Beaulieu College
Mathematics Department

GRADE 12

MATHEMATICS: PAPER 3

Preliminary Examination

Memo

Time: 2 Hours

100 marks

Date: 1st August 2013

Examiner: Mrs Richard

Moderator: Ms Smith

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 15 pages.
2. Answer all the questions on this question paper.
3. All necessary working details must be shown.
4. Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.
5. If necessary answers should be corrected to 3 decimal digits, unless otherwise stated.
6. It is in your own interest to write legibly and to present your work neatly.

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

QUESTION 1

The recursive formula for T_{k+1} of a particular sequence is $T_{k+1} = T_k + 4k - 2$

where $T_1 = 2$ and $k \geq 1$

a) Determine the first 4 terms of the sequence. (3) K

$T_1 = 2$

$T_2 = T_1 + 4(1) - 2 = 2 + 4 - 2 = 4$ ✓

$T_3 = T_2 + 4(2) - 2 = 4 + 8 - 2 = 10$ ✓

$T_4 = T_3 + 4(3) - 2 = 10 + 12 - 2 = 20$ ✓

b) Determine the formula for T_k , the general (k^{th}) term of the sequence. (4)

2 ; 4 ; 10 ; 20

+2 +6 +10

+4 +4

$T_k = ak^2 + bk + c$ ✓

$= 2k^2 + bk + c$ ✓

$\therefore 2 = 2(1)^2 + b(1) + c$ ✓

$2 = 2 + b + c$

$0 = b + c \quad \ldots (1)$

$4 = 2(2)^2 + b(2) + c$ ✓

$4 = 8 + 2b + c$

$-4 = 2b + c \quad \ldots (2)$

(1) − (2) : $4 = -b$

$b = -4$ ✓ $\therefore c = 4$ ✓

OR $2a = 4 \therefore a = 2$ ✓

$3a + b = 2$ ✓

$3(2) + b = 2$

$b = -4$ ✓

$a + b + c = 2$ ✓

$2 - 4 + c = 2$

$c = 4$ ✓

[7]

$T_k = 2k^2 - 4k + 4$ ✓

QUESTION 2

Given: $T_k = 3k^2 + 3k - 1$, the k^{th} term of a sequence

Determine the recursive formula for T_{k+1} for this sequence in the form $T_{k+1} = \cdots$

$T_1 = 3(1)^2 + 3(1) - 1 = 5$

$T_2 = 3(2)^2 + 3(2) - 1 = 17$ $\quad +12 \quad 2\times 6$

$T_3 = 3(3)^2 + 3(3) - 1 = 35$ $\quad +18 \quad 3\times 6$ ✓

$T_4 = 3(4)^2 + 3(4) - 1 = 59$ $\quad +24 \quad 4\times 6$ ✓

$\therefore T_{k+1} = T_k + 6(k+1)$ ✓

$T_{k+1} = T_k + 6k + 6$ ✓

$T_1 = 5$ ✓ $\quad k \geq 1$ ✓

3 parts to recursive formula.

