



# basic education

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Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P2  
FEBRUARY/MARCH 2012  
MEMORANDUM**

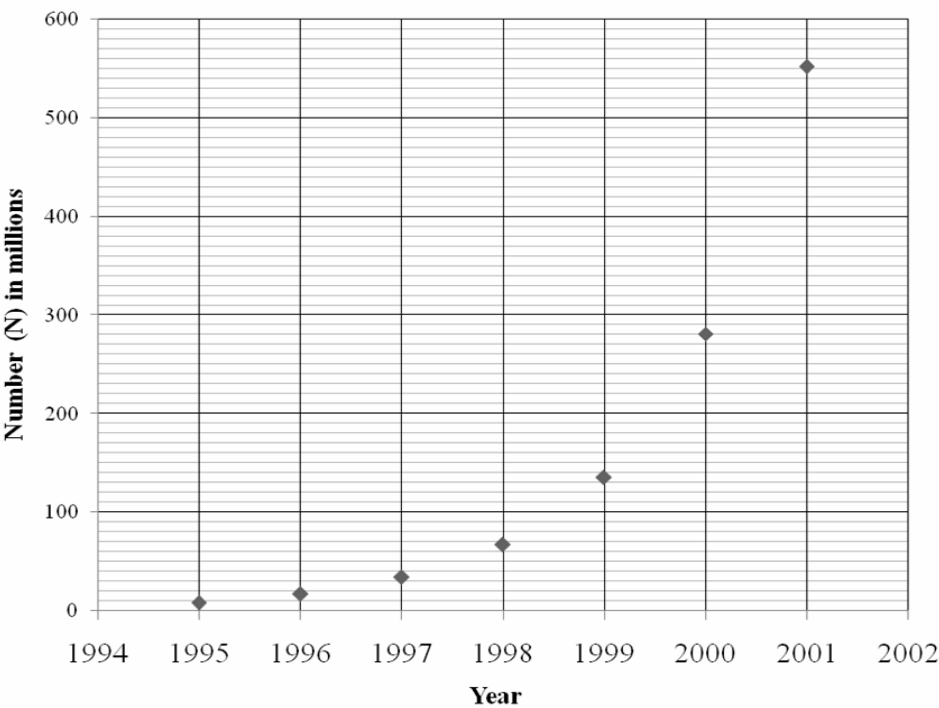
**MARKS: 150**

**This memorandum consists of 18 pages.**

**QUESTION 1**

1.1	<p>Mean</p> $\frac{\sum_1^n x_1}{n} = \frac{102100}{9}$ $= R11\ 344,44$	<p>✓ 102100 ✓ answer (2)</p>
1.2	<p>Standard deviation</p> $\sqrt{\frac{\sum_1^n (x_1 - \bar{x})^2}{n}} = R4\ 460,97$	<p>✓✓ answer (2)</p>
1.3	<p>Value of one standard deviation above mean = R11 344,44 + R4 460,97 = R15 805,41 Only one person earned a commission of more than R 15 805,41. Therefore only 1 person received a rating of good.</p>	<p>✓ adding mean and std. dev. ✓ deduction (2) <b>[6]</b></p>

**QUESTION 2**

2.1	<p style="text-align: center;"><b>Scatter plot of Internet usage</b></p> 	<p>✓ at least four points correct ✓ all points correct (2)</p>
2.2	<p>Exponential (The increase in growth is showing a virtual doubling for each year).</p>	<p>✓ exponential (1)</p>

2.3

YEAR	1995	1996	1997	1998	1999	2000	2001
<b>N (Number in millions)</b>	8	17	34	67	135	281	552
<b>Log N (correct to 1 decimal place)</b>	6,9	7,2	7,5	7,8	8,1	8,4	8,7

✓ at least four values correct  
✓ all values correct  
(2)

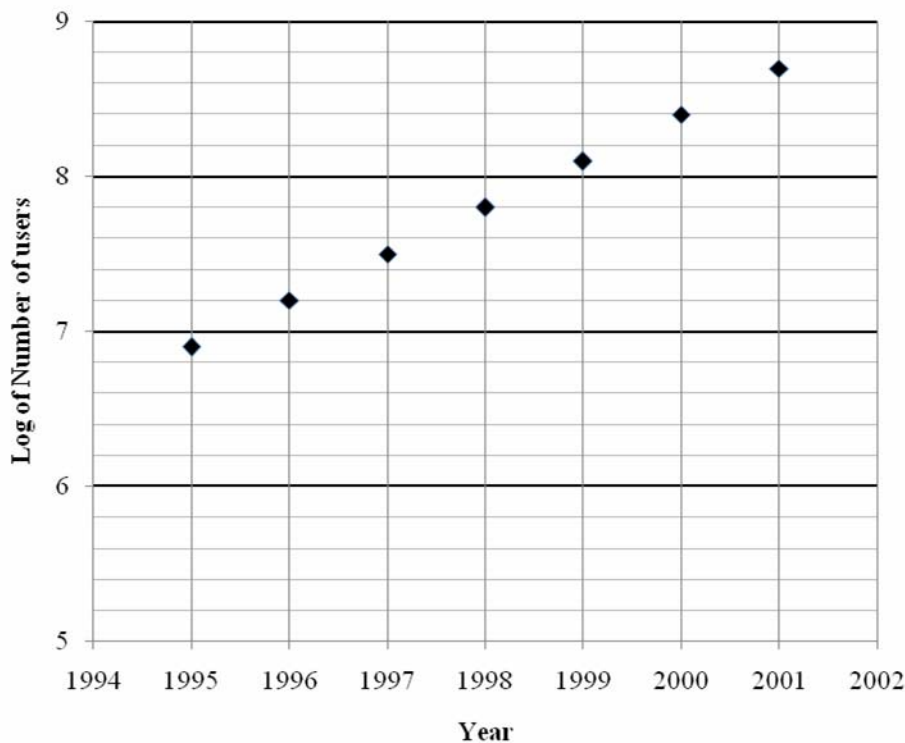
OR (if only log of values in table taken in account)

