NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2012

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $m_{EF} = \frac{3-(-1)}{4-0} = 1$ $\qquad$ $m_{EG} = \frac{3-1}{4-t} = \frac{2}{4-t}$

$\therefore \quad \frac{2}{4-\text{t}} = 1$

$\therefore \quad \text{t} = 2$

(4)

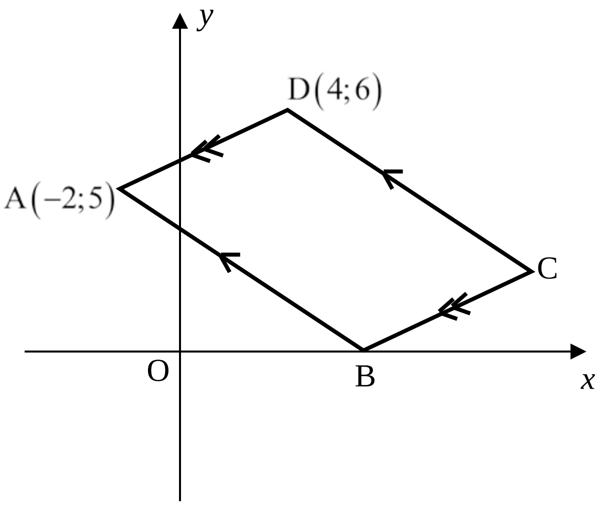
(2) $m_{EF} \times m_{FG} = -1$

$\therefore \quad 1 \times \frac{2}{t} = -1$

$\therefore \quad t = -2$

(2)

(b) (1)



$2y = -x + 16$

$\therefore \quad y = -\frac{x}{2} + 8$

$\therefore \quad m = -\frac{1}{2}$ $\qquad$ Since AB // CD

$\therefore \quad \tan\left(180 - \text{A}\hat{\text{B}}\text{O}\right) = -\frac{1}{2}$ $\qquad$ (0,5 0r -0,5 accepted)

$\therefore \quad \text{A}\hat{\text{B}}\text{O} = 26,6°$

(3)

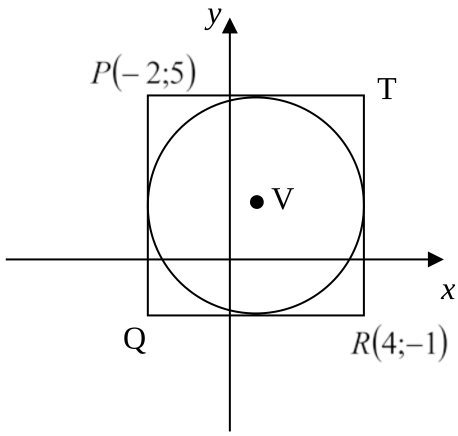
(2) $y - 5 = -\frac{1}{2}(x+2)$ $\qquad$ Choice of formula substitution

$\therefore \quad y = -\frac{1}{2}x + 4$

(3)

(3) $\text{B}(8;0)$ (2)

(c) (1) $T(4;5)$ and $Q(-2;-1)$ (2)



(2) $(1;2)$

$(x-1)^2+(y-2)^2=9$ (4)

(3)

$VS^2 = (3-1)^2+(4-2)^2$

$= 4+4$

$\therefore$ VS $= \sqrt{8}$

$\therefore$ VS $= < 3$ $\sqrt{\ }$

Therefore, the point $(3;4)$ lies inside the circle (4)

[24]

QUESTION 2

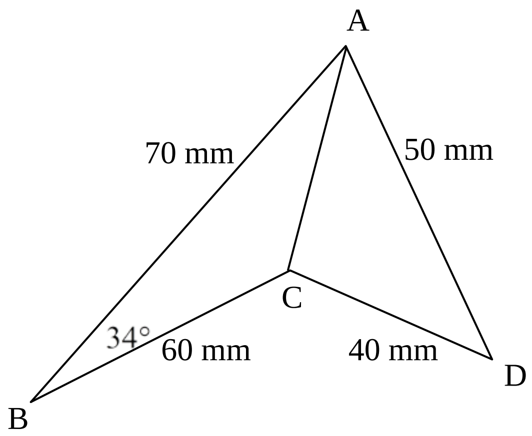
(a) (1) $AC^2 = AB^2 + BC^2 - 2.AB.BC.\cos\hat{B}$

$AC^2 = 60^2 + 70^2 - 2\times 60\times 70\times \cos 34°$

$\therefore$ $AC^2 = 1536,084...$

$\therefore$ $AC = 39,2$ (3)