$T_{k+1} = 2T_k - T_{k-1} + 6 \qquad k+1 \geq 3$

$\therefore k \geq 2$

$T_1 = 5$

$T_2 = 17.$

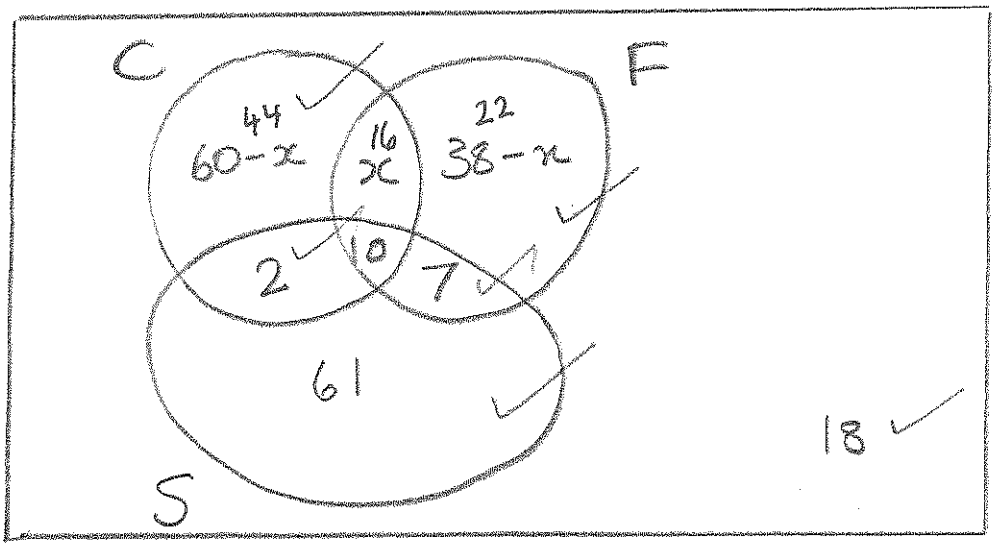
[6]

QUESTION 3

A survey was conducted among 180 learners about their preferences for Coca-Cola, Fanta and Sparletta cool drinks. The findings were:

72 drink Coca-Cola.
55 drink Fanta.
80 drink Sparletta.
10 drink all three types
17 drink Sparletta and Fanta
12 drink Coke and Sparletta
18 do not drink any cool drinks at all.
x prefer Coke and Fanta but not Sparletta

a) Draw a Venn diagram to summarize the information. (5)



b) How many only prefer Coke? (3)

$60 - x + x + 38 - x + 2 + 10 + 7 + 61 + 18$

$= 180$ ✓

$\therefore \quad x = 16$ ✓

only coke $60 - 16 = 44$ ✓

c) What is the probability that if a student is randomly selected, he/she would like any two types of cool drinks? (2)

$$\frac{16+2+7}{180} = \frac{25}{180} = 0,139$$

[10]

QUESTION 4

This frequency table gives the time spent in hours by learners in grade 11 and grade 12 on their maths homework in a typical week (excluding study time)

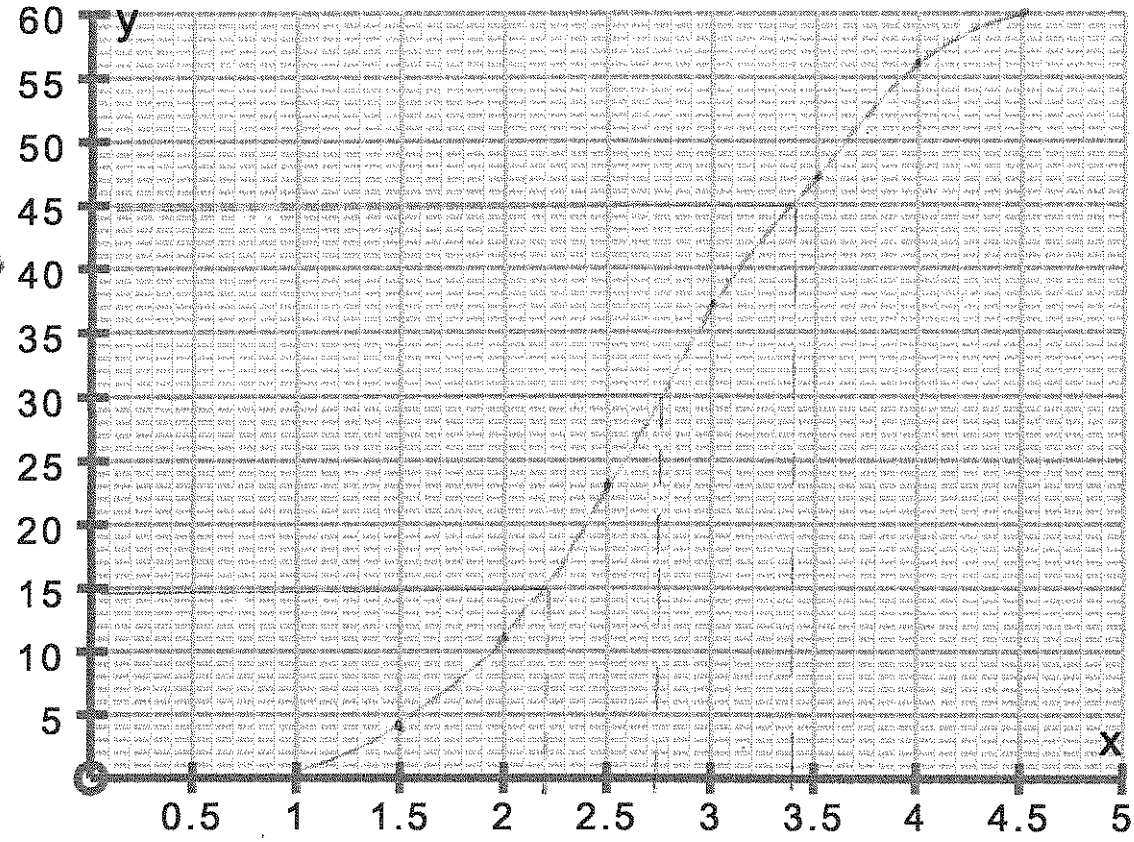
a) Complete the cumulative frequency table for the data, on the grid below. (2)