YEAR	1995	1996	1997	1998	1999	2000	2001
<b>N (Number in millions)</b>	8	17	34	67	135	281	552
<b>Log N (correct to 1 decimal place)</b>	0,9	1,2	1,5	1,8	2,1	2,4	2,7

✓ at least four values correct  
✓ all values correct  
(2)

2.4

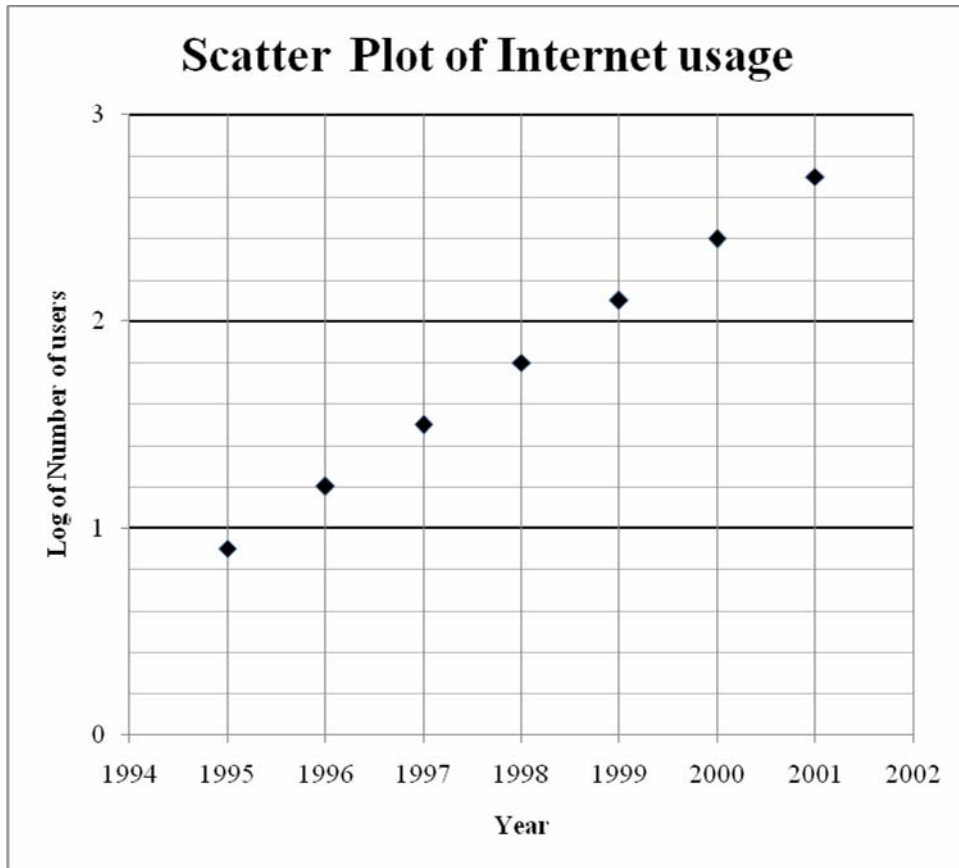
**Scatter Plot of Internet usage**



✓ at least 4 points correctly plotted  
✓ all points correct

(2)

OR (if only log of values in table taken in account)



✓ at least 4 points correctly plotted  
 ✓ all points correct

(2)

2.5 The graph representing log N is a straight line. That is,  
 $\log N = mx + c$   
 $N = 10^{mx+c}$   
 Therefore exponential graph.

