

α MATHEMATICS

Grade 12 Examination Paper

October 2012

Time: 3 hours

Total: 200 marks

INSTRUCTIONS AND INFORMATION

1. Number the answers as the questions are numbered.
2. Write neatly and legibly.
3. Non-programmable calculators may be used.
4. The diagrams in the question paper are not necessarily drawn to scale.
5. All necessary calculations must be shown clearly.
6. All angles are given in radians. Answers must also be given in radians where required.
7. All answers must be rounded off to two decimal places where necessary, unless specified otherwise.
8. This question paper consists of 8 pages of questions and 3 formula sheets.

QUESTION 1

- 1.1 The following formula models the number of years (t), from now in terms of the number of people (P) that stay in a town at that time:

$$t = 100 \times \ln\left(\frac{4}{3} - \frac{P}{60000}\right)$$

- (a) Determine how many people initially live in the town, when $t = 0$. (4)
- (b) As a result of migration to the cities, the town's population is decreasing. Calculate after how many years there will be no residents left in the town. (2)
- (c) Change the subject of the formula to P , hence write the formula as $P = ..$ (4)
- (d) Hence, or otherwise, determine the initial tempo at which the population decreases, this is when $t = 0$. (4)
- (e) State in your own words the meaning of the answer of 1.1(d). (2)
- 1.2 Use mathematical induction and prove that, for all natural numbers n , the following statement is true:

$$1 \times 5 + 2 \times 6 + 3 \times 7 + \dots + n(n + 4) = \frac{1}{6}n(n + 1)(2n + 13) \quad (10)$$

[26]**QUESTION 2**

- 2.1 Determine the coefficient of x^2 in the expansion of $\left(3x - \frac{4}{x}\right)^8$ (6)
- 2.2 Solve for x in the complex number system if $x^4 + 4x^3 + 3x^2 + 4x + 2 = 0$ and $-2 - \sqrt{2}$ is a root. (6)
- 2.3 Solve for $x \in \mathbb{R}$:
- (a) $|x - 2| < 4$ (3)
- (b) $|x - 2| = x^2$ (7)

[22]

QUESTION 3

3.1 The following system of equations is given:

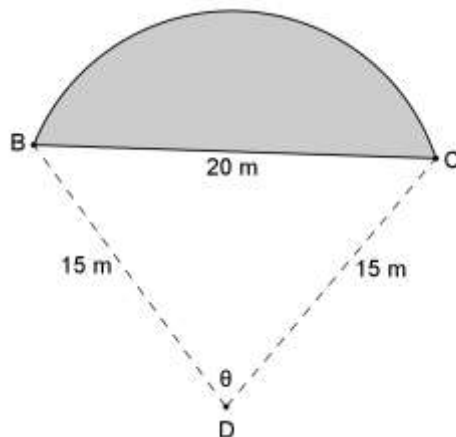
$$\begin{aligned} 3x - 2y + 9z &= 4 \\ x + 2y - 5z &= 7 \\ x + y - 2z &= -2 \end{aligned}$$

The system does not have a solution. Use Cramer's rule to show this. (6)

3.2 Given the complex number $z = cis\left(\frac{7\pi}{6}\right)$. Determine the imaginary part of $z - i$ (4)

3.3 $z = \cos\theta + i\sin\theta$. Use De Moivre's theorem and simplify $z + \frac{1}{z}$
(Hint: $\frac{1}{z} = z^{-1}$) (5)

3.4 The shaded area in the sketch represents a garden. One border of the garden consists of a straight garden path BC with length 20 m. The other border is in the form of the arc of a circle BC with centre D. $DB = DC = 15$ m.



(a) Determine the size of θ and give the answer correct to four decimal digits. (4)

Use $\theta = 1,46$ for further calculations.