for ogive. only for $\bar{x}$

Hours ↓	Frequency f	Midpoint	Cumulative
$1.0 \leq w < 1.5$	4	1,25	4
$1.5 \leq w < 2.0$	7	1,75	11
$2.0 \leq w < 2.5$	12	2,25	23
$2.5 \leq w < 3.0$	14	2,75	37
$3.0 \leq w < 3.5$	10	3,25	47
$3.5 \leq w < 4.0$	9	3,75	56
$4.0 \leq w < 4.5$	4	4,25	60
$4.5 \leq w$	0	✓	✓
	Total = 60		

plot (1,5;4) not (1,25;4)

b) Draw a cumulative frequency curve (ogive) for this data. (2)



Frequency

not joined: join

Revise ogives.

c) Estimate, from your graph, the median and quartiles for the hours spent

$M = 2,7$ ✓

$Q_1 = 2,2$ ✓

$Q_3 = 3,4$ ✓ (3)

d) Calculate the estimated mean time spent by learners

$\bar{x} = 2,767$ ✓✓ 3d.p.

(2)

e) Determine the standard deviation.

$\sigma = 0,811$ ✓✓ 3d.p.

(2)

[11]

QUESTION 5

a) The probability that a hockey team has all its players fit to play is 70%.
The probability that they will win if all the players are fit is 90%.
When they are not fit the probability of them winning becomes 45%.
Calculate the probability of winning their next game. (5)

$0,7 \times 0,9 + 0,3 \times 0,45$ ✓ ✓ ✓ ✓

$= 0,765$ ✓

leave answers as decimals not %.

b) The following data was obtained from the financial office at a certain university:

	Receiving financial aid	Not receiving financial aid	**TOTAL**
Undergraduates	4 222	3 898	**8 120**
Postgraduates	1 879	731	**2 610**
TOTAL	**6 101**	**4 629**	**10 730**

1) Determine the probability that a student selected at random is....

i) receiving financial aid. (2)

$\frac{6101}{10730}$ ✓ = 0,569 ✓

ii) a postgraduate and not receiving financial aid. (2)

$\frac{731}{10730}$ ✓ = 0,068 ✓

iii) undergraduate and receiving financial aid. (2)

$\frac{4222}{10730}$ ✓ = 0,393 ✓

2) Are the events of being an undergraduate and receiving financial aid independent?