✓ linear  
 ✓ reason (2)

[9]

**QUESTION 3**

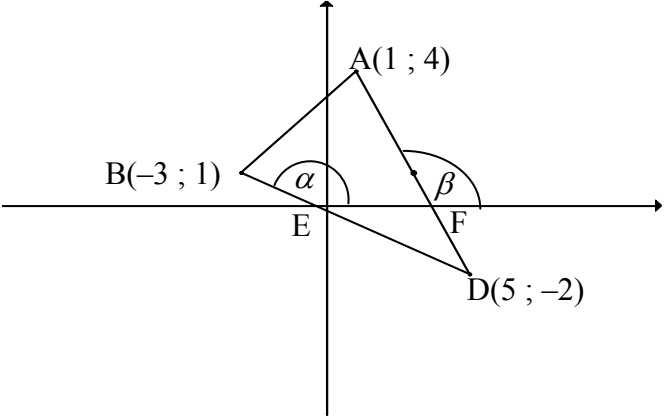
3.1	40	✓ 40 (1)												
3.2	<table border="1"> <thead> <tr> <th>Time, <math>t</math>, in minutes</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td><math>0 \leq t &lt; 5</math></td> <td>3</td> </tr> <tr> <td><math>5 \leq t &lt; 10</math></td> <td>5</td> </tr> <tr> <td><math>10 \leq t &lt; 15</math></td> <td>10</td> </tr> <tr> <td><math>15 \leq t &lt; 20</math></td> <td>15</td> </tr> <tr> <td><math>20 \leq t &lt; 25</math></td> <td>7</td> </tr> </tbody> </table>	Time, $t$ , in minutes	Frequency	$0 \leq t < 5$	3	$5 \leq t < 10$	5	$10 \leq t < 15$	10	$15 \leq t < 20$	15	$20 \leq t < 25$	7	✓ for intervals in table ✓ for first three correct frequencies ✓ for last two correct frequencies (3)
Time, $t$ , in minutes	Frequency													
$0 \leq t < 5$	3													
$5 \leq t < 10$	5													
$10 \leq t < 15$	10													
$15 \leq t < 20$	15													
$20 \leq t < 25$	7													
3.3		✓ first three bars correct ✓ last two bars correct ✓ no gaps between bars  (3) [7]												

**QUESTION 4**

$a = 7$	$b = 15$	$c = 17$	$d = 23$	$e = 34$	$f = 37$	$g = 42$	✓ each correct answer (7)
<b>OR</b>							
$g = 42 ; a = 7 ; d = 23 ; f = 37 ; b = 15$ $\frac{42 + 7 + 23 + 37 + 15 + 3c}{7} = 25$							
$3c = 51$ $c = 17$ $e = 34$							✓ $g$ ✓ $a$ ✓ $d$ ✓ $f$ ✓ $b$ ✓ $c$ ✓ $e$
							(7) [7]

**QUESTION 5**

5.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2 - 4}{5 - 1}$ $= -\frac{6}{4} = -\frac{3}{2}$	✓ for substitution ✓ for answer (2)
5.2	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 1)^2 + (-2 - 4)^2}$ $= \sqrt{16 + 36}$ $= \sqrt{52}$	✓ for substitution ✓ $\sqrt{52}$ (2)
5.3	$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $M = \left( \frac{1 + 5}{2}, \frac{4 - 2}{2} \right)$ $M = (3; 1)$	✓ x-value ✓ y-value (2)
5.4	<p><math>m_{BC} = m_{AD}</math>                      Lines are parallel</p> $= -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ $y - 1 = -\frac{3}{2}(x + 3)$ $2y - 2 = -3x - 9$ $3x + 2y + 7 = 0$ <p style="text-align: center;"><b>OR</b></p> $y = -\frac{3}{2}x + c$ $1 = -\frac{3}{2}(-3) + c$ $c = -\frac{7}{2}$ $y = -\frac{3}{2}x - \frac{7}{2}$ $3x + 2y + 7 = 0$	✓ value $m_{BC}$ ✓ subst $(-3; 1)$ ✓ equation      (3)  ✓ value $m_{BC}$ ✓ subst $(-3; 1)$ ✓ equation  (3)

<p>5.5.1</p>	$m_{AD} = -\frac{3}{2}$ $\tan \beta = -\frac{3}{2}$ $\beta = 180^\circ - 56,31^\circ$ $\beta = 123,69$ 	<p>✓ <math>\tan \beta = m_{AD}</math></p> <p>✓ <math>123,69^\circ</math></p> <p>(2)</p>
<p>5.5.2</p>	$m_{BD} = \frac{-2-1}{5-(-3)} = \frac{-3}{8}$ $\tan \alpha = -\frac{3}{8}$ $\alpha = 180^\circ - 20,56^\circ$ $\alpha = 159,44^\circ$ $\widehat{FED} = 180^\circ - 159,44^\circ = 20,56^\circ$ $\widehat{EFD} = 123,69^\circ$ $\widehat{FDE} = 180^\circ - (20,56^\circ + 123,69^\circ) = 35,75^\circ$	<p>✓ <math>m_{BD} = \frac{-3}{8}</math></p> <p>✓ <math>159,44^\circ</math></p> <p>✓ <math>20,56^\circ</math></p> <p>✓ <math>123,69^\circ</math></p> <p>✓ <math>35,75^\circ</math></p> <p>(5)</p>
<p>5.6</p>	<p>Co-ordinates of centre M (3 ; 1) Radius of circle: <math>\frac{1}{2}</math> of AD = <math>\frac{1}{2} (2\sqrt{13}) = \sqrt{13} = \frac{1}{2}\sqrt{52}</math> Equation of the circle is: <math>(x-3)^2 + (y-1)^2 = 13</math></p> <p style="text-align: center;"><b>OR</b></p> $r^2 = (3-1)^2 + (1-4)^2 = 13$ <p>Equation of the circle is: <math>(x-3)^2 + (y-1)^2 = 13</math></p>	<p>✓ value of radius</p> <p>✓ substitution into equation of circle centre form (2)</p> <p>✓ value of <math>r^2</math></p> <p>✓ substitution into equation of circle centre form (2)</p>
<p>5.7</p>	<p>M(3 ; 1) B(-3 ; 1) <math>MB = \sqrt{(3+3)^2 + (1-1)^2}</math> MB = 6 Point B lies outside the circle because MB &gt; radius</p> <p style="text-align: center;"><b>OR</b></p> <p>M(3 ; 1) B(-3 ; 1) MB = 3 + 3 = 6 Radius of the circle = <math>\sqrt{13} &lt; 6</math> Point B lies outside the circle because MB &gt; radius</p>	<p>✓ substitution</p> <p>✓ outside (2)</p> <p>✓ substitution</p> <p>✓ outside (2)</p> <p>[20]</p>

