GRADE 12 EXAMINATION
NOVEMBER 2012

ADVANCED PROGRAMME MATHEMATICS

MARKING GUIDELINES

Time: 3 hours 300 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

RTP: $9^n - 8n - 1 = 8p \quad \forall n \in \mathbb{N}; n \geq 2$

Step 1: For $n = 2$ true because $9^2 - 16 - 1 = 64 = 8^2$

Step 2: Assume true for $n = k$, i.e. $9^k - 8k - 1 = 8q$ where $q \in \mathbb{N}$

Step 3: for $n = k + 1$

$$\text{LHS} = 9^{(k+1)} - 8(k+1) - 1$$
$$= 9.9^k - 8k - 8 - 1$$
$$= 9^k - 8k - 1 + 8.9^k - 8$$
$$= 8q + 8\left(9^k - 1\right)$$
$$= 8\left(q + q^k - 1\right)$$

$\therefore$ Statement true $\forall n \in \mathbb{N}; n \geq 2$ by the principle of MI

[14]

QUESTION 2

2.1 $\quad logx = log2 + \log\left(x+5\right) - \log\left(x-1\right)$

$$\therefore logx = \log\left(\frac{2(x+5)}{x-1}\right)$$
$$\therefore x\left(x-1\right) = 2x + 10$$
$$\therefore x^2 - 3x - 10 = 0$$
$$\therefore (x-5)(x+2) = 0$$
$$x = 5 \ but\ x \neq -2$$

(6)

2.2 (a) $e^{(x)}$ is increasing $\therefore f\left(x\right) = \dfrac{1}{1+e^x}$ is decreasing (3)

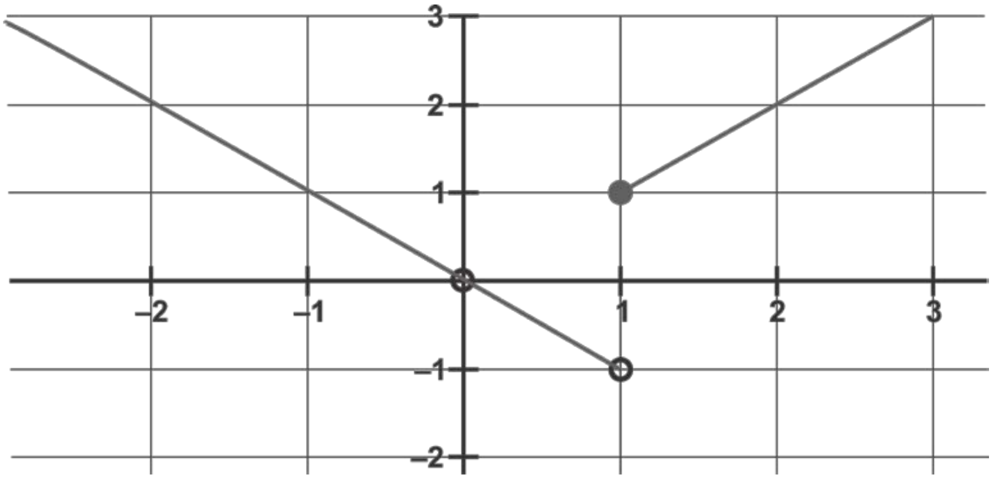
(b) The range of $f(x)$ is (0; 1) (4)

(c) The inverse function:

$$x = \frac{1}{1+e^y}$$
$$\therefore 1 + e^y = \frac{1}{x}$$
$$\therefore e^y = \frac{1}{x} - 1$$
$$\therefore y = \ln(\frac{1}{x} - 1)$$
$$\therefore f^{-1}\left(x\right) = \ln(\frac{1}{x} - 1) \text{ or } \ln\left(\frac{1-x}{x}\right)$$

(4)

2.3 (a)



(4)

(b) Domain of $g(f(x))$ is $(-\infty;0)\cup[1;\infty)$ (4)

[25]