Show all your working to support your answer. (4)

$P(u) = \frac{8120}{10730} = 0,757$

$P(F) = \frac{6101}{10730} = 0,569$

$P(u \text{ and } F) = \frac{4222}{10730} = 0,393$ ✓

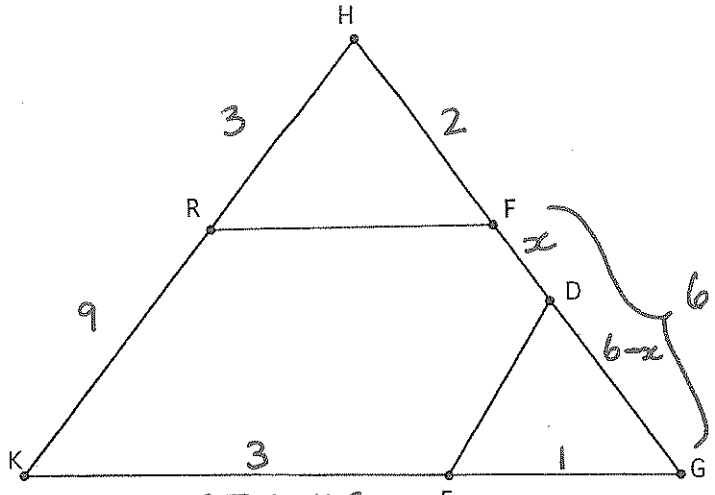
$P(u) \times P(F) = 0,430$ ✓

No, not independent. ✓

[15]

QUESTION 6

RF // KG , ED // KH , RH = 3 units, RK = 9 units , HF = 2 units and GE:EK=1:3.



Calculate:

a) FG (3)

RF // KG

$\frac{9}{3} = \frac{FG}{2}$ ✓ Line // one side $\triangle HKG$ ✓ reasons!

$\frac{18}{3} = FG$ ✓

$FG = 6$ units ✓

b) FD (5)

ED // KH.

