



# basic education

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Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

## **NATIONAL SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P1**

**NOVEMBER 2011**

**MEMORANDUM**

**MARKS: 150**

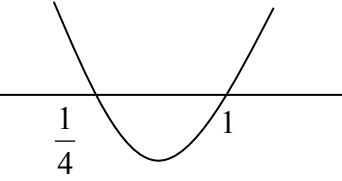
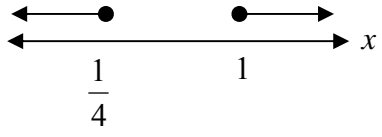
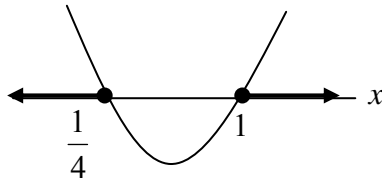
**This memorandum consists of 28 pages.**

**NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in all aspects of the marking memorandum.

**QUESTION 1**

1.1.1	$x(x+1) = 6$ $x^2 + x = 6$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3 \text{ or } 2$ <p><b>OR</b></p> $x^2 + x - 6 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)}$ $x = -3 \text{ or } 2$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> Answers by inspection: award 3/3 marks</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> Answer only of <math>x = 2</math> : award 1/3 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If candidate converts equation to linear: award 0/3 marks</p> </div>	<p>✓ standard form ✓ factors ✓ answers (3)</p> <p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ answers (3)</p>
1.1.2	$3x^2 - 4x = 8$ $3x^2 - 4x - 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$ $= \frac{4 \pm \sqrt{16 + 96}}{6}$ $= \frac{4 \pm \sqrt{112}}{6}$ $= \frac{2 \pm 2\sqrt{7}}{3}$ $= 2,43 \text{ or } -1,10$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> If candidate uses incorrect formula: maximum 1/4 marks (for standard form)</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If an error in subs and gets: <math>\frac{4 \pm \sqrt{-80}}{6}</math> and states “no solution”: maximum 3/4 marks</p> <p>If doesn't conclude with “no solution”: maximum 2/4 marks</p> </div>	<p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ <math>\sqrt{112}</math></p> <p>✓ <math>\frac{4 \pm \sqrt{112}}{6}</math> or decimal answer (4)</p>

	<p><b>OR</b></p> $3x^2 - 4x = 8$ $3x^2 - 4x - 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$ $= 2,43 \text{ or } -1,10$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><b>Note:</b> Penalise 1 mark for inaccurate rounding off to ANY number of decimal places if candidate gives decimal answers</p> </div>	<p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>																				
<p>1.1.3</p>	<p><math>4x^2 + 1 \geq 5x</math></p> <p><math>4x^2 - 5x + 1 \geq 0</math></p> <p><math>(4x - 1)(x - 1) \geq 0</math></p> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">+</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">+</td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;"><math>\frac{1}{4}</math></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">1</td> <td style="padding: 5px;"></td> </tr> </table>  </div> <p><math>x \leq \frac{1}{4}</math> or <math>x \geq 1</math>    <b>OR</b>    <math>\left(-\infty; \frac{1}{4}\right] \cup [1; \infty)</math></p> <p><b>OR</b></p>  <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><b>Note:</b> If candidate gives either of these correct graphical solutions but writes down the incorrect intervals or uses AND: max 3/4 marks</p> </div> <p><b>OR</b></p>  <p><b>NOTES:</b></p> <p>If a candidate gives an answer of <math>1 \leq x \leq \frac{1}{4}</math> then max 3/4 marks.</p> <p>If a candidate gives an answer of <math>\frac{1}{4} \leq x \leq 1</math> then max 2/4 marks.</p> <p>If a candidate gives an answer of <math>x \leq \frac{1}{4}</math> <b>and</b> <math>x \geq 1</math> then max 3/4 marks.</p> <p>If the candidate leaves out the equality of the notation then penalty of 1 mark.</p> <p>If a candidate gives an answer of <math>x \leq \frac{1}{4}; x \geq 1</math> then max 3/4 marks.</p> <p>If candidate gives <math>x \geq \frac{1}{4}</math> and/or <math>x \geq 1</math>, BREAKDOWN: max 2/4 marks.</p> <p>If candidate gives : award 3/4 marks</p> <table border="1" style="border-collapse: collapse; text-align: center; margin-top: 10px;"> <tr> <td style="padding: 5px;">+</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">+</td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;"><math>\frac{1}{4}</math></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">1</td> <td style="padding: 5px;"></td> </tr> </table>	+	0	-	0	+		$\frac{1}{4}$		1		+	0	-	0	+		$\frac{1}{4}$		1		<p>✓ factors</p> <p>✓ both critical values of <math>\frac{1}{4}</math> and 1</p> <p>✓ or <b>OR</b> <math>\cup</math></p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
+	0	-	0	+																		
	$\frac{1}{4}$		1																			
+	0	-	0	+																		
	$\frac{1}{4}$		1																			

1.2.1	$x^2 + 5xy + 6y^2 = 0$ $(x + 3y)(x + 2y) = 0$ $x + 3y = 0 \qquad x + 2y = 0$ $x = -3y \quad \text{OR} \quad x = -2y$ $\frac{x}{y} = -3 \qquad \frac{x}{y} = -2$ <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p><b>Note:</b> If a candidate gives <math>-\frac{x}{y} = 3</math> or <math>-\frac{x}{y} = 2</math> award 2/3 marks</p> </div> <p><b>OR</b></p> <p>Let <math>k = \frac{x}{y}</math></p> $x^2 + 5xy + 6y^2 = 0$ $\left(\frac{x}{y}\right)^2 + 5\left(\frac{x}{y}\right) + 6 = 0$ $k^2 + 5k + 6 = 0$ $(k + 3)(k + 2) = 0$ $k = -3 \quad \text{or} \quad k = -2$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p><b>OR</b></p> $x^2 + 5xy + 6y^2 = 0$ $x = \frac{-5y \pm \sqrt{(5y)^2 - 4(1)(6y^2)}}{2(1)}$ $x = \frac{-5y \pm \sqrt{y^2}}{2}$ $x = \frac{-5y \pm y}{2}$ $x = -3y \qquad x = -2y$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p><b>OR</b></p> $x^2 + 5xy + 6y^2 = 0$ $x^2 + 5xy + \left(\frac{5}{2}y\right)^2 = -6y^2 + \left(\frac{5}{2}y\right)^2$ $\left(x + \frac{5}{2}y\right)^2 = \frac{1}{4}y^2$ $x + \frac{5}{2}y = \pm \frac{1}{2}y$ $x = -\frac{5}{2}y \pm \frac{1}{2}y$	<p>✓ factors</p> <p>✓✓ answers (3)</p> <p>✓ factors</p> <p>✓✓ answers (3)</p> <p>✓ substitutes correctly into correct formula</p> <p>✓✓ answers (3)</p> <p>✓ completing the square</p>
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	$x = -3y \quad x = -2y$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p><b>OR</b></p> <p>Let <math>k = \frac{x}{y}</math>  <math>x = ky</math></p> $x^2 + 5xy + 6y^2 = 0$ $(ky)^2 + 5y(ky) + 6y^2 = 0$ $k^2y^2 + 5y^2k + 6y^2 = 0$ $y^2(k^2 + 5k + 6) = 0$ $(k^2 + 5k + 6) = 0$ $(k + 3)(k + 2) = 0$ $k = -3 \quad \text{or} \quad k = -2$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p>Note: <math>(x;y) = (0;0)</math> is also a solution, but in this case <math>\frac{x}{y}</math> is undefined</p> <p><b>OR</b></p> <p>Let <math>y = 1</math>,</p> $x^2 + 5x + 6 = 0$ $(x + 2)(x + 3) = 0$ $x = -2 \quad \text{or} \quad x = -3$ $\frac{x}{y} = -2 \quad \text{or} \quad \frac{x}{y} = -3$	<p>✓✓ answers (3)</p> <p>✓ factors</p> <p>✓✓ answers (3)</p> <p>✓ factors</p> <p>✓✓ answers (3)</p>
<p>1.2.2</p>	$x + y = 8 \quad x + y = 8$ $-3y + y = 8 \quad -2y + y = 8$ $-2y = 8 \quad \text{OR} \quad -y = 8$ $y = -4 \quad y = -8$ $x = 12 \quad x = 16$ <p><b>OR</b></p> $\frac{8 - y}{y} = -3 \quad \text{OR} \quad \frac{8 - y}{y} = -2$ $8 - y = -3y \quad 8 - y = -2y$ $8 = -2y \quad 8 = -y$ $y = -4 \quad y = -8$ $x = 12 \quad x = 16$	<p>✓ substitution  <math>x = -3y</math>                  ✓ subs <math>x = -2y</math>                  ✓✓ y values                  ✓ both x values correct                  (5)</p> <p>✓ <math>x = 8 - y</math>                  ✓ substitution                  ✓✓ y values                  ✓ both correct x values                  (5)</p>