**QUESTION 6**

6.1	Coordinates of centre M $(-2 ; 1)$ $(1+2)^2 + (-2-1)^2 = 18 = r^2$ Radius = $\sqrt{18}$ or $3\sqrt{2}$	✓✓ coordinates of centre ✓ calculation ✓ value (4)
6.2	$m_{MS} = \frac{-3}{3} = -1$ $m_{MS} \times m_{RS} = -1 \quad \text{OR} \quad \text{tangent} \perp \text{radius}$ $m_{RS} = 1$ $y - y_1 = m(x - x_1)$ $y + 2 = 1(x - 1)$ $y = x - 3$ <p style="text-align: center;"><b>OR</b></p> $m_{MS} = \frac{-3}{3} = -1$ $m_{MS} \times m_{RS} = -1$ $m_{RS} = 1$ $y = x + c$ $-2 = 1 + c$ $c = -3$ $y = x - 3$	✓ gradient MS  ✓ gradient RS ✓ subst (1 ; -2) ✓ equation (4)
6.3	$\frac{MS}{MP} = \frac{1}{3}$ $\therefore MP = 3MS$ $MP^2 = 9MS^2$ $(a+2)^2 + (b-1)^2 = 9(3^2 + 3^2) = 162 \quad (1)$ $MS \perp SR \text{ and } PS \perp SR \quad \therefore m_{PS} = m_{MS}$ $\frac{b+2}{a-1} = \frac{3}{-3} = -1$ $b+2 = -a+1$ $b = -a-1 \quad (2)$ <p>Subst (2) into(1)</p>	✓ $MP = 3MS$  ✓ equation ✓ equal gradients ✓ gradient  ✓ $b = -a - 1$



$$(a + 2)^2 + (-a - 1 - 1)^2 = 162$$

$$(a + 2)^2 + (a + 2)^2 = 162$$

$$2(a + 2)^2 = 162$$

$$(a + 2)^2 = 81$$

$$a + 2 = 9 \text{ or } -9$$

$$a = 7 \text{ or } -11$$

$$b = -a - 1 = -8$$

$$P(7; -8)$$

✓ substitution

✓  $a = 7$

✓  $b = -8$

(8)

**OR**

$$\frac{MS}{MP} = \frac{1}{3}$$

$$\therefore MP = 3MS$$

$$MP^2 = 9MS^2$$

$$(a + 2)^2 + (b - 1)^2 = 9(3^2 + 3^2) = 162 \quad (1)$$

✓  $MP = 3MS$ 

✓ equation

$$MS \perp SR \text{ and } PS \perp SR \quad \therefore m_{PS} = m_{MS}$$

$$\frac{b + 2}{a - 1} = \frac{3}{-3} = -1$$

$$b + 2 = -a + 1$$

$$b = -a - 1 \quad (2)$$

✓ equal gradients

✓ gradient

✓  $b = -a - 1$ 

Subst (2) into(1)

$$a^2 + 4a + 4 + a^2 + 4a + 4 = 162$$

$$2a^2 + 8a - 154 = 0$$

$$a^2 + 4a - 77 = 0$$

$$(a + 11)(a - 7) = 0$$

$$a = 7 \text{ or } -11$$

$$\text{But } a > 0$$

$$\therefore a = 7$$

$$b = -a - 1 = -8$$

$$P(7; -8)$$

✓ substitution

✓  $a = 7$

✓  $b = -8$

(8)