QUESTION 3

3.1 $(2+i)(a+bi+3i)=8i+6$

$\therefore a+i(b+3)=\frac{8i+6}{2+i}\times\frac{2-i}{2-i}$

$=\frac{16i-8i^2+12-6i}{4-i^2}$

$=\frac{20+10i}{5}$

$a+i(b+3)=4+2i$

$\therefore a=4$ and $b=-1$

OR

$(2a-b-3)+(2b+6+a)i=6+8i$

$\therefore 2a-b-3=6$ and $2b+6+a=8$

$\therefore 2a-b=9$, $a+2b=2$ } solve simultaneously

$\therefore -5b=5$ and $a=4$

$b=-1$

(7)

3.2 $x^3-5x^2+kx-13=0$ and $x=2+3i$

$(x-2)^2-(3i)^2=x^2-4x+4+9=x^2-4x+13$

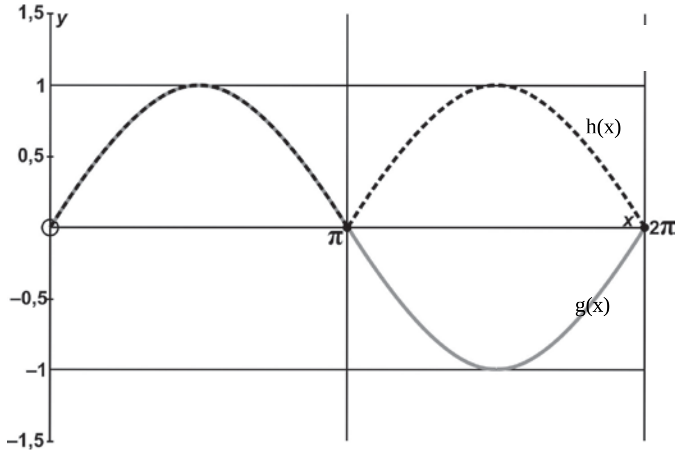
Thus $(x^2-4x+13)(x-1)=0$ $x^3-5x^2+17x-13=0$

Thus $k=17$ (8)

[15]

QUESTION 4

4.1 (a)



(-1)For no labels of graphs

(8)

(b) $|\sin x| \geq 0 \; \forall x \in \mathbb{R}$ but $-1 \leq \sin|x| \leq 1$ since there exist values of $x \geq 0$ where $\sin x < 0$ (2)

4.2 (a) (i)

$$f(x) = \begin{cases} px & \text{if} \quad x < \pi \\ -2\cos x & \text{if} \quad \pi \leq x \leq \frac{3\pi}{2} \\ \frac{1}{\pi}\left(x - \frac{5\pi}{2}\right)^2 - 2 & \text{if} \quad x > \frac{3\pi}{2} \end{cases}$$

For f to be continuous at $x = \pi$ we need $p\pi = -2\cos\pi$

Thus $p\pi = 2$

$\therefore p = \frac{2}{\pi}$ (4)

(ii) $f\left(\frac{3\pi}{2}\right) = -2\cos\frac{3\pi}{2} = 0$

But

$$\lim_{x \downarrow \frac{3\pi}{2}} f(x) = \frac{1}{\pi}\left(\frac{3\pi}{2} - \frac{5\pi}{2}\right)^2 - 2 = \pi - 2$$

$\therefore f$ not continuous at $x = \frac{3\pi}{2}$ (5)

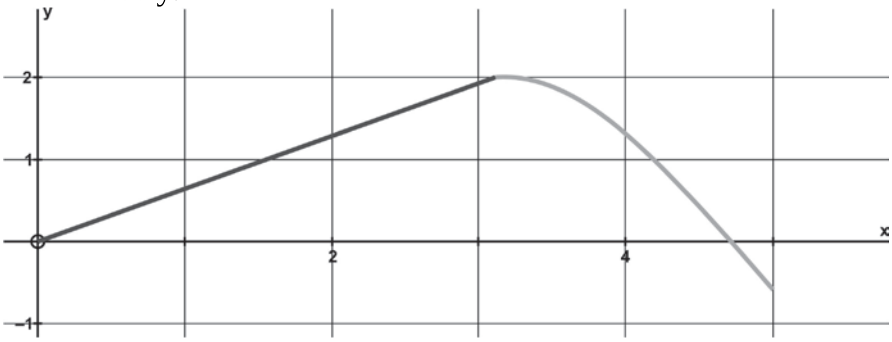
(b) (i) the gradient of f as $x \to \pi^-$ is $\frac{2}{\pi}$ which is constant (straight line)

the gradient of f as $x \to \pi^+$ is $-2(-sin\pi) = 0$

thus the gradient from left $\neq$ gradient from right

not differentiable

Alternatively:



Clearly see the gradient from right $\neq$ gradient from left at π

$\therefore$ not differentiable (5)

(ii) $\lim_{x \to \left(\frac{3\pi}{2}\right)^-} f'(x) = -2\left(-\sin\frac{3\pi}{2}\right) = -2$

$\lim_{x \to \left(\frac{3\pi}{2}\right)^+} f'(x) = \frac{2}{\pi}\left(\frac{3\pi}{2} - \frac{5\pi}{2}\right) = -2$ (6)

(iii) No, as although they have the same gradient the function is discontinuous at $x = \frac{3\pi}{2}$. (2)

[32]

QUESTION 5

5.1 (a) You can't take the log of a negative number. (2)

(b) $\ln x = 0$

$\therefore x = e^0 = 1$ (2)

(c) $\dfrac{d}{dx}\left(\dfrac{x}{\ln x}\right) = \dfrac{1.\ln x - \frac{1}{x}.x}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$

Thus turning point when $\ln x - 1 = 0$ which is when $x = e$ which gives $y = \dfrac{e}{\ln e} = e$

Thus coordinates of TP are $(e;e)$ (8)

5.2 (a) Oblique asymptote: $2x^2 - 2x + 5 = (x+1)(2x-4) + 9$

Thus: $\dfrac{2x^2 - 2x + 5}{x+1} = 2x - 4 + \dfrac{9}{x+1}$ so the oblique asymptote is $y = 2x - 4$ (6)

(b) No. For them to touch $2x - 4 + \dfrac{9}{x+1} = 2x - 4$

$\therefore \dfrac{9}{x+1} = 0$ which is not possible. (3)

[21]

QUESTION 6

Width $= \dfrac{3}{n}$ and $x_i = \dfrac{3i}{n}$

Approximate area $= \sum_{i=1}^{n}\left(3\left(\dfrac{3i}{n}\right)^2 + 1\right).\dfrac{3}{n}$

$= \sum_{i=1}^{n}\left(\dfrac{81i^2}{n^3} + \dfrac{3}{n}\right)$

$= \dfrac{81}{n^3}\sum_{i=1}^{n} i^2 + \dfrac{3}{n}\sum_{i=1}^{n} 1$

$= \dfrac{81}{n^3}\left(\dfrac{n^3}{3} + \dfrac{n^2}{2} + \dfrac{n}{6}\right) + \dfrac{3}{n}.n$

$= 27 + \dfrac{81}{2n} + \dfrac{81}{6n^2} + 3$

$\therefore area = \lim_{n\to\infty}\left(27 + \dfrac{81}{2n} + \dfrac{81}{6n^2} + 3\right) = 30$

[14]

QUESTION 7

7.1 $r^2 = (r-6)^2 + 15^2$

$\therefore 12r = 261$

$\therefore r = 21,75$ cm (6)

7.2 $sin\frac{\theta}{2} = \frac{15}{21,75}$

$\therefore \frac{\theta}{2} = 0,761$

$\therefore \theta = 1,522$

So the arclength ACB $= 21,75 \times 1,522 = 33,1041$ cm (5)

sin; cos or tan or cosine rule could be used.

7.3 Area of segment $= 0.5 \times (21,75)^2 \times 1,522 - 0.5 \times (21,75)^2\, sin 1,522 = 123,7566 \text{ cm}^2$ (5)

[16]

QUESTION 8

8.1 LHS: $2^3 - 3.2.(-1)^2 + (-1)^3 = 8 - 6 - 1 = 1 = RHS$ (2)

8.2 $3x^2 - 3\left(y^2 + x.2y.\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx}\left(-6xy + 3y^2\right) = -3x^2 + 3y^2$

$\therefore \frac{dy}{dx} = \frac{x^2 - y^2}{2xy - y^2}$ (8)