	<p><b>OR</b></p> $x + y = 8$ $y = 8 - x$ $\frac{x}{8-x} = -3 \quad \text{OR} \quad \frac{x}{8-x} = -2$ $x = -3(8-x) \quad x = -2(8-x)$ $x = -24 + 3x \quad x = -16 + 2x$ $-2x = -24 \quad -x = -16$ $x = 12 \quad x = 16$ $y = -4 \quad y = -8$ <p><b>OR</b></p> $(x+2y)(x+3y) = 0$ $x + y = 8$ $x = 8 - y$ $(y+8)(2y+8) = 0$ $y = -8 \quad \text{or} \quad y = -4$ $x = 16 \quad x = 12$ <p><b>OR</b></p> $x = 8 - y$ $(8-y)^2 + 5(8-y)y + 6y^2 = 0$ $64 - 16y + y^2 + 40y - 5y^2 + 6y^2 = 0$ $2y^2 + 24y + 64 = 0$ $y^2 + 12y + 32 = 0$ $(y+8)(y+4) = 0$ $y = -8 \quad \text{or} \quad y = -4$ $x = 16 \quad x = 12$ <p><b>OR</b></p>	<p>✓ <math>y = 8 - x</math></p> <p>✓ substitution</p> <p>✓✓ <math>x</math> values correct</p> <p>✓ both <math>y</math> values correct</p> <p>(5)</p> <p>✓ <math>x = 8 - y</math></p> <p>✓ substitution</p> <p>✓✓ <math>y</math> values correct</p> <p>✓ both <math>x</math> values correct</p> <p>(5)</p> <p>✓ <math>x = 8 - y</math></p> <p>✓ substitution</p> <p>✓ factors</p> <p>✓ both <math>y</math> values correct</p> <p>✓ both <math>x</math> values correct</p> <p>(5)</p>
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<p><b>OR</b></p> $x = 8 - y$ $(8 - y)^2 + 5(8 - y)y + 6y^2 = 0$ $64 - 16y + y^2 + 40y - 5y^2 + 6y^2 = 0$ $2y^2 + 24y + 64 = 0$ $y^2 + 12y + 32 = 0$ $y = \frac{-12 \pm \sqrt{12^2 - 4(1)(32)}}{2(1)}$ $= \frac{-12 \pm \sqrt{16}}{2}$ $y = -8 \quad \text{or} \quad y = -4$ $x = 16 \quad \quad x = 12$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p><b>Note:</b> If a candidate uses the formula and replaces <math>x</math> for <math>y</math> and then answers are swapped: maximum 4/5 marks</p> </div>	<ul style="list-style-type: none"> <li>✓ <math>x = 8 - y</math></li> <li>✓ substitution</li>   <li>✓ substitutes into correct formula</li> <li>✓ both <math>y</math> values correct</li> <li>✓ both <math>x</math> values correct</li> </ul> <p style="text-align: right;">(5)</p>
<p><b>OR</b></p> $y = 8 - x$ $x^2 + 5x(8 - x) + 6(8 - x)^2 = 0$ $x^2 + 40x - 5x^2 + 6(64 - 16x + x^2) = 0$ $2x^2 - 56x + 384 = 0$ $x^2 - 28x + 192 = 0$ $(x - 16)(x - 12) = 0$ $x = 12 \quad \quad x = 16$ $y = -4 \quad \text{or} \quad y = -8$		<ul style="list-style-type: none"> <li>✓ <math>y = 8 - x</math></li> <li>✓ substitution</li>   <li>✓ factors</li> <li>✓ both <math>x</math> values correct</li> <li>✓ both <math>y</math> values correct</li> </ul> <p style="text-align: right;">(5)</p>
<p><b>OR</b></p> $y = 8 - x$ $x^2 + 5x(8 - x) + 6(8 - x)^2 = 0$ $x^2 + 40x - 5x^2 + 6(64 - 16x + x^2) = 0$ $2x^2 - 56x + 384 = 0$ $x^2 - 28x + 192 = 0$ $x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(192)}}{2(1)}$ $= \frac{28 \pm \sqrt{416}}{2}$ $x = 12 \quad \quad x = 16$ $y = -4 \quad \text{or} \quad y = -8$		<ul style="list-style-type: none"> <li>✓ <math>y = 8 - x</math></li> <li>✓ substitution</li>   <li>✓ substitutes into correct formula</li> <li>✓ both <math>x</math> values correct</li> <li>✓ both correct <math>y</math> values</li> </ul> <p style="text-align: right;">(5) <b>[19]</b></p>

## QUESTION 2

2.1.1	$x - 4 = 32 - x$ $2x = 36$ $x = 18$ <p><b>OR</b></p> $a = 4$ $a + 2d = 32$ $2d = 28$ $d = 14$ $x = 14 + 4$ $x = 18$ <p><b>OR</b></p> $x = \frac{4 + 32}{2} = 18$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> If answer only: award 2/2 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If candidate writes <math>x - 4 \quad 32 - x</math> only (i.e. omits equality) : 0/2 marks</p> </div>	<p>✓ <math>T_2 - T_1 = T_3 - T_2</math></p> <p>✓ answer (2)</p> <p>✓ <math>a + 2d = 32</math> and <math>a = 4</math></p> <p>✓ answer (2)</p> <p>✓ substitutes correctly into arithmetic mean formula i.e. <math>\frac{4 + 32}{2}</math></p> <p>✓ answers (2)</p>
2.1.2	$\frac{x}{4} = \frac{32}{x}$ $x^2 = 128$ $x = \pm\sqrt{128}$ $x = \pm 8\sqrt{2} \quad \text{OR} \quad x = \pm 11,31 \quad \text{OR} \quad x = \pm 2^{\frac{7}{2}}$ <p><b>OR</b></p> $a = 4$ $r = \frac{x}{4}$ $ar^2 = 4\left(\frac{x}{4}\right)^2$ $32 = 4\left(\frac{x}{4}\right)^2$ $x^2 = 128$ $x = \pm\sqrt{128}$ $x = \pm 8\sqrt{2} \quad \text{or} \quad x = \pm 11,31 \quad \text{or} \quad x = \pm 2^{\frac{7}{2}}$ <p><b>OR</b></p> $x = \pm\sqrt{4 \times 32}$ $x = \pm\sqrt{128} \quad \text{or} \quad x = \pm 8\sqrt{2} \quad \text{or} \quad x = \pm 11,31 \quad \text{or} \quad x = \pm 2^{\frac{7}{2}}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> If candidate writes <math>\frac{x}{4} \quad \frac{32}{x}</math> only (i.e. omits equality) : 0/2 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If only <math>x = \sqrt{128}</math> then penalty 1 mark</p> </div>	<p>✓ <math>\frac{T_2}{T_1} = \frac{T_3}{T_2}</math></p> <p>✓ <math>x^2 = 128</math></p> <p>✓ both answers (surd or decimal or exponential form) (3)</p> <p>✓ <math>32 = 4\left(\frac{x}{4}\right)^2</math></p> <p>✓ <math>x^2 = 128</math></p> <p>✓ both answers (surd or decimal or exponential form) (3)</p> <p>✓✓ substitutes correctly into geometric mean formula i.e. <math>\pm\sqrt{4 \times 32}</math></p> <p>✓ both answers (surd or decimal or exponential form) (3)</p>