**OR**

$P(a ; b)$   
MSP is a straight line (MS  $\perp$  SR)

$$m_{PM} = -1$$

$$\frac{b-1}{a+2} = -1$$

$$b-1 = -a-2$$

$$b = -a-1 \dots\dots(1)$$

$$PS = 2MS = 2\sqrt{9+9} = 2\sqrt{18}$$

$$PS^2 = 4(18) = 72$$

$$(a-1)^2 + (b+2)^2 = 72 \dots\dots(2)$$

$$(a-1)^2 + (-a-1+2)^2 = 72$$

$$2a^2 - 4a - 70 = 0$$

$$a^2 - 2a - 35 = 0$$

$$(a-7)(a+5) = 0$$

$$a = 7 \text{ or } a = -5$$

$$b = -7-1 = -8$$

$$P(7 ; -8)$$

**OR**

$$2(a-1)^2 = 72$$

$$(a-1)^2 = 36$$

$$a-1 = 6 \text{ or } -6$$

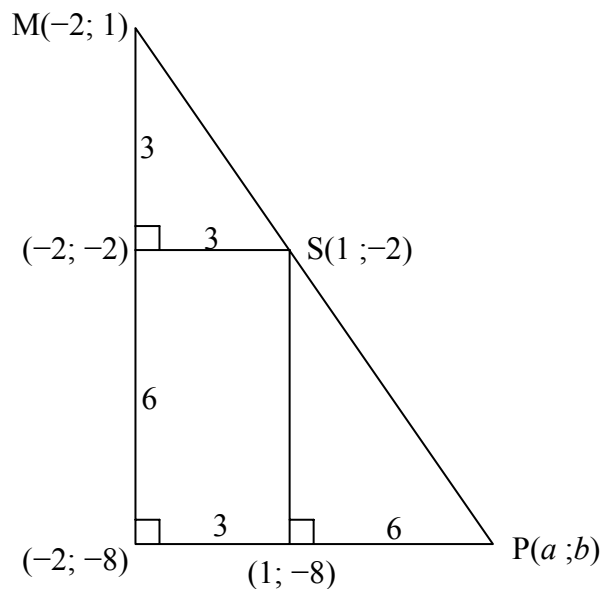
$$a = 7 \text{ or } -5$$

$$a = 7$$

$$b = -8$$

$$P(7 ; -8)$$

**OR**



✓ MSP a straight line

✓  $m_{PM} = -1$

✓  $\frac{b-1}{a+2}$

✓ equation 1

✓ equation 2

✓ substitution of equation 1 into equation 2

✓✓ coordinates

(8)

✓✓ diagram

✓✓ (-2; -8)

✓ (-2; -2)

✓ (1; -8)

✓✓ P(7 ; -8)

(8)

✓✓ division of line segment into

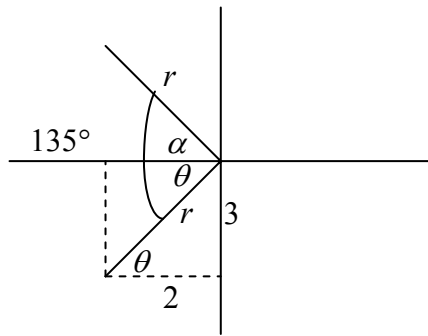
<p>P(a ; b)</p> $\frac{x_S - x_M}{x_P - x_M} = \frac{y_S - y_M}{y_P - y_M} = \frac{1}{3}$ $\frac{-3}{b-1} = \frac{3}{a+2} = \frac{1}{3}$ $-9 = b-1$ $b = -8$ $9 = a+2$ $a = 7$ <p>P(7;-8)</p>	<p>given ratio                  ✓✓ substitution                  ✓ equation</p> <p>✓ equation                  ✓ coordinates</p> <p style="text-align: right;">(8)  <b>[16]</b></p>
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**QUESTION 7**

7.1		<p>For correct coordinates and label of each image:</p> <ul style="list-style-type: none"> <li>✓ K'</li> <li>✓ L'</li> <li>✓ M'</li> <li>✓ N'</li> </ul> <p style="text-align: right;">(4)</p>
7.2.1	Transformation is not rigid, because the area is not preserved under enlargement.	<ul style="list-style-type: none"> <li>✓ not rigid</li> <li>✓ size not preserved</li> </ul> <p style="text-align: right;">(2)</p>
7.2.2	$N''(-2; -2)$	<ul style="list-style-type: none"> <li>✓✓ coordinates of <math>N''</math></li> </ul> <p style="text-align: right;">(2)</p>
7.3	$(x ; y) \rightarrow (-y ; x) \rightarrow (-2y ; 2x)$	<ul style="list-style-type: none"> <li>✓ <math>-y</math></li> <li>✓ <math>x</math></li> <li>✓ <math>-2y</math></li> <li>✓ <math>2x</math></li> </ul> <p style="text-align: right;">(4)</p>
7.4	Area of KLMN : area of $K''L''M''N'' = 1 : 4$	<ul style="list-style-type: none"> <li>✓✓ answer</li> </ul> <p style="text-align: right;">(2)</p>
7.5	<p>If the point that is furthest away from the origin is sent into the circle, the whole quadrilateral is sent into the circle. K is furthest away.</p> $KO = \sqrt{3^2 + 3^2} = \sqrt{18}$ $p.KO = 1, p = \frac{1}{\sqrt{18}}$	<ul style="list-style-type: none"> <li>✓ K – furthest</li> <li>✓ <math>KO = \sqrt{18}</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(3)  <b>[17]</b></p>

