AP MATHS
GRADE 12
PRELIM EXAMINATION
22 JULY 2013

QUESTION 1

$n=1$: LHS $= 1^2 = 1$ ✓ RHS $= \frac{1(1+1)(2(1)+1)}{6} = 1$ ✓

LHS = RHS

∴ True for $n=1$ ✓

Assume true for $n=k$:

$\therefore 1^2+2^2+3^2+\ldots+k^2 = \frac{k(k+1)(2k+1)}{6}$ ✓✓

Prove true for $n=k+1$:

$1^2+2^2+3^2+\ldots+k^2+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ ✓✓

$$\text{RHS} = \frac{2k^3+3k^2+k}{6} + k^2+2k+1 \quad ✓$$

$$= \frac{2k^3+3k^2+k+6k^2+12k+6}{6} \quad ✓$$

$$= \frac{2k^3+9k^2+13k+6}{6} \quad ✓$$

$$= \frac{(k+1)(2k^2+7k+6)}{6} \quad ✓$$

$$= \frac{(k+1)(k+2)(2k+3)}{6} \quad ✓$$

$$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6} \quad ✓✓$$

∴ True for $n=k+1$, if true for $n=k$.

∴ True for $n=1$, $n=2$, $n=3$ ... ✓

(16) T1 T2

[16]

QUESTION 2

2.1 $f(0) = \frac{1}{2}(e^0 + e^0)$ ✓

$= 1 \text{ m}$ ✓

(2) T1

2.2 The cable is hanging at a dangerously low height above the ground as most people and even children will be able to touch it. ✓✓

(2) T1

2.3 $f(d) = 2$

$\frac{1}{2}(e^d + e^{-d}) = 2$ ✓

$e^d + \frac{1}{e^d} = 4$

$e^{2d} - 4e^d + 1 = 0$ ✓

$e^d = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$ ✓

$= 2 \pm \sqrt{3}$ ✓✓

$e^d = 2 + \sqrt{3}$

$d = \ln(2 + \sqrt{3})$ ✓

$d = 1,32 \text{ m}$ ✓✓

(8) T3

[12]

QUESTION 3

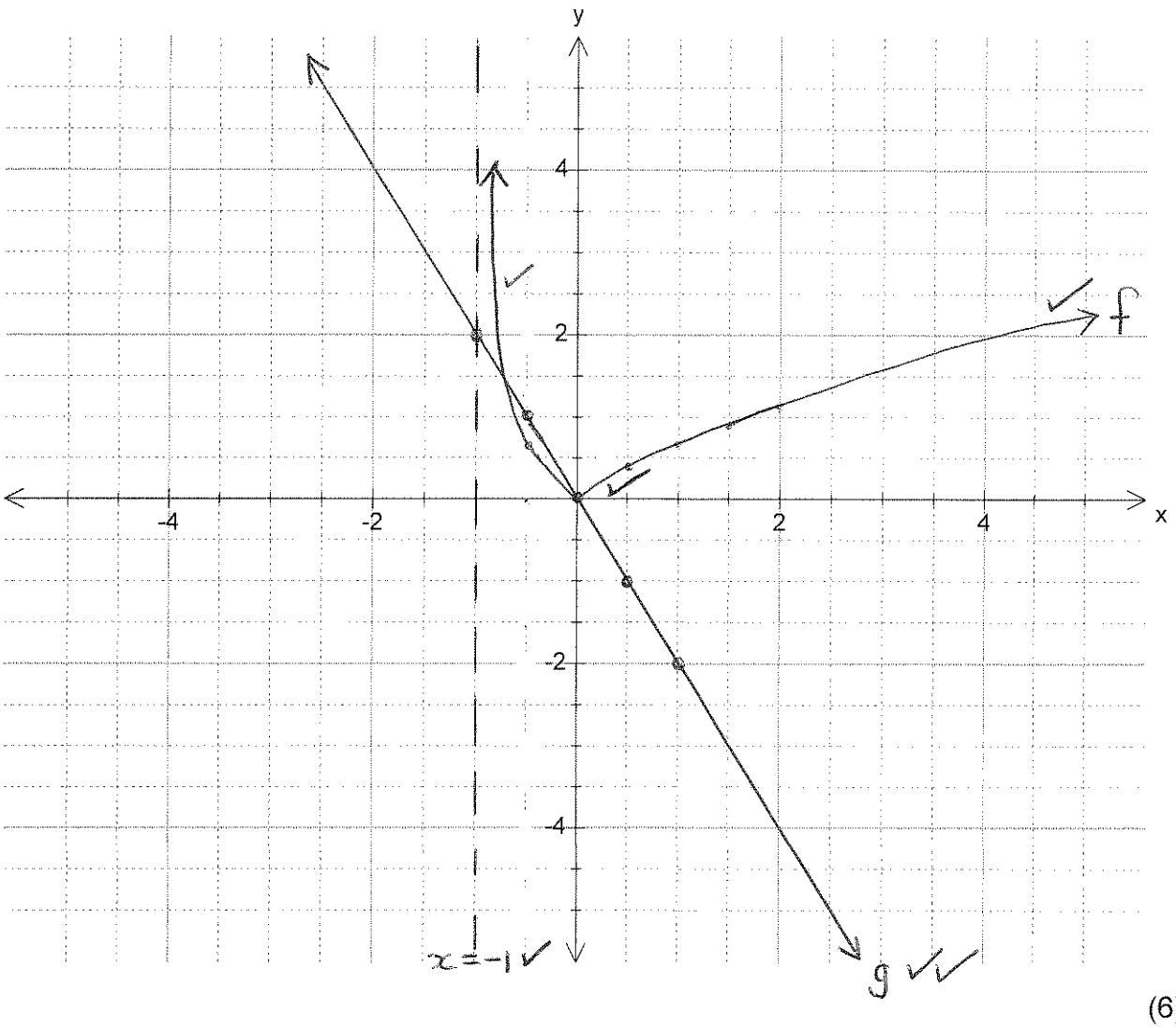
3.1 ON ANSWER SHEET.