(2)



$AC^2 = CD^2 + AD^2 - 2.CD.AD.\cos\hat{D}$

$\therefore$ $1536,084 = 1600 + 2500 - 4000.\cos\hat{D}$

$\therefore$ $\cos\hat{D} = 0,64098$

$\therefore$ $\hat{D} = 50,1°$ (3)

(b) $2\sin(90° - \theta) = \frac{1}{8}$

$\cos\theta = \frac{1}{16}$

$\therefore$ $\theta = 86,4°$ *or* $\theta = -86,4°$ (3)

[9]

QUESTION 3

(a) $\sin 20°.\cos 320° + \cos(-20°).\sin 400°$

$= \sin 20°.\cos 40° + \cos 20°.\sin 40°$

$= \sin 60°$

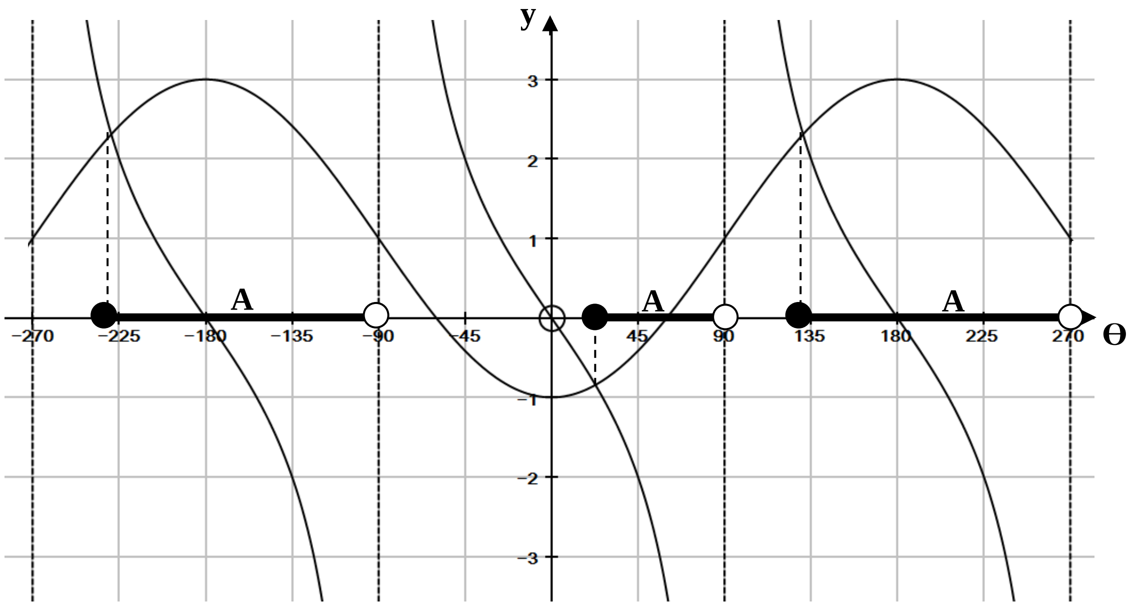
$= \dfrac{\sqrt{3}}{2}$ (5)

(b) (1) 2 (1)

(2) $y \in [-1;3]$ (1)

(3) $p = -2$ and $q = 1$ (2)

(4) asymptotes
shape
intercepts (3)



(5) See graph in (4)…one for each interval….(subtract one if inclusion/exclusion at end points not indicated)

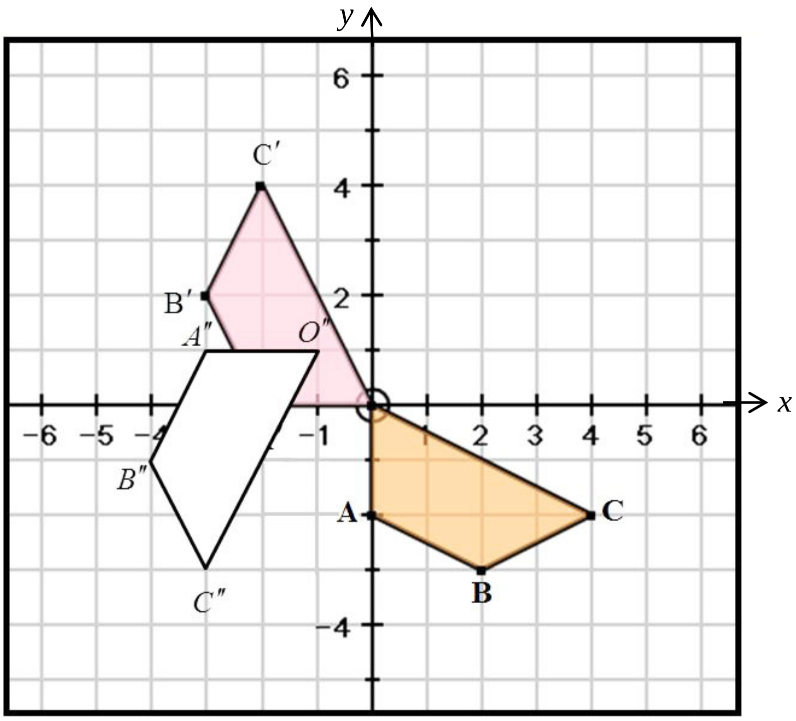
(3)
[15]