$\frac{KE}{EG} = \frac{HD}{DG}$ ✓ Line // one side $\triangle GHK$

let $FD = x$

$\frac{3p}{1p} = \frac{x+2}{6-x}$ ✓ ✓

$3(6-x) = x+2$ ✓

$18 - 3x = x + 2$

$-4x = -16$

$x = 4$ units

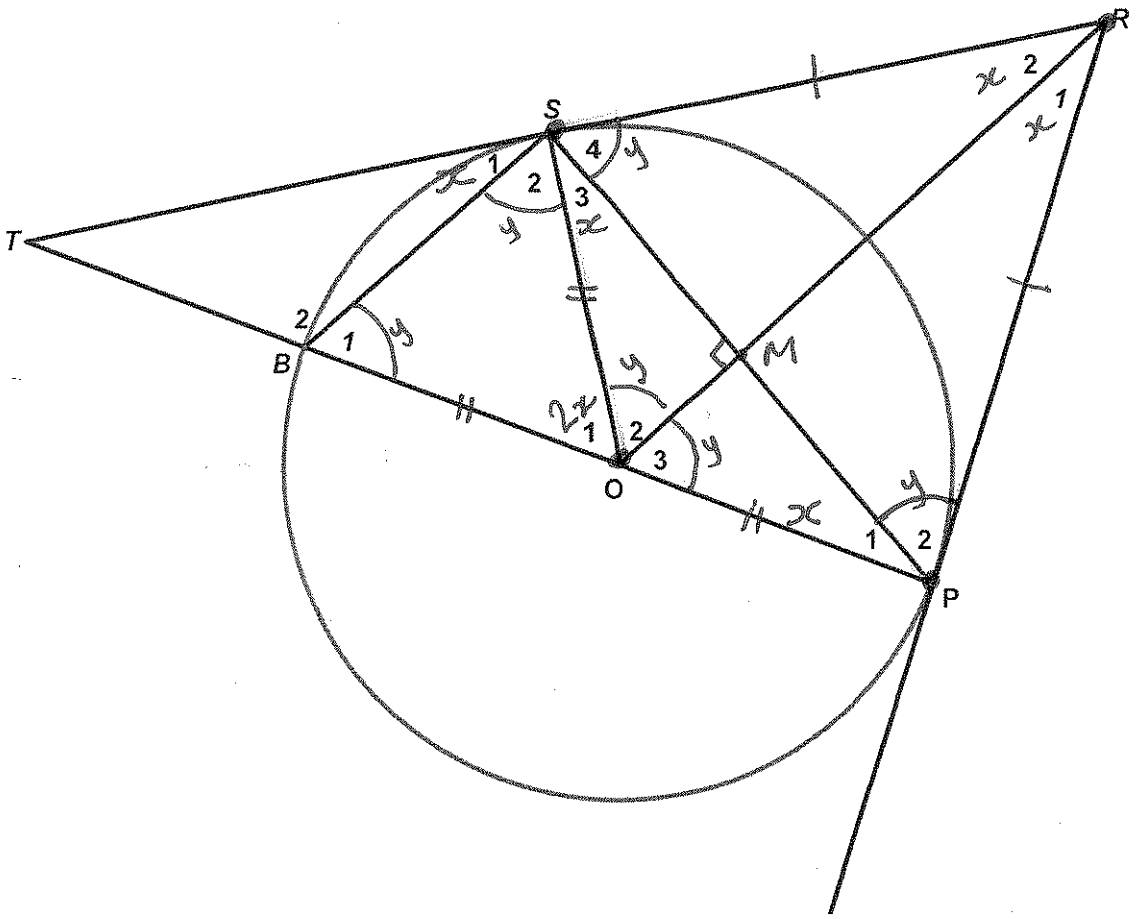
$\therefore FD = 4$ units ✓

[8]

QUESTION 7

Two tangents RP and RS are drawn from a point R outside the circle with centre O.

PO produced cuts the circle at B and meets the tangent RS produced at T. Let $\hat{S}_1 = x$



Prove

a) POSR a cyclic quadrilateral. (3)

$\hat{S}_1 = x = \hat{P}_1$ tan chord

$\hat{P}_2 = \hat{B}_1 \longrightarrow \hat{P}_2 = \hat{S}_2$ tan chord ✓

tan chord

$\therefore \hat{S}_1 + \hat{S}_2 = \hat{P}_1 + \hat{P}_2$ ✓

$\hat{B}_1 = \hat{S}_2$ radii $\therefore$ POSR is cyclic

ext $\angle$ = int opp $\angle$ ✓

OR

$\hat{S}_3 + \hat{S}_4 = 90°$ } tan ⊥ radius. ✓ ✓
$\hat{P}_1 + \hat{S}_2 = 90°$ }

$\therefore S_3 + S_4 + P_1 + P_4 = 180°$

$\therefore$ POSR is a cyclic quad ✓
opp $\angle$'s suppl.

converse reason. Discuss converses!

$\hat{S}_1 \neq \hat{O}_1$ tan chord
$\hat{P}_2 \neq \hat{O}_3$ tan chord
} subtends < at circle not CENTRE.

b) SB // OR (6)

$\hat{P}_2 = \hat{O}_2$ ✓ <'s in same segment ✓
$\hat{P}_2 = \hat{S}_2$ ✓ tan chord ✓
$\therefore \hat{O}_2 = \hat{S}_2$ ✓
$\therefore$ SB // OR alt <'s = ✓

c) $\widehat{R_1} = \frac{1}{2}\hat{O}_1$ (6)

$\hat{S}_3 = \hat{P}_1 = x$ ✓ OS = OP radii ✓
$\hat{S}_3 = \hat{R}_1 = x$ ✓ <'s in same segm
$\hat{O}_1 = 2x$ < at centre = 2x < at circ. ✓
But $\hat{R}_1 = x$ ✓
$\therefore \hat{R}_1 = \frac{1}{2}\hat{O}_1$ ✓

d) $B\hat{O}R = 90^0 + x$ (5)