<p>2.2</p>	$P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \dots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \dots + 3^8$ $= \frac{3^{-4}(3^{13} - 1)}{3 - 1}$ $= 9841,49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$ <p><b>OR</b></p> $P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \dots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \dots + 3^8$ $= \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \dots + 6561$ $= 9841,49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$ <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p><b>Note:</b> Correct answer only: 1/4 marks only</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p><b>Note:</b> If the candidate rounds off and gets 9841,46 (i.e. correct to one decimal place): DO NOT penalise for the rounding off.</p> </div>	<p>✓ <math>a = 3^{-4}</math> or <math>\frac{1}{81}</math></p> <p>✓ <math>r = 3</math> ✓ subs into correct formula</p> <p>✓ answer (4)</p> <p>✓✓ expand the sum ✓ 13 terms in expansion</p> <p>✓ answer (4)</p>
<p>2.3</p>	$S_n = a + [a + d] + [a + 2d] + \dots + [a + (n - 2)d] + [a + (n - 1)d]$ $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots + [a + d] + a$ $2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \dots + [2a + (n - 1)d] + [2a + (n - 1)d]$ $= n[2a + (n - 1)d]$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ <p><b>OR</b></p> $S_n = a + [a + d] + [a + 2d] + \dots + (T_n - d) + T_n$ $S_n = T_n + (T_n - d) + \dots + [a + d] + a$ $2S_n = a + T_n + a + T_n + a + T_n + \dots + a + T_n$ $= n[a + a + (n - 1)d]$ $= [2a + (n - 1)d]$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p><b>Note:</b> If a candidate uses a circular argument (eg <math>S_{n+1} = S_n + T_n</math>): max 1/4 marks (for writing out <math>S_n</math>)</p> </div> <p><b>Note:</b> If a candidate uses a specific linear sequence, then NO marks.</p>	<p>✓ writing out <math>S_n</math> ✓ “reversing” <math>S_n</math></p> <p>✓ expressing <math>2S_n</math> ✓ grouping to get <math>2S_n = n[2a + (n - 1)d]</math> (4)</p> <p>✓ writing out <math>S_n</math> ✓ “reversing” <math>S_n</math></p> <p>✓ expressing <math>2S_n</math> ✓ grouping to get <math>2S_n = n[a + a + (n - 1)d]</math> (4)</p> <p style="text-align: right;"><b>[13]</b></p>

**QUESTION 3**

3.1	21; 24	<div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If candidate writes <math>T_8 = 21</math> <math>T_7 = 24</math>: award 1/2 marks</p> </div>	<p>✓ 21 ✓ 24</p> <p style="text-align: right;">(2)</p>
3.2	<p><math>T_{2k} = 3.2^{k-1}</math> and so <math>T_{52} = 3.2^{26-1} = 100663296</math></p> <p><math>T_{2k-1} = 3 + 6(k-1) = 6k - 3</math> and so <math>T_{51} = 6(26) - 3 = 153</math></p> <p><math>T_{52} - T_{51} = 100663296 - 153</math> <math>= 100663143</math></p> <p><b>OR</b></p> <p>Consider sequence <math>P</math>: 3 ; 6 ; 12 ... <math>P_n = 3.2^{n-1}</math> <math>P_{26} = 3.2^{26-1} = 100663296</math></p> <p>Consider sequence <math>Q</math>: 3 ; 9 ; 15 ... <math>Q_n = 6n - 3</math> <math>Q_{26} = 6(26) - 3 = 153</math> <math>T_{52} - T_{51} = P_{26} - Q_{26}</math> <math>= 100663296 - 153</math> <math>= 100663143</math></p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> If candidate writes out all 52 terms <b>and</b> gets correct answer: award 5/5 marks</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> If candidate used <math>k = 52</math>: max 2/5</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> if candidate interchanges order i.e. does <math>T_{51} - T_{52}</math>: max 4/5 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> writes out all 52 terms and subtracts <math>T_{51} - T_{52}</math>: max 4/5 marks</p> </div>	<p>✓ <math>3.2^{k-1}</math> ✓ <math>T_{52}</math>  ✓ <math>6k - 3</math> ✓ <math>T_{51}</math>  ✓ answer</p> <p style="text-align: right;">(5)</p> <p>✓ <math>P_n = 3.2^{n-1}</math> ✓ <math>P_{26}</math>  ✓ <math>Q_n = 6n - 3</math> ✓ <math>Q_{26}</math>  ✓ answer</p> <p style="text-align: right;">(5)</p>