8.3 at $(2;-1)$ $\frac{dy}{dx} = \frac{4-(-1)^2}{2(2)(-1)-(-1)^2} = \frac{-3}{5}$

$\therefore y = -\frac{3}{5}x + c$

$-1 = -\frac{3}{5}.2 + c$

$\therefore c = \frac{1}{5}$

$\therefore y = -\frac{3}{5}x + \frac{1}{5}$ (5)

[15]

QUESTION 9

9.1 $\int 2x^3 - \sec^2\left(\frac{x}{2}\right) - \frac{1}{\sqrt[3]{x}}\,dx = \frac{1}{2}x^4 - 2\tan\left(\frac{x}{2}\right) - \frac{3}{2}x^{\frac{2}{3}} + c$ (6)

9.2 $\int(\cos^2 3x)(\sin 3x)\,dx$

Let $\cos 3x = u$

Then: $\frac{du}{dx} = -\sin 3x.3$ or $-\frac{du}{3} = \sin 3x dx$

Thus: $-\frac{1}{3}\int u^2 du = -\frac{1}{9}u^3 + c = -\frac{1}{9}\cos^3 3x + c$ (7)

9.3 $\int cos 4\theta.\sin 5\theta d\theta = \frac{1}{2}\int sin 9\theta + \sin(\theta)\,d\theta$

$= -\frac{1}{2}.\frac{\cos 9\theta}{9} - \frac{1}{2}.\cos\theta + c$ (6)

9.4 $\int y\sqrt{y+3}\,dy$

Let $u = y+3$ $then \frac{du}{dy} = 1$ or $du = dy$

Thus $\int y\sqrt{y+3}\,dy = \int(u-3).u^{\frac{1}{2}}du = \int u^{\frac{3}{2}} - 3u^{\frac{1}{2}}du$

$= \frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + c = \frac{2}{5}(y+3)^{\frac{5}{2}} - 2(y+3)^{\frac{3}{2}} + c$

OR

Let $f(y) = y \quad \therefore f'(y) = 1$

$g'(y) = \sqrt{y+3} \qquad \therefore g(y) = \frac{2}{3}(y+3)^{\frac{3}{2}}$

$\therefore \int fg'dy = fg - \int f'gdy$

$= \frac{2y}{3}(y+3)^{\frac{3}{2}} - \frac{2}{3}\int(y+3)^{\frac{3}{2}}dy$

$= \frac{2y}{3}(y+3)^{\frac{3}{2}} - \frac{2.2}{3.5}(y+3)^{\frac{5}{2}} + c$

$= \frac{2y}{3}(y+3)^{\frac{3}{2}} - \frac{4}{15}(y+3)^{\frac{5}{2}} + c$ (9)

(-1 for no “c”)

[28]

QUESTION 10

10.1 $Vol = \pi \int_1^a \left(\frac{1}{x}\right)^2 dx = \pi \left[-\frac{1}{x}\Big|_1^a \right] = -\frac{\pi}{a} + \pi\, units^3$ (8)

10.2 $Vol\left(a \to \infty\right) = \pi \quad units^3$ (2)

[10]

QUESTION 11

$S = \frac{8k}{x^2} + \frac{k}{\left(15-x\right)^2}$

Thus $\frac{dS}{dx} = -2.8kx^{-3} - 2.k\left(15-x\right)^{-3}.\left(-1\right) = 0$

$\therefore 8\left(15-x\right)^3 - x^3 = 0$

$\therefore \left(2\left(15-x\right)-x\right)\left(4\left(15-x\right)^2 + 2x\left(15-x\right) + x^2\right) = 0$ or $\therefore 8\left(15-x\right)^3 = x^3$

$\therefore 30 - 3x = 0$ or $x = 10$ km from A $\therefore 2\left(15-x\right) = x$

$\therefore x = 10$

(**Not necessary for the way question was stated**:

Check for min: $\frac{d^2S}{dx^2} = 48kx^{-4} + 6k\left(15-x\right)^{-4} > 0$ for $x = 10$ thus minimum.)

