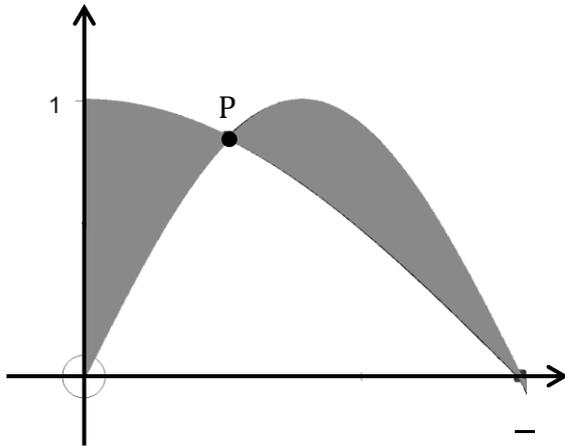
The graph below is of the function



Use a Riemann Sum to determine the area between this curve and the  $x$ -axis from  $x = -1$  to  $x = 1$ . (11)

**QUESTION 8**

The sketch shows the graphs of  $y = \cos x$  and  $y = \sin x$  for the interval  $0 \leq x \leq \frac{\pi}{2}$ .



- a) Show that the  $x$ -coordinate of the point of intersection  $P$  is  $\frac{\pi}{6}$ . (5)
  - b) Hence showing all working, and without the use of a calculator, calculate the area of the **unshaded** region. (10)
- [15]**

**QUESTION 9**

Evaluate the following integrals:

- a)  $\int_0^1 x^2 dx$  (2)
  - b)  $\int_0^1 x^3 dx$  (8)
  - c)  $\int_0^1 x^4 dx$  (8)
  - d)  $\int_0^1 x^5 dx$  (9)
- [27]**

**QUESTION 10**

- a) The function  $f(x) = \sin x$  is defined as follows:

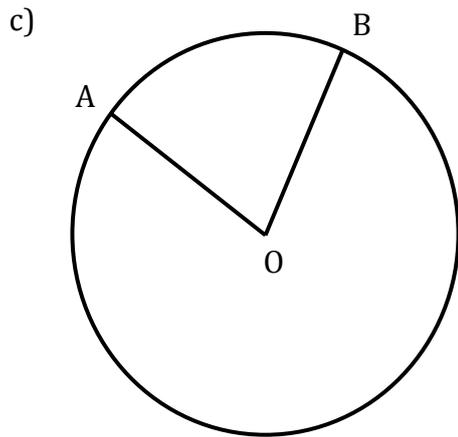
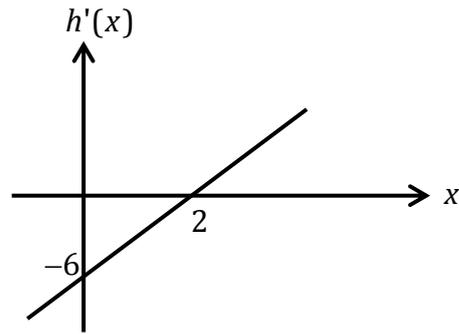
—

Determine the value(s) of  $x$  and if  $f(x)$  is differentiable at  $x = \frac{\pi}{2}$ . (8)

b) The graph of  $h'(x)$ , a linear function, is shown. Draw a rough sketch of the following:

i)  $h''(x)$  (4)

ii)  $h(x)$  (4)



Points A and B lie on the circle with centre O.

The area of the minor sector AOB is  $72 \text{ cm}^2$

The perimeter of the minor sector is 36 cm.

Calculate the radius of the circle.

(9)  
[25]

### QUESTION 11

Biologists have determined that when a fish swims a distance  $x$  metres at a speed of  $v$  m/s relative to the water and against a current flowing at the rate of  $u$  m/s (with  $u < v$ ), then the fish expends a total amount of energy given by:

$\frac{1}{2}kx^3$ , where  $k$  is a constant.

If the current is flowing at 2 m/s, find the speed  $v$  at which the fish must swim in order to minimise the total energy expended.

[9]

## SECTION B: STATISTICS

Answers to 4 decimal places where necessary.

### QUESTION 1

A hardware manufacturer produces 95 471 different kinds of hardware. A systems designer is asked to design an inventory system to identify each item. Each identity number must begin with 2 capital letters. Any letter can be used except I and O. The rest of the identity number must consist of the digits 0 to 9. For example, DQ7099, RR25, etc. how many digits must the systems designer include in the identity number format to ensure that each kind of hardware has a unique identity number?

[6]

### QUESTION 2

The probability that a certain beginner golfer hits a good shot if he chooses the correct club is  $\frac{1}{5}$ . The probability of a good shot with an incorrect club is  $\frac{1}{10}$ . In his bag, he has 5 different clubs, only one of which is correct for the shot in question. If he chooses a club at random, what is the probability that he hits a good shot?

[6]

### QUESTION 3

A study was done to determine the efficiency of 3 different drugs: A, B and C in relieving exam stress. Over the period of research, 80 patients were given the chance to use all 3 drugs. The following results were obtained:

- 40 reported relief from drug A
- 35 reported relief from drug B
- 40 reported relief from drug C
- 21 reported relief from both drugs A and C
- 18 reported relief from both drugs B and C
- 68 reported relief from at least one of the drugs
- 7 reported relief from all 3 drugs

- a) Record this information in a Venn diagram. (6)
- b) How many of the patients got relief from none of the drugs? (2)
- c) How many of the patients got relief from drugs A and B, but not C? (2)
- d) What is the probability that a randomly chosen patient got relief from at least two of the drugs? (4)

[14]

#### QUESTION 4

- a) How many distinct permutations can be formed with the letters of the word SOCIOLOGICAL? (6)
- b) In how many of these permutations will the three O's be together? (6)
- [12]**

#### QUESTION 5

The probability that a patient recovers from a rare blood disorder is 0,4. If 10 people are known to have contracted this disorder, find the probability that:

- a) Exactly 7 survive. (5)
- b) At least 2 survive. (6)
- [11]**

#### QUESTION 6

In a test which was marked out of 40, the average mark was 25, with a standard deviation of 5 marks. The test was written by 124 candidates. Assume that the marks are normally distributed.

- a) Determine the number of candidates who obtained between 16 and 32 marks. (12)
- b) Determine the range of the marks of the bottom 25% of the candidates. (10)
- c) If each mark is multiplied by 2,5 to obtain a percentage, what effect does this have on the mean and standard deviation? (4)
- [26]**

#### QUESTION 7

Adam and Ben find a jumble of  $n$  ropes lying on the floor. Each takes hold of one loose end. What is the probability that they are both holding the same rope? (write down the letter of the correct answer only):

- A) —
- B) —
- C) -
- D) —
- E) —

**[4]**

**TOTAL: 281 MARKS**