(6) T3 T1

ANSWER SHEET

Name: MEMO Grade 12

Question 3.1



(6)

3.2 $-\ln(x+1) = -2x$ ✓✓

$\ln(x+1) = 2x$ ✓

$e^{2x} = x+1$ ✓✓

(5) T2

3.3 $e^{2x} - x - 1 = 0$

Let $f(x) = e^{2x} - x - 1$ ✓

$f'(x) = 2.e^{2x} - 1$ ✓✓

$$x_{r+1} = x_r - \frac{e^{2x_r} - x_r - 1}{2.e^{2x_r} - 1}$$ ✓✓

Set $x_1 = -0,9$ ✓

$x = -0,79681$ ✓✓✓

(9) T3

3.4 $-0,79681 < x < 0$ ✓✓

(2) T1

[22]

QUESTION 4

$[x-(1+3i)][x-(1-3i)]$ is a factor of $g(x)$ ✓✓

$= [(x-1)-3i][(x-1)+3i]$

$= (x-1)^2 - 9i^2$

$= x^2 - 2x + 1 + 9$

$= x^2 - 2x + 10$ ✓✓

$g(x) = x^4 - 2x^3 + 14x^2 - 8x + 40$

$= (x^2 - 2x + 10)(x^2 + \boxed{0}x + 4)$

$-2x^3$

$\boxed{0}x^3$

$-2x^3$ ✓✓

$= (x-1-3i)(x-1+3i)(x^2+4)$ ✓✓

$= (x-1-3i)(x-1+3i)(x+2i)(x-2i)$ ✓✓

[10] T2 T3

QUESTION 5

5.1 (a) $\lim_{x \to -5} \frac{(x-3)}{(x+5)(x-3)}$ d.n.e. ✓✓

$f(-5)$ d.n.e. ✓

Jump discontinuity ✓

(4) T1 T2

(b) $\lim_{x \to 3^-} f(x) = \frac{1}{3+5}$

$= \frac{1}{8}$ ✓

$\lim_{x \to 3^+} f(x) = 2^{-3}$

$= \frac{1}{8}$ ✓

$\therefore \lim_{x \to 3} f(x) = \frac{1}{8}$ ✓

$f(3) = 2^{-3}$

$= \frac{1}{8}$ ✓

$\therefore \lim_{x \to 3} f(x) = f(3)$ ✓

and f is continuous at $x = 3$. ✓

(6) T1 T2

5.2 $x = -1$ ✓ is a local maximum. ✓

Before $x = -1$ $g'(x) > 0$ and after $g'(x) < 0$. ✓

$x = 4$ ✓ is a point of horizontal inflection. ✓

Before $x = 4$ $g'(x) < 0$ and after $g'(x) < 0$ ✓

also before $x = 4$ $g''(x) > 0$ and after $g''(x) < 0$. ✓✓

(7) T4

[17]