[10]

Total for Module 1: 200 marks

MODULE 2 STATISTICS

QUESTION 1

1.1 $X \sim bin(20\ ;7)$

$P(x=7)=\binom{20}{7}(0,2)^7(0,8)^{13}=0,0545$ (6)

1.2 (a) $\binom{12}{4}=495$ (1)

(b) $\binom{6}{2}\binom{6}{2}=225$ (3)

(c) $\binom{6}{1}\binom{5}{1}\times 2=60$ (4)

1.3 $\dfrac{\binom{9}{3}\binom{6}{3}\binom{3}{3}}{3!}=280$ (5)

1.4 $P(\text{at least one})=1-P(\text{none})$

$=1-\dfrac{\binom{4}{0}\binom{16}{10}}{\binom{20}{10}}=0,9567$ or $=1-\dfrac{\binom{10}{0}\binom{10}{4}}{\binom{20}{4}}=0,9567$ (7)

[26]

QUESTION 2

$X \sim N(100;15^2)$

2.1 $P(120<x<129)=P\left(\dfrac{120-100}{15}<z<\dfrac{129-100}{15}\right)$

$=P(1,33<z<1,93)$

$=0,4732-0,4082$

$=0,065$

$\therefore 6,5\%$ (10)

2.2 $P(X<a)=0,022$ or $\mu=100-2,01(15)=130,15\ \mu=100-2,01(15)=130,15$

$-2,01=\dfrac{a-100}{15}$

$a=69,5$

$a\approx 70$

$\therefore$ less than 70% (6)

[16]

QUESTION 3

3.1 $\bar{x} = 57{,}8\%$

$\bar{y} = 60{,}7\%$ (2)

3.2 $b = \dfrac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2} = \dfrac{10(39070) - (578)(607)}{10(38378) - (578)^2}$

or $= \dfrac{39070 - 10(57{,}8)(60{,}7)}{38378 - 10(57{,}8)^2}$

$= 0{,}802$

$sub(57{,}8\ ; 60{,}7)$

$60{,}7 = a + 0{,}802(57{,}8)$

$a = 14{,}344$

$\therefore\ y = 14{,}344 + 0{,}802x$ (6)

3.3 $r = \dfrac{10(39070) - (578)(607)}{\sqrt{\left[10(38378) - (578)^2\right]\left[10(41221) - (607)^2\right]}} = 0{,}8546$ (3)

3.4 Allie, $y = 14{,}344 + 0{,}802(85)$

$= 82{,}5$ (3)

3.5 This is a reliable estimate as a strong correlation exists. (2)

[16]

QUESTION 4

4.1 (a) 2010: $\bar{x} = 62 \quad \sigma = 49{,}452$ 2011: $\bar{y} = 82{,}4 \quad \sigma = 96{,}8055$ (4)

(b) $H_0 : \mu_y - \mu_x = 10$

$H_1 : \mu_y - \mu_x > 10$

Rejection Region:
Reject H_0 if $Z > 1{,}28$

Test statistic: $z = \dfrac{(82{,}4 - 62) - (10)}{\sqrt{\dfrac{(96{,}8055)^2}{5} + \dfrac{(49{,}452)^2}{5}}} = 0{,}2139$

Conclusion: We fail to reject H_0 at the 10% l.o.s and suggest insufficient evidence to support the claim. (10)

4.2 $1{,}88\sqrt{\dfrac{(0{,}25)(0{,}75)}{n}} < 0{,}04$

$\sqrt{\dfrac{(0{,}25)(0{,}75)}{n}} < \dfrac{1}{47}$

$\dfrac{3}{16n} < \dfrac{1}{2209}$

$414{,}1 < n \quad n = 415$ (6)

[20]