(b) Calculate the circumference of the garden. (3)

(c) Determine the area of the garden. (5)

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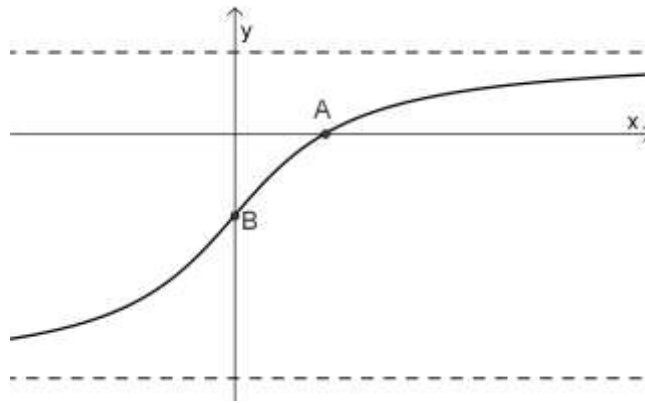
QUESTION 4

$$4.1 \quad f(x) = \begin{cases} \arcsin x & \text{if } x < 0 \\ x^2 + x & \text{if } x \geq 0 \end{cases}$$

Use algebraic methods and determine whether f is

- (a) continuous at the point $x = 0$ (4)
- (b) differentiable at the point $x = 0$ (4)

4.2 The sketch shows the graph of $f(x) = \arctan x - \pi/4$:



- (a) Give the equations of the two asymptotes. (2)
- (b) Determine the coordinates of the intercepts, A and B, of f with the axis. (4)

[14]

QUESTION 5

5.1 Determine the derivative of $f(x) = \sqrt{x+1}$ by working from first principles. (7)

5.2 Differentiate the following functions as asked:

(a) $\frac{d}{dx} [\cos(2x)]$ (2)

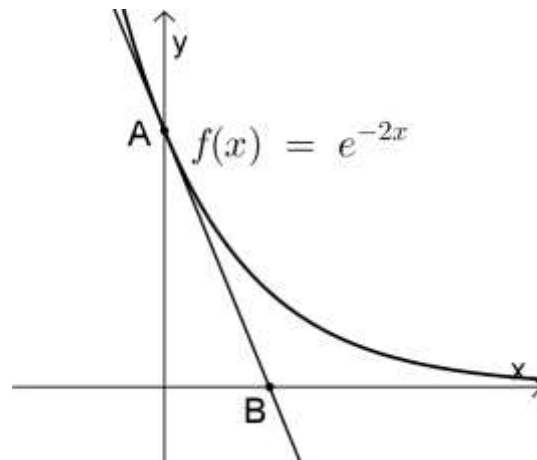
(b) $D_x \left[\arccos x \times \frac{1}{1-x^2} \right]$ (5)

(c) Determine $\frac{dy}{dx}$ if $\sin(xy) + \cot(y^2) = 10x$ (Tip: implicit differentiation.) (8)

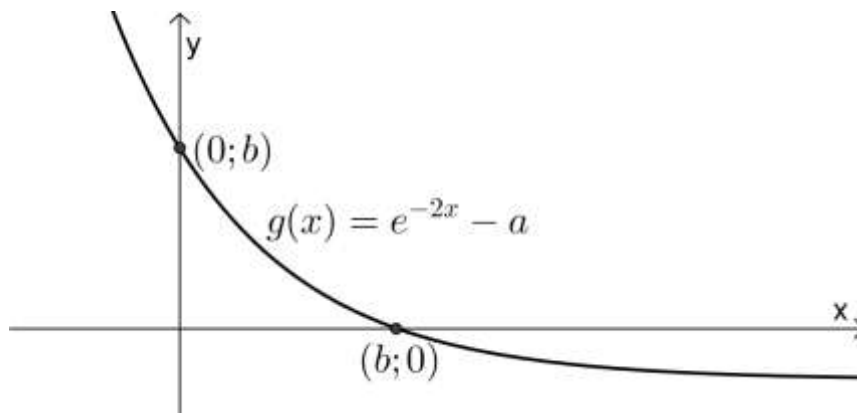
[22]

QUESTION 6

The sketch shows the function of $f(x) = e^{-2x}$. A is the intercept of f with the y -axis. The tangent to f at A intercepts the x -axis at B .