QUESTION 6

6.1 (a) $LHS = \frac{\sec A + 1 + \sec A - 1}{(\sec A - 1)(\sec A + 1)}$ ✓✓

$= \frac{2 \sec A}{\sec^2 A - 1}$ ✓

$= \frac{2}{\cos A} \div \tan^2 A$ ✓

$= \frac{2}{\cos A} \times \frac{\cos^2 A}{\sin^2 A}$ ✓

$= 2 \cos A \operatorname{cosec}^2 A$ ✓ $= RHS$

(6) T2

(b) $\lim_{A \to 0} (2\cos A . \operatorname{cosec}^2 A) A^2$

$= \lim_{A \to 0} (2\cos A)\left(\frac{A^2}{\sin^2 A}\right)$

$= 2 \times 1$

$= 2$

(5) T1

6.2 (a) $f'(x) = 6x . \cos 4x + 3x^2 . (-\sin 4x)(4)$

$= 6x . \cos 4x - 12x^2 . \sin 4x$

(5) T3

(b) $f(x) = (x + x^{1/2})^{1/2}$

$f'(x) = \frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$

$= \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}$

(6) T3

6.3 (a) $y = (4x^2 + 1)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(4x^2 + 1)^{-1/2}(8x)$

$= \frac{4x}{\sqrt{4x^2 + 1}}$

$= \frac{4x}{y}$

(6) T2

(b) $\frac{dy}{dx} = \frac{4x}{y}$

$\frac{d^2y}{dx^2} = \frac{4y - 4x\left(\frac{dy}{dx}\right)}{y^2}$

$= \frac{4y}{y^2} - \frac{4x\left(\frac{4x}{y}\right)}{y^2}$

$= \frac{4}{y} - \frac{16x^2}{y^3}$

(6) T4

[34]

QUESTION 7

7.1 $A = \frac{1}{2}r^2\theta \checkmark - \frac{1}{2}(OA)(OX)\sin\hat{O} \checkmark$

but $\cos\hat{O} = \frac{OX}{OA}$

$\cos\theta = \frac{OX}{r}$ ✓

$OX = r.\cos\theta$ ✓

$\therefore A = \frac{1}{2}r^2\theta - \frac{1}{2}r.r\cos\theta.\sin\theta$ ✓

$= \frac{1}{2}r^2(\theta - \sin\theta\cos\theta)\ cm^2$ ✓✓

(7) T3

7.2 $\sin\hat{O} = \frac{AX}{OA}$

$AX = r.\sin\theta$ ✓ and $XB = r - r\cos\theta$ ✓

$AB = r\theta$ ✓

$\therefore P = r\sin\theta + r - r\cos\theta + r\theta$

$= 12\sin\frac{\pi}{6} + 12 - 12\cos\frac{\pi}{6} + 12\left(\frac{\pi}{6}\right)$ ✓

$= 12\left(\frac{1}{2}\right)$ ✓ $+ 12 - 12\left(\frac{\sqrt{3}}{2}\right)$ ✓ $+ 2\pi$

$= 18 - 6\sqrt{3} + 2\pi\ cm$ ✓✓

(8) T3

[15]

QUESTION 8

8.1 $\int_0^3 x(x-1)\,dx = \int_0^3 x^2 - x\,dx$ ✓

$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^3$ ✓

$= \frac{3^3}{3} - \frac{3^2}{2} - 0$ ✓

$= \frac{9}{2}$ ✓

(4) T1

8.2 $\int \frac{x}{\sqrt{x+4}}\,dx$

Set $u = x+4$ ✓ and $x = u-4$

$\frac{du}{dx} = 1$

$du = dx$ ✓

$= \int \frac{u-4}{\sqrt{u}}\,du$ ✓

$= \int u^{1/2} - 4u^{-1/2}\,du$ ✓

$= \frac{u^{3/2}}{3/2}$ ✓ $- \frac{4u^{1/2}}{1/2}$ ✓ $+ c$

$= \frac{2(x+4)^{3/2}}{3} - 8(x+4)^{1/2} + c$ ✓✓

(8) T3

8.3 $\int x \cdot \cos 2x\,dx$

Let $f(x) = x$ and $g'(x) = \cos 2x$

$f'(x) = 1$ $g(x) = \frac{\sin 2x}{2}$

$= x \cdot \frac{\sin 2x}{2}$ ✓ $- \int \frac{\sin 2x}{2} \cdot 1\,dx$ ✓ $+ c$

$= \frac{x \cdot \sin 2x}{2} - \frac{1}{2}\left(\frac{-\cos 2x}{2}\right)$ ✓✓ $+ c$