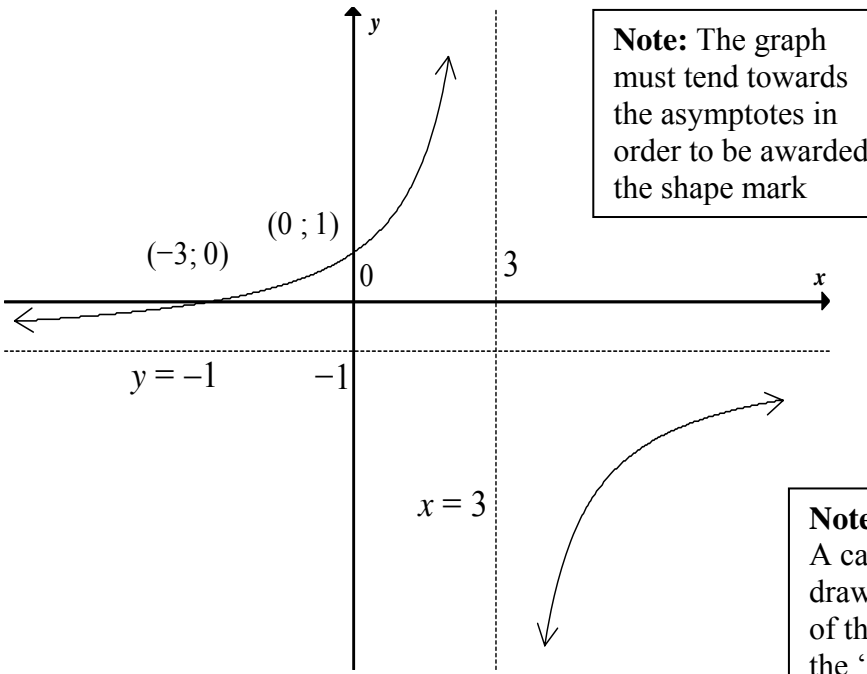
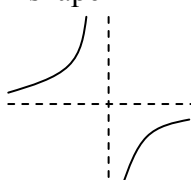
3.3	<p>For all <math>n \in \mathbf{N}</math>, <math>n = 2k</math> or <math>n = 2k - 1</math> for some <math>k \in \mathbf{N}</math></p> <p>If <math>n = 2k</math> :</p> $T_n = T_{2k} = 3 \cdot 2^{k-1}$ <p>If <math>n = 2k - 1</math> :</p> $T_n = T_{2k-1}$ $= 6k - 3$ $= 3(2k - 1)$ <p>In either case, <math>T_n</math> has a factor of 3, so is divisible by 3.</p> <p><b>OR</b></p> $P_n = 3 \cdot 2^{n-1}$ <p>Which is a multiple of 3</p> $Q_n = 6n - 3$ $= 3(2n - 1)$ <p>Which is also a multiple of 3</p> <p>Since <math>T_n = Q_{2k-1}</math> or <math>T_n = P_{2k}</math> for all <math>n \in \mathbf{N}</math>, <math>T_n</math> is always divisible by 3</p> <p><b>OR</b></p> <p>The odd terms are odd multiples of 3 and the even terms are 3 times a power of 2. This means that all the terms are multiples of 3 and are therefore divisible by 3.</p>	<p>✓ factors <math>3 \cdot 2^{k-1}</math></p> <p>✓ factors <math>3(2k - 1)</math></p> <p>(2)</p> <p>✓ factors <math>3 \cdot 2^{n-1}</math></p> <p>✓ factors <math>3(2n - 1)</math></p> <p>(2)</p> <p>✓ odd multiples of 3</p> <p>✓ 3 times a power of 2</p> <p>(2)</p> <p><b>[9]</b></p>
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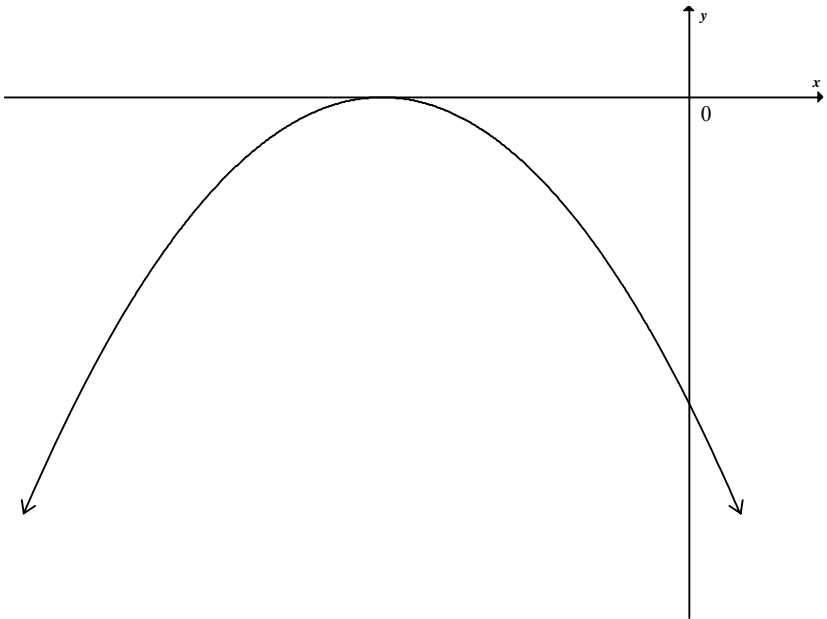
**QUESTION 4**

<p>4.1</p>	<p>The second, third, fourth and fifth terms are 1 ; -6 ; <math>T_4</math> and -14</p> <p>First differences are: <math>-7</math> ; <math>T_4 + 6</math> ; <math>-14 - T_4</math>          So <math>T_4 + 6 + 7 = -14 - 2T_4 - 6</math>  <math>T_4 = -11</math>  <math>d = -11 + 6 + 7 = 2</math> or <math>-14 + 22 - 6 = 2</math></p> <p><b>OR</b></p> <div style="text-align: center;"> </div> <p><math>T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)</math>  <math>-15 = (-7 + 2d) + (-7 + d) + -7</math>  <math>-15 = -21 + 3d</math>  <math>6 = 3d</math>  <math>d = 2</math></p> <p><b>OR</b></p> $4a + 2b + c = 1$ $9a + 3b + c = -6$ $5a + b = -7$ $25a + 5b + c = -14$ $16a + 2b = -8$ $10a + 2b = -14$ $6a = 6$ $a = 1$ $d = 2a = 2$ <p><b>OR</b></p> <div style="text-align: center;"> </div> <p><math>T_4 + 13 = -20 - 2T_4</math>  <math>3T_4 = -33</math>  <math>T_4 = -11</math>  <math>d = -11 + 13</math>  <math>d = 2</math></p>	<p>✓ -7          ✓ <math>T_4 + 6</math>          ✓ <math>-14 - T_4</math></p> <p>✓ setting up equation  <math>T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)</math>          ✓ answer (5)</p> <p>✓ -7          ✓ <math>-7 + d</math>          ✓ <math>-7 + 2d</math></p> <p>✓ setting up equation  <math>T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)</math>          ✓ answer (5)</p> <p>✓ <math>4a + 2b + c = 1</math>          ✓ <math>9a + 3b + c = -6</math></p> <p>✓ <math>25a + 5b + c = -14</math></p> <p>✓ solved simultaneously</p> <p>✓ answer (5)</p> <p>✓ -7          ✓ <math>T_4 + 6</math>          ✓ <math>-14 - T_4</math></p> <p>✓ setting up equation          ✓ answer (5)</p>
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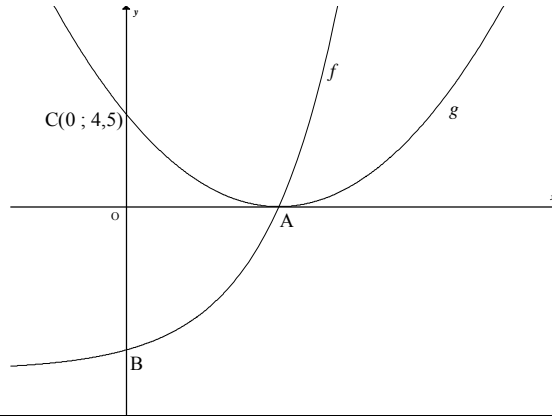
	<p><b>OR</b></p> $  \begin{array}{ccccccccc}  T_1 & & T_2 & & T_3 & & T_4 & & T_5 \\  x & & 1 & & -6 & & y & & -14 \\  & \searrow & / & \searrow & / & \searrow & / & \searrow & / \\  & 1-x & & -7 & & y+6 & & -14-y & \\  & / & \searrow & / & \searrow & / & \searrow & / & \searrow \\  & -8+x & & y+13 & & -20-2y & & &   \end{array}  $ <p> <math>y + 13 = -20 - 2y</math>  <math>3y = -33</math>  <math>y = -11</math> </p> <p>Second difference = <math>y + 13 = -11 + 13 = 2</math></p>	<p> <math>\checkmark -7</math>  <math>\checkmark y + 6</math>  <math>\checkmark -14 - y</math> </p> <p> <math>\checkmark</math> setting up equation  <math>\checkmark</math> answer                 </p> <p>(5)</p>
<p>4.2</p>	$  \begin{array}{ccc}  T_1 & & 1 & & -6 \\  & \searrow & / & \searrow & / \\  & -9 & & -7 & \\  & / & \searrow & / & \searrow \\  & 2 & & &   \end{array}  $ <p> <math>T_1 = 10</math> </p> <p><b>OR</b></p> <p> <math>a = 1</math>  <math>5a + b = -7</math>  <math>5(1) + b = -7</math>  <math>b = -12</math>  <math>a + b + c = 1</math>  <math>4(1) + 2(-12) + c = 1</math>  <math>c = 21</math>  <math>T_n = n^2 - 12n + 21</math>  <math>T_1 = (1)^2 - 12(1) + 21</math>  <math>= 10</math> </p> <p><b>OR</b></p> <p> <math>T_4 + 13 = -8 + T_1</math>                      <math>y + 13 = -8 + x</math>  <math>-11 + 13 = -8 + T_1</math>                      <b>OR</b>   <math>-11 + 13 = -8 + x</math>  <math>T_1 = 10</math>    <math>x = 10</math> </p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p><b>Note:</b> Answer only: award 2/2 marks</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p><b>Note:</b> If incorrect <math>d</math> in 4.1, 2/2 CA marks for <math>T_1 = d + 8</math> (since <math>1 - T_1 = -7 - d</math>)</p> </div>	<p> <math>\checkmark</math> method  <math>\checkmark T_1 = 10</math> </p> <p>(2)</p> <p> <math>\checkmark</math> method  <math>\checkmark T_1 = 10</math> </p> <p>(2)</p> <p> <math>\checkmark</math> method  <math>\checkmark T_1 = 10</math> </p> <p>(2)</p> <p>[7]</p>