**QUESTION 8**

8.	$x_Q = x \cos \theta + y \sin \theta$ $x_Q = -2 \cos 135^\circ + (-3) \sin 135^\circ$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \text{ or } \frac{-\sqrt{2}}{2} \text{ or } -0,71$ $y_Q = y \cos \theta - x \sin \theta$ $y_Q = -3 \cos 135^\circ - (-2) \sin 135^\circ$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3,54$ $Q\left(\frac{-1}{\sqrt{2}}; \frac{5}{\sqrt{2}}\right)$ <p style="text-align: center;"><b>OR</b></p> $x_Q = x \cos \theta - y \sin \theta$ $x_Q = -2 \cos(-135^\circ) - (-3) \sin(-135^\circ)$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \text{ or } \frac{-\sqrt{2}}{2} \text{ or } -0,71$ $y_Q = y \cos \theta + x \sin \theta$ $y_Q = -3 \cos(-135^\circ) + (-2) \sin(-135^\circ)$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3,54$ $Q\left(\frac{-1}{\sqrt{2}}; \frac{5}{\sqrt{2}}\right)$ <p style="text-align: center;"><b>OR</b></p> $x' = x \cos \theta - y \sin \theta$ $-2 = x \cos 135^\circ - y \sin 135^\circ$ $-2 = \frac{-x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$ $-2\sqrt{2} = -x - y \quad (1)$ $y' = y \cos \theta + x \sin \theta$ $-3 = y \cos 135^\circ + x \sin 135^\circ$ $-3 = \frac{-y}{\sqrt{2}} + \frac{x}{\sqrt{2}}$ $-3\sqrt{2} = x - y \quad (2)$ <p>Solving (1) and (2) simultaneously:</p> $-5\sqrt{2} = -2y$ $y = \frac{5}{\sqrt{2}} \quad \text{and} \quad x = \frac{-1}{\sqrt{2}}$	<ul style="list-style-type: none"> <li>✓ subst -2 and -3 into correct formula for <math>x_Q</math></li> <li>✓ using <math>135^\circ</math></li> <li>✓ <math>x</math> coordinate (in any format)</li>   <li>✓ subst -2 and -3 into correct formula for <math>y_Q</math></li> <li>✓ for <math>y</math> coordinate (in any format) (5)</li>   <li>✓ subst -2 and -3 into correct formula for <math>x_Q</math></li> <li>✓ using <math>-135^\circ</math></li> <li>✓ <math>x</math>-coordinate (in any format)</li>   <li>✓ subst -2 and -3 into correct formula for <math>y_Q</math></li> <li>✓ for <math>y</math>-coordinate (in any format) (5)</li>   <li>✓ subst -2 and <math>135^\circ</math> into correct formula for <math>x'</math></li> <li>✓ simplification</li>   <li>✓ subst -2 and <math>135^\circ</math> into correct formula for <math>y'</math></li>   <li>✓ <math>y</math>-coordinate</li> <li>✓ <math>x</math>-coordinate (5)</li> </ul>
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**OR**Using first principles:  $Q = (-r \cos \alpha; r \sin \alpha)$ 

$$Q' = (-2; -3)$$

$$\tan \theta = \frac{3}{2}$$

$$r = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\theta = 56,31^\circ$$

$$\therefore \alpha = 135^\circ - 56,31^\circ = 78,69^\circ$$

$$Q = (-r \cos \alpha; r \sin \alpha)$$

$$= (-0,71; 3,54)$$

$$\checkmark \tan \theta = \frac{3}{2}$$

$$\checkmark r = \sqrt{13}$$

$$\checkmark \theta = 56,31^\circ$$

✓

$$Q = (-r \cos \alpha; r \sin \alpha)$$

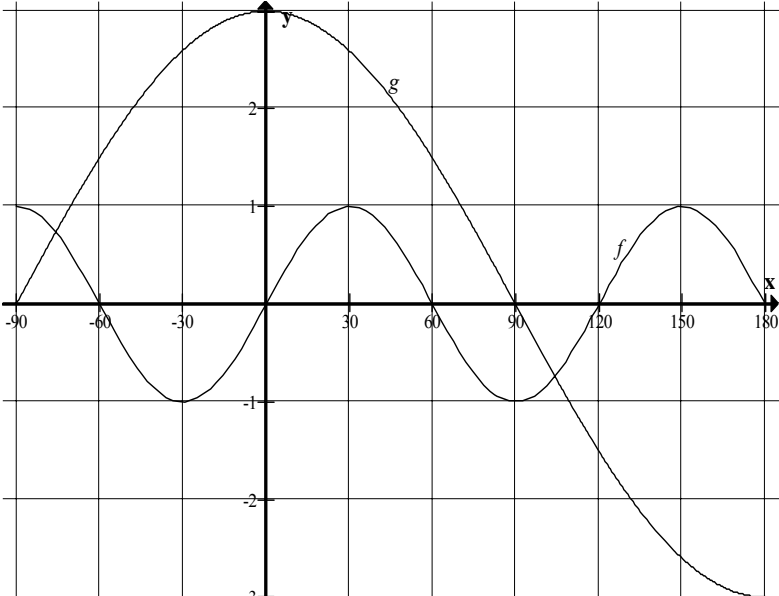
✓ answer

(5)  
[5]