- 6.1 Calculate the x -coordinate of B . (6)
- 6.2 The graph of g results from the graph of f above which moves a units down, with $0 < a < 1$. The graph of g then intercepts the x -axis as well as the y -axis. These intercepts are equidistant from the origin, at $(0; b)$ and $(b; 0)$. The following sketch shows the graph of this function:



Determine an equation that can be used to calculate b . The equation must contain only b as unknown and not a . (4)

- 6.3 The following equation can be used to calculate b : $e^{-2b} - 1 + b = 0$.
Use Newton's method and calculate this value, correct to four decimal digits.
Use $b = 0,5$ as first approximation. (5)
- 6.4 Hence give the value of a correct to four decimal digits. (1)

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QUESTION 7

7.1 Determine the following integral: $\int \left(\sin(2x + 4) + \frac{2}{x^2} + e^{4x} \right) dx$ (5)

7.2 If $\int_0^a \sec^2(2x) dx = \frac{1}{2}$ with $a \in (0; \frac{\pi}{2})$, calculate the value(s) of a . (6)

7.3 $u = \sqrt{1 - x^2}$

(a) Determine $\frac{du}{dx}$ and simplify your answer to root form without any negative exponents. (3)

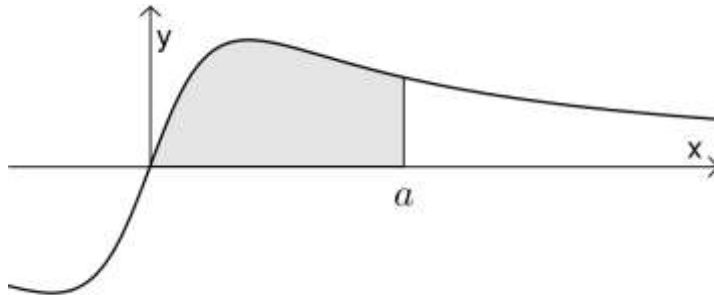
(b) Hence, or otherwise, determine $\int \arcsin x dx$.
Use integration by parts. (Hint: determine $\int 1 \cdot \arcsin x dx$ and use 7.3 (a)) (6)

[20]

QUESTION 8

8.1 Use 'n Riemann sum and calculate the value of $\int_0^1 (2x^2 + 1) dx$ (12)

8.2 The following sketch shows the graph of the function $f(x) = \frac{\arctan x}{\sqrt{1+x^2}}$:



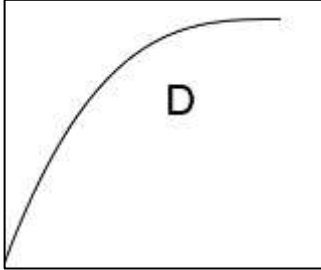
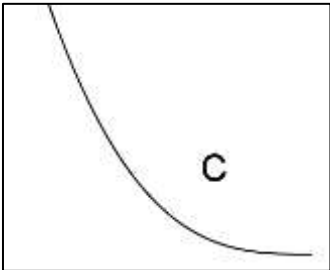
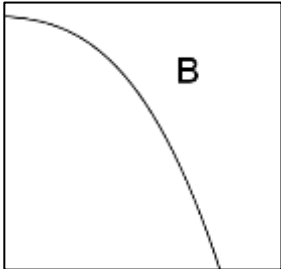
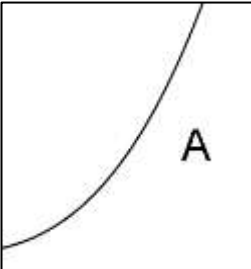
The shaded area, which is enclosed by f , the x - axis, and the line $x = a$, rotates around the x - axis. The volume of the solid of revolution obtained is equal to $\frac{\pi^4}{192}$.