**QUESTION 5**

<p>5.1.1</p>	$y = f(0)$ $= \frac{-6}{0-3} - 1$ $= 1$ <p>(0 ; 1) <b>OR</b> <math>x = 0</math> and <math>y = 1</math></p>	<p><b>Note:</b> Mark 5.1.1 and 5.1.2 as a single question. If the intercepts are interchanged: max 3/5 marks</p>	<p>✓ <math>y = 1</math> ✓ <math>x = 0</math></p> <p>(2)</p>
<p>5.1.2</p>	$0 = \frac{-6}{x-3} - 1$ $1 = \frac{-6}{x-3}$ $x - 3 = -6$ $x = -3$ <p>(-3 ; 0)</p>		<p>✓ <math>y = 0</math> ✓ <math>x - 3 = -6</math> ✓ answer</p> <p>(3)</p>
<p>5.1.3</p>	 <p><b>Note:</b> The graph must tend towards the asymptotes in order to be awarded the shape mark</p>	<p>✓ shape</p>  <p>✓ both intercepts correct ✓ horizontal asymptote ✓ vertical asymptote</p> <p>(4)</p> <p><b>Note:</b> A candidate who draws only one 'arm' of the hyperbola loses the 'shape' mark i.e. max 3/4 marks</p>	
<p>5.1.4</p>	<p><math>-3 &lt; x &lt; 3</math> <b>OR</b> <math>(-3; 3)</math> <b>OR</b> <math>-3 &lt; x</math> and <math>x &lt; 3</math></p> <p><b>Note:</b> if candidate writes <math>-3 &lt; x</math> only: 1/2 marks</p> <p><b>Note:</b> if candidate writes <math>x &lt; 3</math> only: 1/2 marks</p>	<p>✓ <math>-3</math> and <math>3</math> ✓ inequality OR interval notation</p> <p>(2)</p>	

5.1.5	$y = \frac{-6}{-2-3} - 1$ $= \frac{1}{5}$ $m = \frac{1 - \frac{1}{5}}{0 - (-2)}$ $= \frac{2}{5}$ <p><b>OR</b></p> $m = \frac{f(0) - f(-2)}{0 - (-2)}$ $= \frac{1 - \frac{1}{5}}{0 + 2}$ $= \frac{2}{5}$	<p>✓ <math>\frac{1}{5}</math></p> <p>✓ formula ✓ substitution ✓ answer</p> <p>(4)</p> <p>✓ formula</p> <p>✓ <math>f(-2) = \frac{1}{5}</math></p> <p>✓ substitution ✓ answer</p> <p>(4)</p>
5.2	$x = -\frac{b}{2a} < 0 \text{ since } b < 0 \text{ and } a < 0$ 	<p>✓ y-intercept negative</p> <p>✓ turning point on the x axis</p> <p>✓ turning point on the left of the y axis</p> <p>✓ <b>maximum</b> TP and quadratic shape</p> <p>(4)</p> <p><b>[19]</b></p>

**QUESTION 6**



6.1	$0 = 2^x - 8$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$ $A(3 ; 0)$	$f(0) = 2^0 - 8$ $= 1 - 8$ $= -7$ $B(0 ; -7)$	✓ $y = 0$ ✓ answer for A  ✓ $x = 0$ ✓ answer for B  (4)
6.2	$y = -8$ <b>OR</b> $y + 8 = 0$	<b>Note:</b> no CA marks	✓ answer  (1)
6.3	$h(x) = f(2x) + 8$ $= (2^{2x} - 8) + 8$ $= 4^x \text{ or } 2^{2x}$	<b>Note:</b> answer only: award 2/2 marks	✓ $(2^{2x} - 8)$ ✓ answer of $h(x) = 4^x$ or $2^{2x}$  (2)
6.4	$x = 4^y$ <b>OR</b> $x = 2^{2y}$ $y = \log_4 x$ $2y = \log_2 x$ $y = \frac{1}{2} \log_2 x$ <b>OR</b> $y = \log_2 \sqrt{x}$  <b>OR</b> $y = \frac{\log x}{\log 4}$	<b>Note:</b> answer only award 2/2 marks  <b>Note:</b> candidate works out $f^{-1}$ and gets $y = \log_2(x + 8)$ award 1/2 marks	✓ switch $x$ and $y$ ✓ answer in the form $y = \dots$  (2)
6.5	$p(x) = -\log_4 x$ <b>OR</b> $p(x) = \log_{\frac{1}{4}} x$  <b>OR</b> $p(x) = \log_4 \frac{1}{x}$ <b>OR</b> $p(x) = -\frac{1}{2} \log_2 x$  <b>OR</b> $y = -\log_2 \sqrt{x}$		✓ answer  (1)