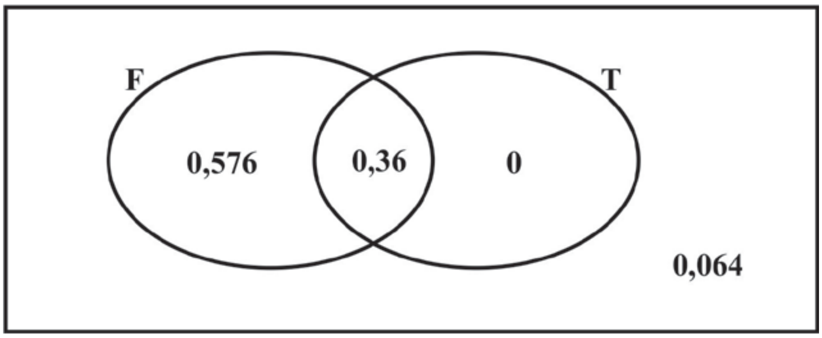
QUESTION 5

5.1 $P(F|T)=1$ $P(F|T')=0,9$ $P(T)=0,36$

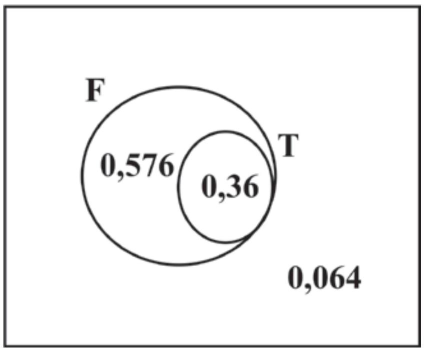
$P(F|T)=\dfrac{P(F\cap T)}{P(T)}$ $P(F|T')=\dfrac{P(F\cap T')}{P(T')}$

$0,36=P(F\cap T)$ $(0,9)(0,64)=P(F\cap T')$

$0,576=P(F\cap T')$



OR



(10)

5.2 (a) $P(F)=0,576+0,36=0,936$ (2)

(b) $P(F'\cap T')=1-0,936=0,064$ (1)

[13]

QUESTION 6

6.1 Alicia: $z=\dfrac{98-56}{18}=2,33$

Rae: $z=\dfrac{40-56}{18}=-0,89$ (2)

6.2 Alicia: $2,33=\dfrac{x-60}{12}$ or $60+\dfrac{7}{3}(12)\,60+\dfrac{7}{3}(12)$

$x=87,99$

Rae: $-0,89=\dfrac{x-60}{12}$ or $60-\dfrac{8}{9}(12)\,60-\dfrac{8}{9}(12)$

$x=49,32$ (6)

6.3 Standard deviation was lowered to reduce the wide spread of values allowing more consistency. (1)

[9]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

1.1 $5\ 600\left(1+\frac{0,0684}{4}\right)^{20} = 7\ 860,70$

$7\ 860,70 - 5\ 600 =$ **2 260,70** (4)

1.2 $10\ 033,38 = x\frac{\left[\left(1+\frac{0,0684}{4}\right)^{20} - 1\right]}{\frac{0,0684}{4}}$ ***x* = 425** (6)

1.3 $\left(1+\frac{0,0684}{12}\right)^{12} = \left(1+\frac{i}{4}\right)^{4}$ ***i* = 0,0688**

$Fv = \frac{420\left[\left(1+\frac{0,0688}{4}\right)^{20} - 1\right]}{\frac{0,0688}{4}}$ **= 9 924,93** (10)

Using $i = 0,069$: $x = 9\ 930,07$
Using $i = 0,06879$: $x = 9\ 924,91$
Using $i = 0,07$: $x = 9\ 954,68$

[20]

QUESTION 2

$(1 - 0,12)^2\ (1 - 0,105)\ (1 - i)^3 = (1 - 0,1)^6$
i = 0,0847 … = 8,47%

OR

$T_6 = A(1 - 0,1)^6 = 0,531\ 441A$
$T_3 = A(1 - 0,12)^2\ (1 - 0,105) = 0,693\ 088A$

$0,531\ 441A = 0,693\ 088A\ (1 - x)^3$ **x = 8,47%** (8)

[8]

QUESTION 3

3.1 $$OB = 850\ 000\left(1+\frac{0,0856}{12}\right)^{80} - \frac{7\ 400\left[\left(1+\frac{0,0856}{12}\right)^{80}-1\right]}{\frac{0,0856}{12}}$$

$$= 1\ 500\ 985,64 - 794\ 495,95 = \mathbf{706\ 489,70}$$

OR

$$OB = \left[850\ 000 - \frac{7\ 400\left[1-\left(1+\frac{0,0856}{12}\right)^{-80}\right]}{\frac{0,0856}{12}}\right]\left(1+\frac{0,0856}{12}\right)^{80} = 706\ 489,70$$ (8)