QUESTION 4

(a) (1) OABC is reflected about the $y = x$ line. (2)

(2) $(x; y) \rightarrow (x; -y)$ and $(x; y) \rightarrow (-y; x)$
OR
$(x; y) \rightarrow (x; -y) \rightarrow (y; x)$ (2)

(3)



$O(-1;1)$ $A''(-3;1)$ $B''(-4;-1)$ $C''(-3;-3)$ (4)

(b) (1)(i) 2 (ii) 3 (iii) 4 (3)

(2)

(i) $\dfrac{Area\ of\ \Delta P'OR'}{Area\ of\ \Delta POR} = 9$ (1)

(ii) $\therefore \dfrac{Area\ of\ \Delta POR + PRR'P'}{Area\ of\ \Delta POR} = 9$

$\therefore PRR'P' = 64$ (2)

[14]

QUESTION 5

(a) (1) $\overline{x} = \dfrac{0\times 59 + 1\times 84 + 2\times 166 + 3\times 102 + 4\times 72 + 5\times 18}{500} = 2,2$

Or

Number of Children (x_i)	**Frequency** (f_i)	$(x_i)(f_i)$
0	58	0
1	84	84
2	166	332
3	102	306
4	72	288
5	18	90
	Total	1 100

$\overline{x} = \dfrac{1100}{500}$

$= 2,2$

(3)

(2) $a = (2,2-0)^2 = 4,84$ $b = 166\times 0,04 = 6,64$ $c = (2,2-3)^2 = 0,64$

$d = 102\times 0,64 = 65,28$ (4)

(3) $\sigma = \sqrt{\dfrac{848}{500}} = 1,3$ (1)

(b)

(1) The median for Type A is 120 min and the median for Type B is 125 min. (2)

(2) Type A (1)

(3) Type B (1)

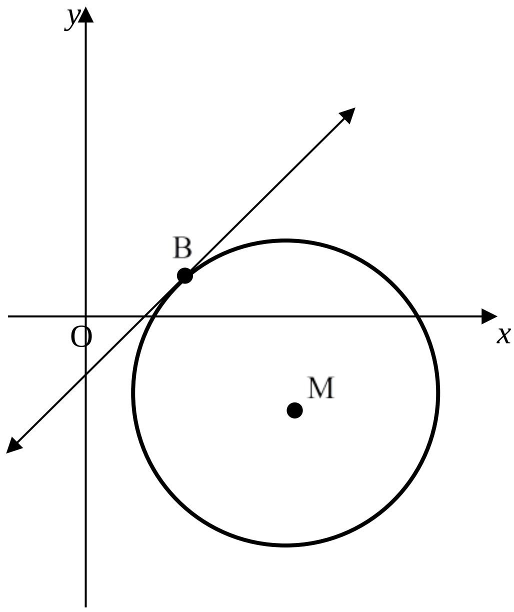
(4) Type B has a greater median.
Type B has a smaller range.
Type B has a smaller interquartile range.
(any two correct statements supporting the claim) (2)

[14]

Total for Section A: 76 marks

SECTION B

QUESTION 6



(a)(1) $x^2 + y^2 - 12x + 4y + 27 = 0$

$\therefore \quad (x-6)^2 - 36 + (y+2)^2 - 4 + 27 = 0$

$\therefore \quad (x-6)^2 + (y+2)^2 = 13$

$\therefore \quad M(6;-2)$

$3y - 2x + 5 = 0$

$y = \frac{2}{3}x - \frac{5}{3}$

$\therefore \quad m_{\tan} = \frac{2}{3}$

$\therefore \quad m_{MB} = -\frac{3}{2}$

Equation of BM: $y + 2 = -\frac{3}{2}(x-6)$

$y = -\frac{3}{2}x + 7$ (6)