<p>6.6</p> $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ <p><math>x = 3</math> is the axis of symmetry of <math>g</math>  <math>\therefore</math> by symmetry  <math>g(2) = g(4)</math> and <math>g(1) = g(5)</math></p> <p>Answer = <math>g(0) + g(3)</math>  <math>= 4,5 + 0</math>  <math>= 4,5</math></p> <p><b>OR</b></p> $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $\sum_{k=0}^3 g(k) = g(0) + g(1) + g(2) + g(3)$ $\sum_{k=4}^5 g(k) = g(4) + g(5)$ <p><math>x = 3</math> is the axis of symmetry of <math>g</math>  <math>\therefore</math> by symmetry</p> $g(4) = g(2)$ $g(5) = g(1)$ $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $= g(0) + g(3)$ $= 4,5 + 0$ $= 4,5$ <p><b>OR</b></p> $g(x) = a(x - 3)^2 + 0$ $4,5 = a(0 - 3)^2 + 0$ $4,5 = 9a$ $a = \frac{1}{2}$ $g(x) = \frac{1}{2}(x - 3)^2$ $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $\sum_{k=0}^3 g(k) = g(0) + g(1) + g(2) + g(3)$ $= 4,5 + 2 + 0,5 + 0$ $= 7$	<p>✓ <math>= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)</math>  ✓ <math>g(2) = g(4)</math> and <math>g(1) = g(5)</math>  ✓ <math>g(0) + g(3)</math>  ✓ answer (4)</p> <p>✓ expansion</p> <p>✓ <math>g(2) = g(4)</math> and <math>g(1) = g(5)</math></p> <p>✓ <math>g(0) + g(3)</math>  ✓ answer (4)</p> <p>✓ <math>g(x) = \frac{1}{2}(x - 3)^2</math></p> <p>✓ expansion</p>
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$$\begin{aligned}\sum_{k=4}^5 g(k) &= g(4) + g(5) \\ &= 0,5 + 2 \\ &= 2,5\end{aligned}$$

$$\begin{aligned}\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k) \\ &= 7 - 2,5 \\ &= 4,5\end{aligned}$$

**OR**

$$g(x) = ax^2 + bx + c$$

$$g(k) = ak^2 + bk + c$$

$$g(0) = c$$

$$g(1) = a + b + c$$

$$g(2) = 4a + 2b + c$$

$$g(3) = 9a + 3b + c$$

$$\sum_{k=0}^3 g(k) = 14a + 6b + 4c$$

$$g(4) = 16a + 4b + c$$

$$g(5) = 25a + 9b + c$$

$$\sum_{k=4}^5 g(k) = 41a + 9b + 2c$$

$$\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k) = -27a - 3b + 2c$$

$$g(x) = a(x-3)^2 + 0$$

$$4,5 = a(0-3)^2 + 0$$

$$4,5 = 9a$$

$$a = \frac{1}{2}$$

$$g(x) = \frac{1}{2}(x-3)^2$$

$$= \frac{1}{2}x^2 - 3x + \frac{9}{2}$$

$$\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k) = -27a - 3b + 2c$$

$$= -27\left(\frac{1}{2}\right) - 3(-3) + 2\left(\frac{9}{2}\right)$$

$$= 4,5$$

✓ 7 - 2,5

✓ answer

(4)

✓✓ -27a - 3b + 2c

✓  $g(x) = \frac{1}{2}(x-3)^2$ 

✓ answer

(4)

[14]

**QUESTION 7**

7.1	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log \frac{1}{2} = n \log 0,93$ $n = \frac{\log \frac{1}{2}}{\log 0,93}$ $= 9,55 \text{ years}$ <p style="text-align: center;"><b>OR</b></p> $A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log 0,93 \frac{1}{2} = n$ $n = 9,55 \text{ years}$	$\checkmark A = \frac{P}{2}$ $\checkmark$ subs into correct formula $\checkmark$ log $\checkmark$ answer <p style="text-align: right;">(4)</p>
	<p><b>Note:</b> If candidate interchanges <math>A</math> and <math>P</math> i.e. uses <math>P = \frac{A}{2}</math> : max 2/4 marks</p>	<p><b>Note:</b> If candidate uses incorrect formula: max 1/4 marks for <math>A = \frac{P}{2}</math></p>

<p>7.2</p>	<p><b>Radesh:</b></p> $A = P(1 + in)$ $= 6\,000(1 + 0,085 \times 5)$ $= 8\,550$ <p>Bonus = <math>0,05 \times 6\,000</math></p> $= 300$ <p>Received = <math>8\,550 + 300</math></p> $= R8\,850$ <p><b>Thandi:</b></p> $A = P(1 + i)^n$ $= 6\,000\left(1 + \frac{0,08}{4}\right)^{20}$ $= R8\,915,68$ <p>Thandi's investment is bigger.</p>	<p>✓ 8 550</p> <p>✓ R8 850</p> <p>✓ <math>n = 20</math></p> <p>✓ <math>i = \frac{0,08}{4}</math></p> <p>✓ answer</p> <p>✓ choice made</p> <p>(6)</p>
<p>7.3</p>	<p><math>F_v</math> = initial deposit with interest + annuity</p> $= 1\,000\left(1 + \frac{0,15}{12}\right)^{18} + 700\left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}}\right)$ $= 1\,250,58 + 14\,032,33$ $= R15\,282,91$ <p><b>OR</b></p> <p><math>F_v</math> = initial deposit with interest + annuity</p> $= 1\,000\left(1 + \frac{0,15}{12}\right)^{18} + 700\left(\frac{1 - \left(1 + \frac{0,15}{12}\right)^{-18}}{\frac{0,15}{12}}\right)\left(1 + \frac{0,15}{12}\right)^{18}$ $= 1\,250,58 + 11\,220,68\left(1 + \frac{0,15}{12}\right)^{18}$ $= 1\,250,58 + 14\,032,33$ $= R15\,282,91$	<p>✓ <math>i = \frac{0,15}{12}</math> or <math>\frac{1}{80}</math> or 0,0125</p> <p>✓ <math>n = 18</math></p> <p>✓ <math>n = 18</math></p> <p>✓ <math>1\,000\left(1 + \frac{0,15}{12}\right)^{18}</math></p> <p>✓ <math>700\left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}}\right)</math></p> <p>✓ answer</p> <p>(6)</p> <p>✓ <math>i = \frac{0,15}{12}</math> or <math>\frac{1}{80}</math> or 0,0125</p> <p>✓ <math>n = 18</math></p> <p>✓ <math>n = 18</math></p> <p>✓ <math>1\,000\left(1 + \frac{0,15}{12}\right)^{18}</math></p> <p>✓ <math>700\left(\frac{1 - \left(1 + \frac{0,15}{12}\right)^{-18}}{\frac{0,15}{12}}\right)\left(1 + \frac{0,15}{12}\right)^{18}</math></p> <p>✓ answer</p> <p>(6)</p>

	<p><b>OR</b></p> $F_v = 300\left(1 + \frac{0,15}{12}\right)^{18} + 700\left(\frac{\left(1 + \frac{0,15}{12}\right)^{19} - 1}{\frac{0,15}{12}}\right)$ $= 375,17 + 14\,907,74$ $= R15\,282,91$	<p>✓ <math>i = \frac{0,15}{12}</math> or <math>\frac{1}{80}</math> or 0,0125</p> <p>✓ <math>n = 19</math> (corresponding to 700)</p> <p>✓ <math>n = 18</math> (corresponding to 300)</p> <p>✓ <math>300\left(1 + \frac{0,15}{12}\right)^{18}</math></p> <p>✓ <math>700\left(\frac{\left(1 + \frac{0,15}{12}\right)^{19} - 1}{\frac{0,15}{12}}\right)</math></p> <p>✓ answer</p> <p style="text-align: right;">(6) <b>[16]</b></p>
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**QUESTION 8**