$= \frac{x \cdot \sin 2x}{2} + \frac{\cos 2x}{4} + c$ ✓✓

(8) T3

8.4 $\int \cos 5x \cos 8x\,dx = \frac{1}{2}\int \cos(-3x) + \cos 13x\,dx$ ✓

$= \frac{1}{2}\int \cos 3x + \cos 13x\,dx$ ✓✓

$= \frac{1}{2}\left(\frac{\sin 3x}{3}\text{ ✓} + \frac{\sin 13x}{13}\text{ ✓}\right) + c$

$= \frac{\sin 3x}{6} + \frac{\sin 13x}{26} + c$ ✓✓

(7) T2

[27]

QUESTION 9

9.1 $x = 3$ ✓✓

$$f(x) = \frac{x^2 - x - 2}{x - 3}$$ ✓✓

$$\begin{array}{r|l} & x + 2 \\ \hline x - 3 & x^2 - x - 2 \\ & x^2 - 3x \\ \hline & 2x - 2 \\ & 2x - 6 \\ \hline & 4 \end{array}$$ ✓✓

$$f(x) = \frac{4}{x-3} + x + 2$$

$y = x + 2$ ✓✓

9.2 $$f'(x) = \frac{(2x-1)(x-3) - 1(x^2 - x - 2)}{(x-3)^2}$$ ✓ ✓ ✓

(8) T2

$$\frac{2x^2 - 7x + 3 - x^2 + x + 2}{(x-3)^2} = 0$$ ✓ ✓

$x^2 - 6x + 5 = 0$ ✓

$(x - 5)(x - 1) = 0$ ✓✓

$x = 5$ ✓ or $x = 1$ ✓

(10) T3

9.3 The one part of the graph is always concave down and the other part concave up. ✓

BUT There is no point on the graph where the concavity changes. ✓ Concavity only changes AFTER the asymptote.

(2) T1

[20]

QUESTION 10

$$F = \frac{1000V}{0,006V^2 + 35}$$

$$F' = \frac{1000(0,006V^2 + 35) - 1000V(0,012V)}{(0,006V^2 + 35)^2}$$

$$\frac{6V^2 + 35\,000 - 12V^2}{(0,006V^2 + 35)^2} = 0$$

$$-6V^2 = -35\,000$$

$$\sqrt{V^2} = \sqrt{\frac{17\,500}{3}}$$

$$V = 76,38 \text{ km/h}$$

[12] T4

QUESTION 11

11.1 $y^2 = x$

$$\left(\frac{1}{x}\right)^2 = x$$

$$\frac{1}{x^2} = x$$

$$1 = x^3$$

$x = 1$ and $y = 1$

$A(1 ; 1)$

(4) T2

11.2 $$V = \pi \int_0^1 x\,dx + \pi \int_1^3 \left(\frac{1}{x}\right)^2 dx$$

$$= \pi \int_0^1 x\,dx + \pi \int_1^3 x^{-2}\,dx$$

$$= \pi \left[\frac{x^2}{2}\right]_0^1 + \pi \left[\frac{-1}{x}\right]_1^3$$

$$= \pi\left(\frac{1}{2} - 0\right) + \pi\left(\frac{-1}{3} - \frac{-1}{1}\right)$$

$$= \frac{7}{6}\pi \text{ units}^3$$

(11) T3

[15]

