APM Memo Gr 12 Prelims BHS 2013.

Q1

a(i) $12,6 = r\theta$ ✓Formula ✓subst

$12,6 = 40,2\theta$ ✓

$\frac{21}{67} = \theta$ ✓

4.

(ii) Area $= \frac{1}{2}\theta r^2 - \frac{1}{2}(40.2)^2 \sin\theta = 253,26 - 249,13$ ✓Formula ✓subst ✓Formula ✓subst

$= 4,13\ cm^2$ ✓

5

b.) LHS: $\tan A + \frac{1}{\tan A}$ ✓

$\frac{\tan^2 A + 1}{\tan A}$ ✓

$= \frac{\tan^2 A + 1}{\tan A} \times \frac{1}{\tan^2 A + 1}$ ✓

$= \frac{1}{\tan A}$ ✓

$= \cot A$ ✓

6

Q2.

(i) $k = 4 + \frac{45}{k}$ ✓

$k^2 - 4k - 45 = 0$. ✓

✓

$k = -5$ ✓ and $k = 9$

N/V $|2x+1| = 9$ ✓

$2x+1 = 9$ ✓ OR $2x+1 = -9$ ✓

$x = 4$ ✓ $x = -5$ ✓

8.

(ii) $\frac{x^2+1}{3-x} \leq x$ $x \neq 3$. ✓

$x^2 + 1 \leq 3x - x^2$ ✓

$2x^2 - 3x + 1 \leq 0$

$(x - \frac{1}{2})(x - 1) \leq 0$ ✓ ✓ ✓

$\frac{1}{2} \leq x \leq 1 \cup x > 3$ ✓ ✓

$(3-x)$	+	+	+	+	0	−	−	−
$(x-1)$	−	−	0	+	+	+	+	+
$(x-\frac{1}{2})$	−	0	+	+	+	+	+	+
		$\frac{1}{2}$		1		3		
	+	0	−	0	+	✕	−	

8.

(iii) $e^{2x} - 2e^x - 15 = 0$ ✓

✓ ✓

$e^x = -3$ ✓ OR $e^x = 5$.

N/V. $\ln 5 = x$ ✓ 7

$x = 1,61$ ✓

iv. $\ln\left(\frac{a^4 \times a^2 x}{x^2}\right) = 0$ ✓

$\therefore \ln\left(\frac{a^6}{x}\right) = 0$ ✓

$1 = \frac{a^6}{x}$ ✓

$x = a^6$ ✓ 7

b.

(i) $f(i) = i^4 + i^3 + i - 1$ ✓

$= +1 + -1i + i - 1$ ✓✓

$= 0$ ✓ 4

(ii) $f(i) = 0$ therefore i is a root and

$-i$ complex conjugate $-i$

$\therefore f(x) = x^4 + x^3 + x - 1$

$= (x - i)(x + i)(\ldots)$

$= (x^2 + 1)(x^2 + x - 1)$

for $0 = (x^2 + 1)(x^2 + x - 1)$

$x = i$ or $x = -i$ or $x = \frac{1}{2}(-1 - \sqrt{5})$

or $\frac{1}{2}(\sqrt{5} - 1)$ ✓

10

c Let $n = 1$ ✓

$5 \mid 7^1 - 2^1 = 5$ ✓

Let us assume that $5 \mid 7^k - 2^k$ ✓

$\therefore$ there exist an $a \in \mathbb{Z}$

$$5a = 7^k - 2^k \checkmark$$

Now for $n = k+1$

$= 7^{k+1} - 2^{k+1}$ ✓

$= 7^k \times 7 - 2^k \times 2$

$= (5a + 2^k) 7 - 2^k \times 2$ ✓

$= 5a \times 7 + 2^k \times 7 - 2^k \times 2$

$= 7 \times 5a + 2^k (7 - 2)$ ✓

$= 7 \times 5a + 5 \times 2^k$

$= 5 (7a + 2^k)$ ✓

$\therefore 5 \mid 7^{k+1} - 2^{k+1}$ ✓

$\therefore$ By principle of MI $5 \mid 7^n - 2^n \ \forall \ n \in \mathbb{N}$. ✓