8.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4(x+h)^2 - (-4x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4(x^2 + 2xh + h^2) + 4x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-4x^2 - 8xh - 4h^2 + 4x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $= \lim_{h \rightarrow 0} (-8x - 4h)$ $= -8x$ <p><b>OR</b></p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> Incorrect notation: no lim written: penalty 2 marks</p> <p>lim written before equals sign: penalty 1 mark</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> A candidate who gives <math>-8x</math> only: 0/5 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> A candidate who omits brackets in the line <math>\lim_{h \rightarrow 0} (-8x - 4h)</math> : NO penalty</p> </div> <p>✓ formula</p> <p>✓ substitution</p> <p>✓ expansion</p> <p>✓ <math>-8x - 4h</math></p> <p>✓ answer</p> <p style="text-align: right;">(5)</p>
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	$f(x) = -4x^2$ $f(x+h) = -4(x+h)^2$ $= -4x^2 - 8xh - 4h^2$ $f(x+h) - f(x) = -8xh - 4h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $= \lim_{h \rightarrow 0} (-8x - 4h)$ $= -8x$	<ul style="list-style-type: none"> <li>✓ substitution</li> <li>✓ expansion</li> <li>✓ formula</li> <li>✓ <math>-8x - 4h</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(5)</p>
8.2.1	$y = \frac{3}{2x} - \frac{x^2}{2}$ $= \frac{3}{2}x^{-1} - \frac{1}{2}x^2$ $\frac{dy}{dx} = -\frac{3}{2}x^{-2} - x$ $= -\frac{3}{2x^2} - x$	<ul style="list-style-type: none"> <li>✓ <math>\frac{3}{2}x^{-1}</math></li> <li>✓ <math>-\frac{3}{2}x^{-2}</math></li> <li>✓ <math>-x</math></li> </ul> <p style="text-align: right;">(3)</p>
8.2.2	$f(x) = (7x+1)^2$ $= 49x^2 + 14x + 1$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$ <p><b>OR</b></p> $f(x) = (7x+1)^2$ $f'(x) = 2(7x+1)(7) \quad \text{By the chain rule}$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> <p><b>Note:</b> Incorrect notation in 8.2.1 and/or 8.2.2: Penalise 1 mark</p> </div> <ul style="list-style-type: none"> <li>✓ multiplication</li> <li>✓ <math>98x</math></li> <li>✓ <math>14</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> <li>✓✓ chain rule</li> <li>✓✓ answer</li> </ul> <p style="text-align: right;">(4) <b>[12]</b></p>

**QUESTION 9**

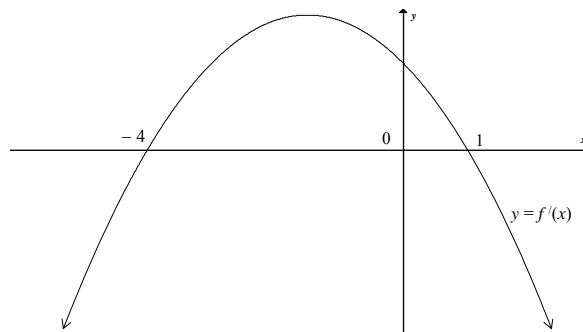
<p>9.1</p>	$f(x) = -2x^3 + ax^2 + bx + c$ $f'(x) = -6x^2 + 2ax + b$ $= -6(x-5)(x-2)$ $= -6(x^2 - 7x + 10)$ $= -6x^2 + 42x - 60$ $2a = 42$ $a = 21$ $b = -60$ $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$ $f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$ $-9 = -52 + c$ $c = 43$	<div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> A candidate who substitutes the values of <math>a</math>, <math>b</math> and <math>c</math> and then checks (by substitution) that <math>T(2; -9)</math> and <math>S(5; 18)</math> lie on the curve: award max 2/7 marks</p> </div>	$\checkmark f'(x) = -6x^2 + 2ax + b$ $\checkmark \checkmark -6(x-5)(x-2)$ $\checkmark b = -60$ $\checkmark 2a = 42$ $\checkmark \text{subs } (5; 18) \text{ or } (2; -9)$ $\checkmark c = 43$ <p style="text-align: right;">(7)</p>
<p><b>OR</b></p>	$a = 21; b = -60; c = 43$	<div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> A candidate who substitutes the values of <math>a</math>, <math>b</math> and <math>c</math> into the function i.e. gets <math>f(x) = -2x^3 - 21x^2 - 60x + 43</math> and then shows by substitution that <math>T(2; -9)</math> and <math>S(5; 18)</math> are on the curve <b>and</b> works out the derivative i.e. gets <math>f'(x) = -6x^2 - 42x - 60</math> <b>and</b> shows (by substitution into the derivative) that the turning points are at <math>x = 2</math> and <math>x = 5</math> (assuming what s/he sets out to prove and proving what is given): <b>award max 4/7 marks</b> as follows:  <math>\checkmark x = 2</math> from <math>f'(x) = 0</math> OR subs <math>x = 2</math> into the derivative and gets 0  <math>\checkmark x = 5</math> from <math>f'(x) = 0</math> OR subs <math>x = 5</math> into the derivative and gets 0  <math>\checkmark</math> substitution of <math>x = 2</math> in <math>f</math> and gets <math>-9</math>  <math>\checkmark</math> substitution of <math>x = 5</math> in <math>f</math> and gets 18</p> </div>	
	$f'(x) = -6x^2 + 2ax + b$ $f'(2) = -6(2)^2 + 2a(2) + b$ $0 = -24 + 4a + b$ $b = 24 - 4a$ $f'(5) = -6(5)^2 + 2a(5) + b$ $0 = -150 + 10a + b$ $0 = -150 + 10a + (24 - 4a)$ $0 = -126 + 6a$ $6a = 126$ $a = 21$ $b = -60$ $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$ $a = 21; b = -60; c = 43$	<div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If derivative equal to zero is not written: penalize once only</p> </div>	$\checkmark f'(x) = -6x^2 + 2ax + b$ $\checkmark f'(2) = 0$ $\checkmark f'(5) = 0$ $\checkmark 6a = 126$ $\checkmark b = -60$ $\checkmark \text{subs } (5; 18) \text{ or } (2; -9)$ $\checkmark c = 43$ <p style="text-align: right;">(7)</p>

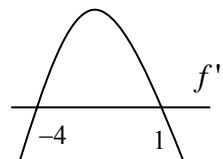
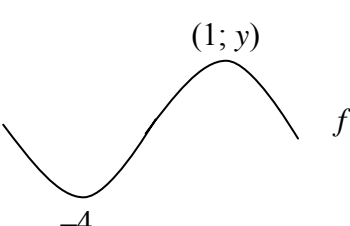
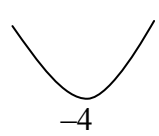
	<p><b>OR</b></p> $f(2) = -9 \text{ i.e. } -16 + 4a + 2b + c = -9$ $4a + 2b + c = 7$ $f(5) = 18 \text{ i.e. } -250 + 25a + 5b + c = 18$ $25a + 5b + c = 268$ $21a + 3b = 261$ $f'(x) = -6x^2 + 2ax + b \text{ and } f'(2) = 0 \quad \text{OR} \quad f'(5) = 0$ $4a + b = 24 \qquad 10a + b = 150$ $12a + 3b = 72 \qquad 30a + 3b = 450$ $9a = 189 \qquad 9a = 189$ $a = \frac{189}{9} \qquad \text{OR} \qquad a = \frac{189}{9}$ $a = 21 \qquad a = 21$ $12(21) + 3b = 72$ $3b = -180$ $b = -60$ $4a + 2b + c = 7 \qquad 25a + 5b + c = 268$ $4(21) + 2(-60) + c = 7 \quad \text{OR} \quad 25(21) + 5(-60) + c = 268$ $c = 43 \qquad c = 43$	$\checkmark -16 + 4a + 2b + c = -9$ $\text{and } -250 + 25a + 5b + c = 18$ $\checkmark f'(x) = -6x^2 + 2ax + b$ $\checkmark f'(2) = 0 \text{ or } f'(5) = 0$ $\checkmark 9a = 189$ $\checkmark b = -60$ $\checkmark \text{subs } (5 ; 18) \text{ or } (2 ; -9)$ $\checkmark c = 43$ <p style="text-align: right;">(7)</p>
<p>9.2</p>	$f'(x) = -6x^2 + 42x - 60$ $m_{\text{tan}} = -6(1)^2 + 42(1) - 60$ $= -24$ $f(1) = -2(1)^3 + 21(1)^2 - 60(1) + 43$ $= 2$ <p>Point of contact is (1 ; 2)</p> $y - 2 = -24(x - 1)$ $y = -24x + 26$ <p style="text-align: center;"><b>OR</b></p> $y = -24x + c$ $2 = -24(1) + c$ $c = 26$ $y = -24x + 26$	$\checkmark f'(x) = -6x^2 + 42x - 60$ $\checkmark \text{subs } f'(1)$ $\checkmark m_{\text{tan}} = -24$ $\checkmark f(1) = 2$ $\checkmark y - 2 = -24(x - 1)$ $\text{OR } y = -24x + 26$ <p style="text-align: right;">(5)</p>
<p>9.3</p>	$f'(x) = -6x^2 + 42x - 60$ $f''(x) = -12x + 42$ $0 = -12x + 42$ $x = \frac{7}{2}$ <p><b>OR</b></p>	$\checkmark f''(x) = -12x + 42$ $\checkmark x = \frac{7}{2}$ $\checkmark x = \frac{2+5}{2}$ <p style="text-align: right;">(2)</p>