**QUESTION 9**

9.1.1	$r = 13$ $\cos \alpha = \frac{12}{13}$	$\checkmark 13$ $\checkmark \frac{12}{13}$ (2)
9.1.2	$\hat{T}\hat{O}\hat{R} = 180^\circ - (90^\circ + \alpha)$ $= 90^\circ - \alpha$	$\checkmark 180^\circ - (90^\circ + \alpha)$ $\checkmark 90^\circ - \alpha$ (2)
9.1.3	$\cos \hat{T}\hat{O}\hat{R} = \frac{TR}{OT}$ $\cos(90^\circ - \alpha) = \frac{7,5}{OT}$ $OT = \frac{7,5}{\cos(90^\circ - \alpha)}$ $OT = \frac{7,5}{\sin \alpha}$ $OT = \frac{7,5}{\frac{5}{13}}$ $OT = 19,5$ <p style="text-align: center;"><b>OR</b></p> $\sin(\hat{R}\hat{T}\hat{O}) = \frac{7,5}{OT}$ $\therefore OT = \frac{7,5}{\sin \alpha}$ $OT = \frac{7,5}{\frac{5}{13}}$ $OT = 19,5$	$\checkmark$ $\cos(90^\circ - \alpha) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4) $\checkmark$ $\sin(\hat{R}\hat{Q}\hat{O}) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4)
9.2	$LHS = \frac{\cos x \cdot \cos x(-\tan x)}{-\cos x}$ $= \cos x \cdot \frac{\sin x}{\cos x}$ $= \sin x$ $= RHS$	$\checkmark \cos x$ $\checkmark -\tan x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark \text{answer}$ (4) <b>[12]</b>

**QUESTION 10**

10.1	Period = $120^\circ$	✓ $120^\circ$ (1)
10.2	$\sin 3x = -1$ $x = -30^\circ$ or $x = 90^\circ$	✓ $-30^\circ$ ✓ $90^\circ$ (2)
10.3	Maximum value of $f(x)$ is 1 $\therefore$ Maximum value of $h(x)$ is 0	✓ max of $f(x)$ ✓ answer (2)
10.4		✓ $-90^\circ$ ; $90^\circ$ ✓ $(0^\circ; 3)$ ✓ $(180^\circ; -3)$  (3)
10.5	$\frac{\sin 3x}{3} - \cos x = 0$ $\sin 3x - 3 \cos x = 0$ $\therefore \sin 3x = 3 \cos x$ <p>There are 2 solutions where graphs <math>f</math> and <math>g</math> are equal</p>	✓ $\sin 3x = 3 \cos x$  ✓ answer (2)
10.6	$f(x).g(x) < 0$ $x \in (-60^\circ ; 0^\circ)$ or $(60^\circ ; 90^\circ)$ or $(120^\circ ; 180^\circ)$  <p style="text-align: center;"><b>OR</b></p> $-60^\circ < x < 0^\circ$ or $60^\circ < x < 90^\circ$ or $120^\circ < x < 180^\circ$	✓✓✓ for each interval ✓ correct brackets or correct symbols  (4) <b>[14]</b>

## QUESTION 11

11.1.1	$\sin 61^\circ = \sqrt{p}$ $\sin 241^\circ = \sin (180^\circ + 61^\circ)$ $= -\sin 61^\circ$ $= -\sqrt{p}$		✓ $-\sin 61^\circ$ ✓ answer (2)
11.1.2	$\cos 61^\circ = \sqrt{1 - \sin^2 61^\circ}$ $= \sqrt{1 - p}$		✓ identity ✓ answer (2)
11.1.3	$\cos 122^\circ = \cos 2(61^\circ)$ $= 2\cos^2 61^\circ - 1$ $= 2(\sqrt{1-p})^2 - 1$ $= 2(1-p) - 1$ $= 2 - 2p - 1$ $= 1 - 2p$		✓ double angle ✓ expansion  ✓ answer (3)
11.1.4	$\cos 73^\circ \cos 15^\circ + \sin 73^\circ \sin 15^\circ$ $= \cos(73^\circ - 15^\circ)$ $= \cos 58^\circ = (\cos 180^\circ - 122^\circ)$ $= -(\cos 122^\circ)$ $= -(1 - 2p)$ $= 2p - 1$		✓ $\cos(73^\circ - 15^\circ)$  ✓ $-(\cos 122^\circ)$ ✓ answer (3)
11.2.1	$\text{LHS} = \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{\cos^2 x + 2\cos x \sin x + \sin^2 x - (\cos^2 x - 2\sin x \cos x + \sin^2 x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{4\cos x \sin x}{\cos^2 x - \sin^2 x}$ $= \frac{2\sin 2x}{\cos 2x}$ $= 2\tan x$ $= \text{RHS}$		✓ $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ ✓ numerator ✓ $4\cos x \sin x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $2\sin 2x$ ✓ $\cos 2x$ (6)
11.2.2	$\cos x = \sin x \quad \text{or} \quad \cos x = -\sin x$ $x = 45^\circ \quad \quad \quad x = 135^\circ$		✓✓ for answer (2)
11.3.1	$\sin x = \cos 2x - 1$ $\sin x = 1 - 2\sin^2 x - 1$ $\sin x = -2\sin^2 x$ $2\sin^2 x + \sin x = 0$		✓ $1 - 2\sin^2 x$ (1)