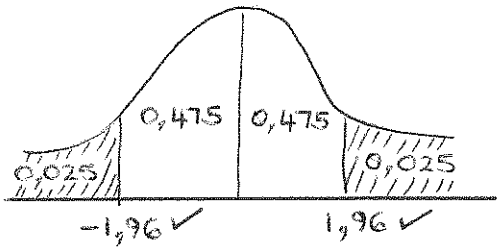
QUESTION 12

12.1 $\bar{x} = \frac{1325}{8}$ ✓

$= 165,625$ ✓

Let $H_0 : \mu = 162$ ✓

$H_1 : \mu \neq 162$ ✓



$z = \frac{165,625 - 162}{\frac{6,35}{\sqrt{8}}}$ ✓

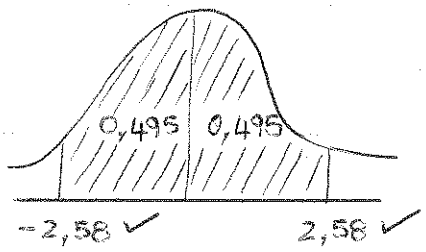
$= 1,6147$ ✓

1,6147 lies within the acceptance region ✓

∴ We accept H_0 and reject H_1 and we say at the 5% significance level that the mean height of women is 162 m. ✓

(10) T3

12.2



$\pm 2,58 = \frac{165,625 - \mu}{\frac{6,35}{\sqrt{8}}}$ ✓

$\mu = 159,8327$ ✓✓ or $\mu = 171,4173$ ✓✓

∴ The 99% confidence interval is $(159,8327 ; 171,4173)$ ✓

(8) T2

[18]

QUESTION 13

13.1 $66 = \frac{990}{n}$ ✓

$n = 15$ ✓

(2) T1

13.2 $\bar{y} = \frac{982}{15}$ ✓

$\bar{y} = 65,4667$ ✓

(2) T1

13.3 $b = \frac{(15)(67680) - (990)(982)}{(15)(68900) - (990)^2}$ ✓✓

$= 0,8056$ ✓✓

$(66 ; 65,4667)$ lies on the line

$y = a + 0,8056x$

$65,4667 = a + 0,8056(66)$ ✓✓

$a = 12,2971$ ✓✓

(8) T3

13.4 $y = 12,2971 + 0,8056(54)$ ✓

$= 55,7995$ ✓

(2) T1

13.5 $r = 0,8056 \times \frac{\sqrt{237,3}}{\sqrt{183,31119...}}$ ✓✓

$= 0,9167$ ✓✓

(4) T3

13.6 There is a very strong positive correlation between the variables ✓✓

(2) T1

[20]

QUESTION 14

14.1 $\binom{12}{7}(0,2)^7(0,8)^5$

$= 0,0033$

(9) T2

14.2 $1 - \dfrac{\binom{12}{0}\binom{48}{5}}{\binom{60}{5}}$ $= 1 - 0,31352...$

$= 0,6865$

(8) T3

[17]

QUESTION 15

15.1 $\int_0^1 c(x - x^3)\,dx = 1$

$\int_0^1 cx - cx^3\,dx = 1$

$\left[\dfrac{cx^2}{2} - \dfrac{cx^4}{4}\right]_0^1 = 1$

$\dfrac{c}{2} - \dfrac{c}{4} = 1$

$2c - c = 4$

$c = 4$

(8) T3

15.2 $f(x) = 4x - 4x^3$

$f'(x) = 4 - 12x^2$

$4 - 12x^2 = 0$

$x^2 = \dfrac{1}{3}$

$x = 0,5774$

(8) T4

[16]

QUESTION 16

16.1 $0,25 = \frac{P(A \cap B)}{0,24}$ ✓

$P(A \cap B) = 0,06$ ✓

(2) T2

16.2 (a) $P(A \cup B) = 0,4 + 0,24 - 0,06$ ✓

$= 0,58$ ✓

(2) T2

(b) $P(A \cap B') + P(A' \cap B)$

$= (0,4 - 0,06) + (0,24 - 0,06)$

$= 0,34$ ✓ $+ 0,18$ ✓

$= 0,52$ ✓

(4) T2

(c) $P(B \mid A) = \frac{0,06}{0,4}$ ✓

$= 0,15$ ✓

(2) T2

16.3 $P(A \cap B) = 0,06$ ✓

$P(A) \times P(B) = 0,096$ ✓

$\therefore P(A \cap B) \neq P(A) \times P(B)$ and the events are not independent. ✓

(3) T2

[13]

QUESTION 17

17.1 G B B G B B G ✓

$3! \times 4!$ ✓

OR B G B G B G B ✓

$4! \times 3!$ ✓

OR B B G G G B B ✓

$4! \times 3!$ ✓

$\therefore 3 \times 3! \times 4! \checkmark \leq 432$ ✓

(14)

(10) T4

17.2 $\binom{9}{2}\binom{7}{3}\binom{4}{4} = 1260$ ✓✓✓

(6) T3

[16]