$\hat{O}_1 = 2x$ ✓
$\hat{B}_1 = \hat{S}_2$ OB = OS radii ✓
Let $\hat{R}_1 = y = \hat{S}_2$ ✓
$\hat{S}_2 = y = \hat{O}_2$ alt <'s; // lines.
$B\hat{O}R + \hat{R}_1 = 180^0$ cons <'s; // lines ✓
$\therefore 2x + y + y = 180^0$ <'s in Δ
$2x + 2y = 180^0$
$x + y = 90^0$ ✓
$\therefore y = 90^0 - x$ and $B\hat{O}R + 90^0 - x = 180^0$
$\therefore B\hat{O}R = 90^0 + x$

[20]

OR see page 16

OR

7d) $\hat{B}_2 = B\hat{S}P + \hat{P}_1$ ext $\angle$ of Δ

but $B\hat{S}P = 90°$ $\angle$ in semi ⊙

$\hat{B}_2 = 90° + x$

$B\hat{O}R = \hat{B}_2$ corr $\angle$'s; SB ∥ OR

$\therefore B\hat{O}R = 90° + x.$

OR

7b) $\hat{S}_4 = \hat{B}_1$ tan chord thm

$\hat{S}_4 = \hat{O}_3$ $\angle$'s in same segment

$\therefore \hat{B}_1 = \hat{O}_3$

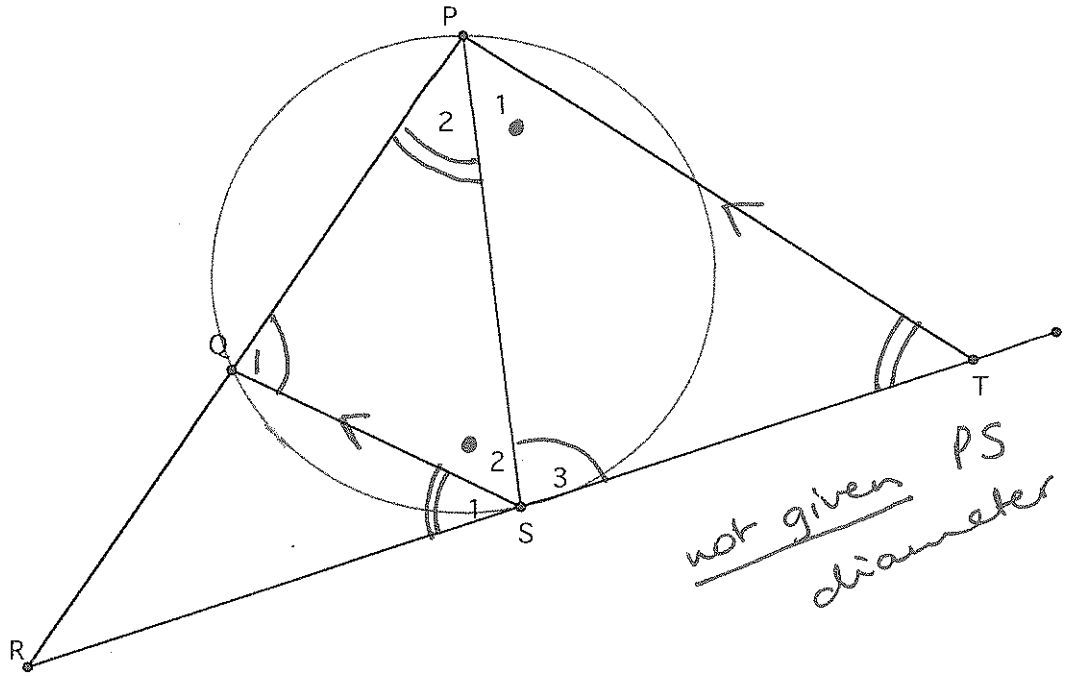
$\therefore$ SB ∥ OR corr $\angle$'s equal.

b) Have to prove ≡ Δ's to get M midpt of SP OR OM ⊥ SP.

Cannot <u>assume</u>.

QUESTION 8

RST is a tangent to the circle at S. Chord PQ produced meets the tangent at R. QS // PT.