11.3.2	$\sin x = \cos 2x - 1$ $2 \sin^2 x + \sin x = 0$ $\sin x (2 \sin x + 1) = 0$ $\sin x = 0 \text{ or } \sin x = -\frac{1}{2}$ $\therefore x = 0^\circ + 180^\circ k; k \in \mathbb{Z} \text{ or } x = \{210^\circ \text{ or } 330^\circ\} + 360^\circ k; k \in \mathbb{Z}$ <p style="text-align: center;"><b>OR</b></p> $x = n \cdot 180^\circ$ $x = n \cdot 360^\circ - 30^\circ$ $x = (2n + 1) \cdot 180^\circ + 30^\circ, n \in \mathbb{Z}$	$\checkmark \sin x (2 \sin x + 1) = 0$ $\checkmark \sin x = 0 \text{ or } \sin x = -\frac{1}{2}$ $\checkmark 0^\circ + 180^\circ k$ $\checkmark 210^\circ$ $\checkmark 330^\circ$ $\checkmark + 360^\circ k; k \in \mathbb{Z}$
		(6)

11.4	$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times \dots \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin 88^\circ}{\cos 88^\circ}\right) \left(\frac{\sin 89^\circ}{\cos 89^\circ}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin(90^\circ - 2^\circ)}{\cos(90^\circ - 2^\circ)}\right) \left(\frac{\sin(90^\circ - 1^\circ)}{\cos(90^\circ - 1^\circ)}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\cos 2^\circ}{\sin 2^\circ}\right) \left(\frac{\cos 1^\circ}{\sin 1^\circ}\right)$ $= \tan 45^\circ$ $= 1$ <p style="text-align: center;"><b>OR</b></p> $\tan 89^\circ = \cot 1^\circ \quad \tan 88^\circ = \cot 2^\circ \dots$ $\therefore \text{product is } (\tan 1^\circ \cdot \cot 1^\circ)(\tan 2^\circ \cdot \cot 2^\circ) \dots (\tan 44^\circ \cdot \cot 44^\circ) \cdot \tan 45^\circ$ $= 1 \times 1 \times 1 \times \dots \times 1 = 1$	$\checkmark \text{ identity}$ $\checkmark \text{ co-ratios}$ $\checkmark \text{ simplification}$ $\checkmark \text{ for answer}$ <p style="text-align: right;">(4)</p> $\checkmark \text{ identity}$ $\checkmark \text{ co-ratios}$ $\checkmark \text{ simplification}$ $\checkmark \text{ for answer}$ <p style="text-align: right;">(4)</p> <p style="text-align: right;"><b>[29]</b></p>
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**QUESTION 12**

12	<p>In <math>\triangle CBG</math> and <math>\triangle CDH</math>:</p> <p><math>CG^2 = x^2 + y^2</math>      Pythagoras</p> <p><math>CH^2 = x^2 + y^2</math>      Pythagoras</p> <p>In <math>\triangle FAE</math></p> <p><math>AE^2 = x^2 + x^2</math></p> <p style="padding-left: 20px;"><math>= 2x^2</math></p> <p style="padding-left: 20px;"><math>= GH^2</math></p> <p>In <math>\triangle CGH</math></p> <p><math>GH^2 = CG^2 + CH^2 - 2 CG \cdot CH \cdot \cos GCH</math></p> <p><math>\cos G\hat{C}H = \frac{CG^2 + CH^2 - GH^2}{2CG \cdot CH}</math></p> <p><math>\cos G\hat{C}H = \frac{x^2 + y^2 + x^2 + y^2 - 2x^2}{2\sqrt{x^2 + y^2} \cdot \sqrt{x^2 + y^2}}</math></p> <p><math>\cos G\hat{C}H = \frac{2y^2}{2(x^2 + y^2)}</math></p> <p><math>\cos G\hat{C}H = \frac{y^2}{x^2 + y^2}</math></p>	<p>✓ <math>CG^2</math></p> <p>✓ <math>CH^2</math></p> <p>✓ <math>AE^2</math></p> <p>✓ <math>AE^2 = GH^2</math></p> <p>✓ use of cos rule</p> <p>✓ manipulation of formula</p> <p>✓ substitution</p> <p>✓</p> <p><math>\cos G\hat{C}H = \frac{2y^2}{2(x^2 + y^2)}</math></p> <p style="text-align: right;">(8)</p> <p style="text-align: right;"><b>[8]</b></p>
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**TOTAL: 150**