3.2 $$12\ 000\left(1+\frac{0,0856}{12}\right)^{63} + 12\ 000\left(1+\frac{0,0856}{12}\right)^{15} = 32\ 128,62$$

706 489,70 – 32 128,62 **= 674 361,08**

OR

$$T_{17} = 850\ 000\left(1+\frac{0,0856}{12}\right)^{17} - \frac{7\ 400\left[\left(1+\frac{0,0856}{12}\right)^{17}-1\right]}{\frac{0,0856}{12}} - 12\ 000$$

$$= 813\ 932,50$$

$$T_{65} = 813\ 932,50\left(1+\frac{0,0856}{12}\right)^{48} - \frac{7\ 400\left[\left(1+\frac{0,0856}{12}\right)^{48}-1\right]}{\frac{0,0856}{12}} - 12\ 000$$

$$= 711\ 074,60$$

$$T_{80} = 711\ 074,60\left(1+\frac{0,0856}{12}\right)^{15} - \frac{7\ 400\left[\left(1+\frac{0,0856}{12}\right)^{15}-1\right]}{\frac{0,0856}{12}} = \mathbf{674\ 361,08}$$ (8)

3.3 $675\ 000\left(1+\frac{0,0856}{12}\right)^{159}-\frac{7\ 100\left[\left(1+\frac{0,0856}{12}\right)^{159}-1\right]}{\frac{0,0856}{12}}$

= 2 089 931,38 – 2 086 399,34 = 3 532,04

$3\ 532,04\left(1+\frac{0,0856}{12}\right)$ **= 3 557,24**

OR

$675\ 000\left(1+\frac{0,0856}{12}\right)^{160}-\frac{7\ 100\left(1+\frac{0,0856}{12}\right)\left[\left(1+\frac{0,0856}{12}\right)^{159}-1\right]}{\frac{0,0856}{12}}$

= 2 104 839,56 – 2 101 282,32 **= 3 557,24**

OR

$675\ 000-\frac{7\ 100\left[1-\left(1+\frac{0,0856}{12}\right)^{-159}\right]}{\frac{0,0856}{12}}$

= 675 000 – 673 859,23 = 1 140,77

$1\ 140,77\left(1+\frac{0,0856}{12}\right)^{160}$ **= 3 557,24**

OR

$675\ 000=\frac{7\ 100\left[1-\left(1+\frac{0,0856}{12}\right)^{-159}\right]}{\frac{0,0856}{12}}+y\left(1+\frac{0,0856}{12}\right)^{-160}$

y **= 3 557,24** (10)

[26]

QUESTION 4

4.1 $3 \times 4 \times 0,6 \times 0,8 - 1/7 =$ **5,617** (6)

4.2 $\boldsymbol{H_{n+1} = 6,6H_n}$ (2)

4.3 $H_0 = 25$ $H_1 = 165$ $\boldsymbol{H_2 = 1\ 089}$ (2)

4.4 $(1 + 5,6) = \left(1+\frac{r}{52}\right)^{52}$ $r = 1,922$ (annual rate, compounded weekly)

$H_1 = 25(1 + 1,922/52) = 25,925$ $\therefore$ **0,925 ≈ 1 hog per week**

OR

$(1 + 5,6) = (1+r)^{52}$ $r = 0,03695$ (weekly rate)

$H_1 = 25(1 + 0,037) = 25,925$ $\therefore$ **0,925 ≈ 1 hog per week** (6)

[16]

QUESTION 5

5.1 (a) antelope: 1 200 – 1 400 leopards: 46 – 48 (2)

(b) 40 < leopards < 53 (2)

(c) antelope = 750 – 800 (2)

(d) 420 = b × 4 500 × 30 ***b = 0,00311*** (5)

(e) *f* × 0,003 × 30 × 4 500 = 4 ***f = 0,00988*** (5)

5.2 (a) Both populations initially increase; formerly only leopards increase. (2)

(b) Range of both populations larger in second plot. (2)

(c) Equilibrium point reached over longer period of time in second plot. (2)

[22]

QUESTION 6

6.1 $T_{n+1} = 1,333\ T_n$, $T_1 = 27$ (4)