(a)(2) For B, $-\frac{3}{2}x + 7 = \frac{2}{3}x - \frac{5}{3}$

$\therefore \quad -9x + 42 = 4x - 10$

$\therefore \quad -13x = -52$

$\therefore \quad x = 4$

$\therefore \quad y = -\frac{3}{2} \times 4 + 7 = 1$

$\therefore \quad B(4;1)$ (3)

(b)(1) For C, Let $x = 0$,

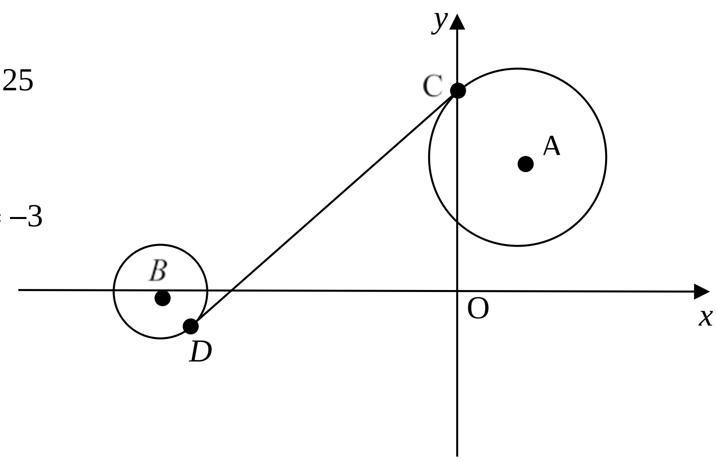
$(0-4)^2 + (y-6)^2 = 25$

$\therefore \quad (y-6)^2 = 9$

$\therefore \quad y - 6 = 3 \ \ or \ \ y - 6 = -3$

$\therefore \quad y = 9 \ \ or \ \ y = 3$ n/a

$\therefore \quad C(0;9)$



(4)

(2) $B(-10;0)$

$BC^2 = (-10-0)^2 + (0-9)^2 = 181$

$CD^2 + BD^2 = BC^2$

$\therefore \quad CD^2 + 1 = 181$

$\therefore \quad CD^2 = 180$

$\therefore \quad CD = \sqrt{180}$ (5)

Mistake in Question…Therefore all students get 5 marks regardless

[18]

QUESTION 7

(a) (1) $\cos(90° + \vartheta) = -\sin\vartheta$

$= \frac{-3}{5}$ (2)

(2) $\sin(-180° - \theta)$

$= \sin\theta$

$= \frac{3}{5}$ (2)

(3) $\cos(\theta - 60°)$

$= \cos\theta.\cos 60° + \sin\theta.\sin 60°$

$= \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$

$= \frac{1}{2}.\frac{4}{5} + \frac{\sqrt{3}}{2}.\frac{3}{5}$ or $-\frac{1}{2}.\frac{4}{5} + \frac{\sqrt{3}}{2}.\frac{3}{5}$ (both answers must be present) (3)

(b) $4\cos^2\theta - \sin\theta\cos\theta = 4$

$\therefore \quad 4\cos^2\theta - \sin\theta.\cos\theta = 4\sin^2\theta + 4\cos^2\theta$ (one mark for using a squares identity correctly)

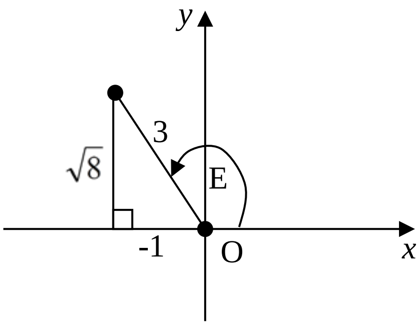
$\therefore \quad -4\sin^2\theta - \sin\theta.\cos\theta = 0$ (for grouping remaining terms)

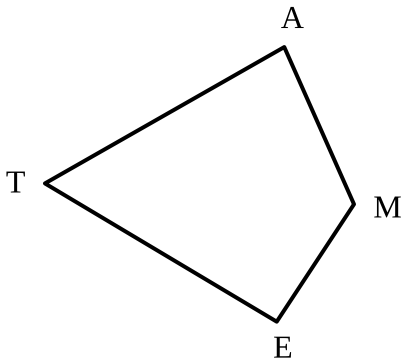
$\therefore \quad \sin\theta(-4\sin\theta - \cos\theta) = 0$ (for factorizing)