Determine the value of a . (9)

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QUESTION 9

Study the following 4 sketches, numbered A,B, C and D:

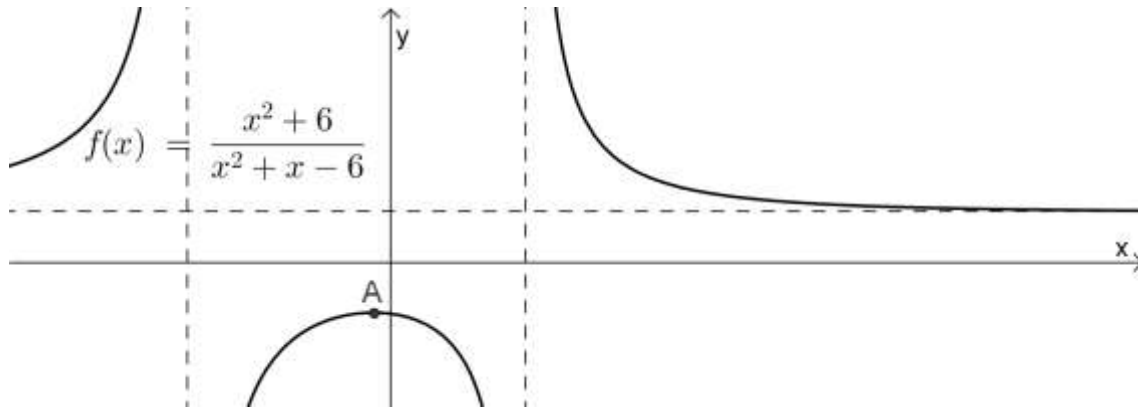


In sketch A $f'(x) > 0$ and $f''(x) > 0$. Hence state whether $f'(x)$ and $f''(x)$ are positive or negative for sketches B, C and D.

[6]

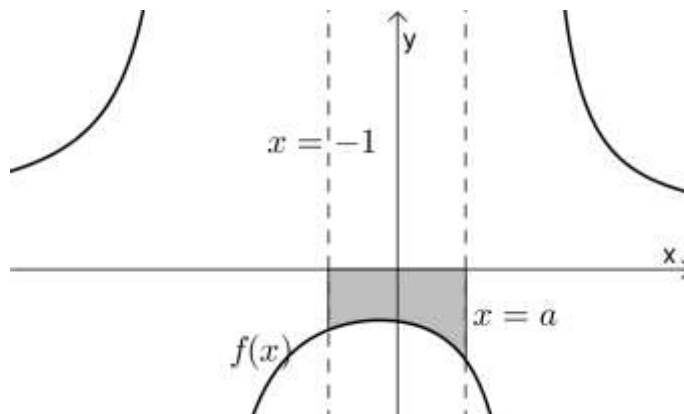
QUESTION 10

The sketch shows the graph of $f(x) = \frac{x^2+6}{x^2+x-6}$. The graph has vertical and horizontal asymptotes and two turning points of which the maximum turning point is at A.



- 10.1 Determine the equations of the vertical and horizontal asymptotes. (5)
- 10.2 Determine the coordinates of the turning point A. (6)
- 10.3 Calculate at which x -value the graph of f intercepts the horizontal asymptote. (4)
- 10.4 Given that $f(x) = \frac{x^2+6}{x^2+x-6} = 1 - \frac{x-12}{x^2+x-6}$. Decompose $f(x)$ in partial fractions. (5)

The following sketch shows a portion of the same graph of $y = f(x)$, with the lines $x = -1$ and $x = a$. The region included by these lines, the graph of f and the x -axis is shaded.



- 10.5 Assume that $f(x) = 1 + \frac{2}{x-2} - \frac{3}{x+3}$.
Write down an expression that can be used to calculate this area. (1)
- 10.6 Hence determine this area in terms of a . (It is not necessary to simplify your answer.) (5)

[26]

TOTAL: 200