Prove:

a) $\Delta SPQ \;///\; \Delta PTS$ (6)

In Δ SPQ and ΔPTS ✓ ✓

$\hat{S}_2 = \hat{P}_1$ alt $\angle$'s; QS // PT ✓

$\hat{Q}_1 = \hat{S}_3$ tan chord ✓

$\hat{P}_2 = \hat{T}$ $\angle$'s in Δ ✓

Δ SPQ /// ΔPTS equiangular. ✓

b) $SP^2 = SQ \cdot PT$ (1)

$\frac{SP}{SQ} = \frac{PT}{PS}$ sides in prop /// Δ's

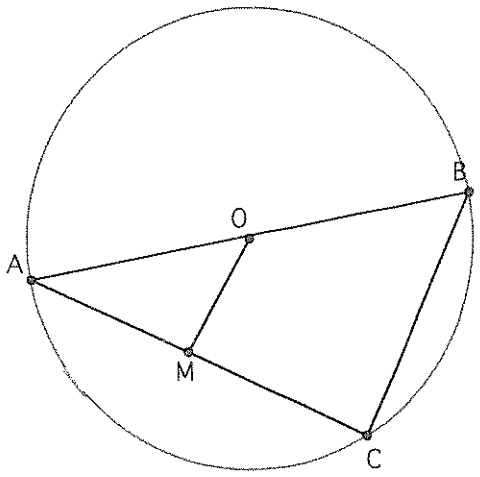
$PS^2 = PT \cdot SQ$ ✓

[7]

confusing /// with $\equiv$

QUESTION 9

O is the centre of the circle. $OM \perp AC$. The radius is equal to 5 cm and BC is 8 cm.



a) Write down the size of $B\hat{C}A$ (1)

$B\hat{C}A = 90°$ ✓ $\angle$ in semi circle

b) Determine the length of AM. (5)

$OM \perp AC \therefore A\hat{M}O = B\hat{C}A = 90°$ ✓

$\therefore OM \parallel BC$

$AC^2 = 10^2 - 8^2$ Pyth ✓

$= 36 \quad \therefore AC = 6$ cm ✓

$AM = MC = 3$ cm ✓ $OM \parallel BC$ in ΔABC

or line from centre $\perp$ to chord ✓

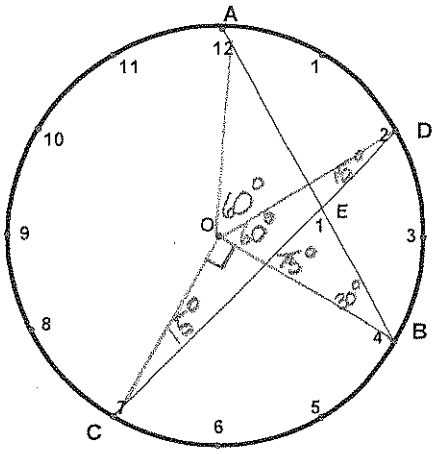
c) $Area\ \Delta AOM : Area\ \Delta ABC$ (3)

1 : 4 ✓✓

[9]

QUESTION 10

The diagram shows the face of a clock.
12 is joined to the 4 and 2 to the 7 with straight lines AB and CD respectively. O is the centre of the clock.



a) If AO and DO are joined, show that $A\hat{O}D = 60^\circ$ (No reasons required)

$A\hat{O}D = \frac{1}{6} \times 360^\circ$ ✓ ✓

$= 60^\circ$

(2)

b) If CO and BO are joined, calculate $C\hat{O}B$

$C\hat{O}B = \frac{3}{12} \times 360^\circ = 90^\circ$ ✓ ✓ ✓

(2)

c) Hence, calculate $\hat{E}_1$, providing reasons.

see diagram ✓

$\hat{E}_1 = 75^\circ$ ✓ E is not on circle OR on centre.

$D\hat{O}B = 60^\circ$, $O\hat{D}C = 15^\circ$

(3)

$\angle$'s in $\triangle$

ext $\angle$ of $\triangle$ ... ✓

[7]

[Total: 100 marks]