$\therefore \quad \sin\theta = 0 \ or \ \tan\theta = -\frac{1}{4}$

$\therefore \quad \theta = k.180 \ or \ \theta = -14° + k.180$ where k is an integer (8)

(c) (1) $\cos\hat{E} < 0$ and $\hat{A} + \hat{E} = 180°$ (2)





(2)

$$\tan\hat{A} = \tan\left(360° - 180° - \hat{E}\right)$$
$$= -\tan\hat{E}$$
$$= -\left(-\sqrt{8}\right)$$
$$= \sqrt{8}$$

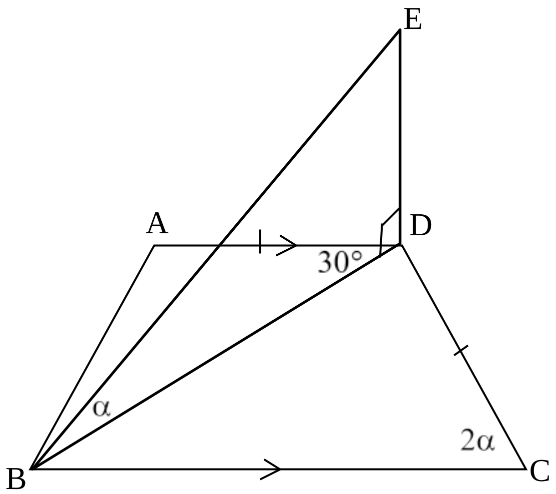
(4)

[21]

QUESTION 8

(a)

$$\frac{Area\ of\ \Delta BDE}{Area\ of\ \Delta ADB}$$
$$= \frac{\frac{1}{2} \times BD \times DE}{\frac{1}{2} \times AD \times BD \times \sin 30°}$$
$$= \frac{\frac{1}{2} \times DE}{\frac{1}{2} \times AD \times \frac{1}{2}} = \frac{2DE}{AD}$$



(3)

(b) In ΔBDE,

$$\frac{DE}{BD} = \tan\alpha \quad \therefore \quad DE = BD\tan\alpha = BD.\frac{\sin\alpha}{\cos\alpha}$$

Also, AD = DC and in ΔBDC,

$$\frac{DC}{\sin 30°} = \frac{BD}{\sin 2\alpha}$$

$$\therefore \quad \frac{AD}{\frac{1}{2}} = \frac{BD}{2\sin\alpha.\cos\alpha}$$

$$\therefore \quad AD = \frac{BD}{4\sin\alpha.\cos\alpha}$$

$$\therefore \frac{Area\ of\ \ \Delta BDE}{Area\ of\ \ \Delta ADB} = \frac{2.BD.\tan\alpha}{\dfrac{BD}{4\sin\alpha.\cos\alpha}}$$ (subs into 8(a)

$$= \frac{\sin\alpha}{\cos\alpha} \times \frac{8\sin\alpha.\cos\alpha}{1}$$

$$= 8\sin^2\alpha$$

(7)

[10]

QUESTION 9

(a)(1) C $(\cos\theta;\sin\theta)$ (2)

(2) A $(-\sin\theta;\cos\theta)$ (2)

(3) B $(\cos\theta-\sin\theta;\cos\theta+\sin\theta)$ (2)

(b) The area of JKLM $= (\sin\theta+\cos\theta)(\cos\theta+\sin\theta)$

$= \cos^2\theta + 2\cos\theta.\sin\theta + \sin^2\theta$

$= 1+\sin 2\theta$ (3)

[9]

QUESTION 10

(a) (1) Highest Possible Final Mark = $\frac{75}{100}\times 90 + \frac{25}{100}\times 100 = 92{,}5 \ \therefore\ 93$ (3)

(2) Since the spread of marks is greater for the lowest scoring 20 learners, one would conclude that the range is greater for the lowest scoring 20 learners. (2)

(3) No. This would be true if the same learner got the lowest mark for each of the assessments.

(4) Final Mark = $\frac{75}{100}\times 46 + \frac{25}{100}\times 78 = 54\%$

New Final Mark = $0{,}75\times 86 + 0{,}25\times 78 = 84\%$

New overall mean mark = $\frac{80\times 60 - 54 + 84}{80} = 60{,}4\%$ (5)

(b) Data set 1 : 60; 80; 80; 80; 80; 100 Range = 40 $\sigma = \sqrt{133{,}3}$

Data set 2 : 62; 62; 62; 98; 98; 98 Range = 36 $\sigma = \sqrt{324}$

$\therefore$ the data sets disprove the learner's claim. (5)

[16]

Total for Section B: 74 marks

Total: 150 marks