6.2 $T_{14} = 1\ 136,495$

OR

$27\left(\frac{4}{3}\right)^{n-1} > 1\ 000$ $n = 14$

$27\left(\frac{4}{3}\right)^{13} = 1\ 136,495$ (4)

[8]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 $2M - 3N = \begin{pmatrix} 12 & -4 \\ -6 & 2 \end{pmatrix} - \begin{pmatrix} 15 & 6 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} -3 & -10 \\ 0 & -7 \end{pmatrix}$ (4)

1.2 not a square matrix. (2)

1.3 one row is a multiple of another OR determinant is zero. (2)

1.4 $\frac{1}{19} \begin{pmatrix} 3 & -2 \\ 2 & 5 \end{pmatrix}$ (4)

[12]

QUESTION 2

2.1 $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} -10 & 17 \\ -4 & 5 \end{pmatrix}$ (6)

2.2 $\tan\theta = 3$ $\theta = 71{,}565^{o}$

$\begin{pmatrix} \cos 2\ (71{,}565) & \sin 2\ (71{,}565) \\ \sin 2\ (71{,}565) & -\cos 2\ (71{,}565) \end{pmatrix} = \begin{pmatrix} -0{,}8 & 0{,}6 \\ 0{,}6 & 0{,}8 \end{pmatrix}$ (6)

2.3 $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ OR $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ AND $\begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$ (4)

2.4 $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 25 \\ -75 \end{pmatrix} = \begin{pmatrix} -45 \\ 65 \end{pmatrix}$

$25\cos\theta + 75\sin\theta = -45$ AND $25\sin\theta - 75\cos\theta = 65$

$\sin\theta = -7/25$ AND $\cos\theta = -24/25$

$\theta = 196{,}26^{o}$ (8)

[24]

QUESTION 3

3.1 $AB = \begin{pmatrix} 1 & 2 & -3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{pmatrix} \begin{pmatrix} 8 & -5 & 6 \\ -12 & -10 & -9 \\ -1 & 5 & 8 \end{pmatrix} = \begin{pmatrix} -13 & -40 & -36 \\ -48 & -5 & -36 \\ 24 & 20 & 53 \end{pmatrix}$

$BA = \begin{pmatrix} 8 & -5 & 6 \\ -12 & -10 & -9 \\ -1 & 5 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & -3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{pmatrix} = \begin{pmatrix} 35 & 0 & 0 \\ 0 & -35 & 0 \\ 0 & 0 & 35 \end{pmatrix}$ (8)

3.2 $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} = \begin{pmatrix} a^2+b^2+c^2 & ad+be+cf & ag+bh+ci \\ ad+be+cf & d^2+e^2+f^2 & dg+eh+fi \\ ag+bh+ci & dg+eh+fi & g^2+h^2+i^2 \end{pmatrix}$ (10)

[18]

QUESTION 4

4.1 Every vertex is used once and once only before returning to start. (2)

4.2 A **B** **J** **B** C D **E** **C** **E** F C

22 **10** **10** 21 16 6 **21** **21** 18 26 16

G B **H** **G** **F** **G** **H** J A

22 17 7 13 13 7 13 20

Doubled edges BJ + FG + GH + CE = 51 **OR** BJ + GC + GH + FE = 51
Circuit with all nine vertices and 15 + 4 edges
Total weight = 248 (original graph) + 51 (doubled edges) **= 299 m** (12)

4.3 300 = 248 (existing edges) + 35 (BH + EF) + CJ
CJ = 17 m (10)

[24]

QUESTION 5

5.1 9 edges (2)

5.2 Kruskal : shortest edges in graph are being chosen (2)

5.3 6 (2)

5.4 JE : will complete a mini-circuit (2)

5.5 DE, AB, IJ, DI **EF / JG** **HG / CI** **BI = 60** (8)

[16]

QUESTION 6

6.1 1 3 6 10

2 3 4

1 1

$\therefore$ **n = 15** (3)

6.2 Eulerian Circuit $\therefore$ needs even degrees at vertices
$\therefore n \approx$ odd amount of vertices. $n \geq 3$ (3)

[6]

Total for Module 4: 100 marks

Total: 300 marks