$x = \frac{2+5}{2}$ $x = \frac{7}{2}$ <p><b>OR</b></p> $x = \frac{-21}{3(-2)}$ $= \frac{7}{2}$	$\checkmark x = \frac{7}{2}$ <p style="text-align: right;">(2)</p> $\checkmark x = \frac{-21}{3(-2)}$ $\checkmark x = \frac{7}{2}$ <p style="text-align: right;">(2)</p> <p style="text-align: right;"><b>[14]</b></p>
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**QUESTION 10**



10.1	<p><i>x</i>-value of turning point:</p> $x = \frac{-4+1}{2}$ $= -\frac{3}{2}$ $\therefore x > -\frac{3}{2} \quad \text{OR} \quad \therefore x \in \left(-\frac{3}{2}; \infty\right)$	$\checkmark x > -\frac{3}{2} \quad \text{OR} \quad \left(-\frac{3}{2}; \infty\right)$ <p style="text-align: right;">(1)</p>
10.2	<p><i>f</i> has a local minimum at <math>x = -4</math> because:</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p><b>OR</b></p> <p><math>f'(x) &lt; 0</math> for <math>x &lt; -4</math>, so <math>f</math> is decreasing for <math>x &lt; -4</math>.  <math>f'(x) &gt; 0</math> for <math>-4 &lt; x &lt; 1</math>, so <math>f</math> is increasing for <math>-4 &lt; x &lt; 1</math>.</p> <p>i.e.  <math>\therefore f</math> has a local minimum at <math>x = -4</math></p> <p><b>OR</b></p>	$\checkmark x = -4$ $\checkmark \checkmark \text{ graph}$ <p style="text-align: right;">(3)</p>  $\checkmark x = -4$ $\checkmark f'(x) < 0 \text{ for } x < -4$ $\checkmark f'(x) > 0 \text{ for } -4 < x < 1$ <p style="text-align: right;">(3)</p>

	<p><b>OR</b> Gradient of <math>f</math> changes from negative to positive at <math>x = -4</math></p> <p><b>OR</b> <math>f'(-4) = 0</math> <math>f''(-4) &gt; 0</math> so graph is concave up at <math>x = -4</math>, so <math>f</math> has a local minimum at <math>x = -4</math>.</p>	<p>✓ <math>x = -4</math> ✓ gradient negative for <math>x &lt; -4</math> ✓ gradient positive for <math>-4 &lt; x &lt; 1</math> (3)</p> <p>✓ <math>f'(-4) = 0</math> ✓ <math>f''(-4) &gt; 0</math> ✓ <math>x = -4</math> (3) <b>[4]</b></p>
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**QUESTION 11**

11.1	$V(0) = 100 - 4(0)$ $= 100$ litres	<p>✓ answer (1)</p>
11.2	<p>Rate in – rate out  <math>= 5 - k</math> l / min</p> <p><math>V'(t) = -4</math> l / min</p>	<p>✓ <math>5 - k</math> ✓ <math>-4</math> ✓ units stated once (3)</p>
11.3	<p><math>5 - k = -4</math>  <math>k = 9</math> l / min</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p><b>Note:</b> Answer only: award 2/2 marks</p> </div> <p><b>OR</b></p> <p>Volume at any time <math>t =</math> initial volume + incoming total – outgoing total  <math>100 + 5t - kt = 100 - 4t</math>  <math>5t - kt = -4t</math>  <math>9t - kt = 0</math>  <math>t(9 - k) = 0</math></p> <p>At 1 minute from start, <math>t = 1</math>, <math>9 - k = 0</math>,  so <math>k = 9</math></p> <p><b>OR</b></p> <p>Since <math>\frac{dV}{dt} = -4</math>, the volume of water in the tank is decreasing by 4 litres every minute. So <math>k</math> is greater than 5 by 4, that is, <math>k = 9</math>.</p>	<p>✓ <math>5 - k = -4</math> ✓ <math>k = 9</math> (2)</p> <p>✓ <math>100 + 5t - kt = 100 - 4t</math> ✓ <math>k = 9</math> (2)</p> <p>✓✓ <math>k = 9</math> (2) <b>[6]</b></p>

**QUESTION 12**

**Note:** If the wrong inequality  $50x + 25y \leq 500$  is used, candidate wrongly says that there are more learners than available seats. Maximum of 10 marks.

12.1	$x, y \in \mathbf{N}$ $x + y \leq 15$ $50x + 25y \geq 500$ $y \leq 8$ <b>OR</b> $y \leq -x + 15$ $y \geq -2x + 20$ $y \leq 8$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin-bottom: 10px;"> <b>Note:</b> If candidate gives <math>50x + 25y = 500</math>: max 5/6 marks                 </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-bottom: 10px;"> <b>Note:</b> for the inequality's marks to be awarded, the LHS and the RHS must be correct                 </div> ✓✓ $x + y \leq 15$ ✓✓ $y \leq 8$ ✓✓ $50x + 25y \geq 500$
12.2		✓ $x + y \leq 15$ ✓ $50x + 25y \geq 500$ ✓ $y \leq 8$ ✓ feasible region
12.3	$C = 600x + 300y$	✓ answer (1)
12.4.1	(6 ; 8) ; (7 ; 6) ; (8 ; 4) ; (9 ; 2) and (10 ; 0) <b>NOTE:</b> The gradient of the search line is $m = -\frac{2}{1}$	3 marks for all correct solutions 2 marks if only 3 or 4 correct solutions 1 mark if only 1 or 2 correct solutions
12.4.2	$C = 6(600) + 8(300) = \text{R}6\,000$ or $C = 7(600) + 6(300) = \text{R}6\,000$ or $C = 8(600) + 4(300) = \text{R}6\,000$ or $C = 9(600) + 2(300) = \text{R}6\,000$ or $C = 10(600) + 0(300) = \text{R}6\,000$	✓ subs ✓ answer (2)
12.5	8 red ; 4 blue	✓ answer (1) <b>[17]</b>

**TOTAL: 150**

**QUESTION 12.2**