1/4

Q3.

a) $$f(x) = \begin{cases} x-3 & x \geq 3 \\ -x+3 & x < 3 \\ x^{1/3} & x \leq 1 \end{cases}$$

$$f'(x) = \begin{cases} 1 \checkmark & x \geq 3 \checkmark \\ -1 \checkmark & x < 3 \checkmark \\ \frac{1}{3}x^{-2/3} & x \leq 1 \end{cases}$$

$f'(2) = -1$ ✓

$f'(0)$ does ~~not~~ exist. ✓ 6.

b) ~~x = 0~~ $x = 1$ ✓ jump discontinuity. ✓ 2.

c) at $x = 0$

$\lim_{x \to 0} f(x) = 0$, continuous

but $f'(0)$ does not exist

∴ Not differentiable at $x = 0$ ✓

at $x = 1$

jump discontinuity ✓

∴ $f(1)$ is not differentiable. ✓

at $x = 3$ ✓

$\lim_{x \to 3^+} f'(x) = -1 \neq \lim_{x \to 3^-} f'(x) = 1$ ✓

∴ $f'(3)$ is not differentiable. ✓

Q4

a. $\lim_{x \to 1} \frac{(3^x - 3)(3^x + 3)}{3^x - 3}$

$= 6.$ ✓✓

4

b. $\lim_{x \to 0} \frac{4}{x \cot 2x.}$

$\lim_{x \to 0} \frac{4 \tan 2x}{x.}$

$\lim_{x \to 0} \frac{2 \times 4 \tan 2x}{2x}$ ✓

$\lim_{x \to 0} 8$ ✓ $= 8$ ✓

5

c. $2x + y + xy' = 3 + 2y \times y'$

$xy' - 2yy' = 3 - 2x - y.$

$y'(x - 2y) = 3 - 2x - y$

$y' = \frac{3 - 2x - y}{x - 2y}$ ✓

10.

$y'(1,1) = 0.\dot{4}3$ ✓

① 5.

$$= \frac{1}{2} - \frac{\cos 4x}{2}$$

$$= \frac{1}{2}\left[1 - \cos 4x\right].$$

$$= \frac{1}{2}\left[1 - (1 - 2\sin^2 2x)\right]$$

$$= \frac{1}{2}\left(+2\sin^2 2x\right)$$

$$= \sin^2 2x$$

8.

Q6.

a. $f(x) = 3x - x^3 + c$

$f(0) = 4$

$\therefore c = 4$

$\therefore f(x) = -x^3 + 3x + 4$

4

b. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ ✓ Formula.

$$= 2.5 - \frac{-(2.5)^3 + 3(2.5) + 4}{3 - 3(2.5)^2}$$

$x_0 = 2.5$

$x_1 = \frac{47}{21}$

$\vdots$

$x_5 = 2.195$

6

Q 7

$2x + 2y = 18$ ✓

$\therefore y = \frac{18 - 2x}{2}$

$= 9 - x$ ✓

$V = x^2 y$ ✓

$V(x) = x^2(9 - x)$

$V(x) = 9x^2 - x^3$ ✓

$V'(x) = 18x - 3x^2$ ✓ ✓

Optimal points

$18x - 3x^2 = 0$ ✓

$x(18 - 3x) = 0$

$x = 0$ or $x = 6$. ✓ ✓

~~$V'(x)$~~

$V(0)$ AND $V(6)$

$= 0$ ✓ $= 108$ units^3 ✓ ✓

$\therefore$ Max Volume at $x = 6$ with 108 units^3.

[12]

Q 8.

a) $\int \sin 4x \cos 3x \, dx$ ✓ Formula. ✓ subst.

$= \frac{1}{2} \int \left(\sin(4x - 3x) + \sin(4x + 3x) \right) dx.$ ✓

$= \frac{1}{2} \int (\sin x + \sin 7x) \, dx$ ✓ ✓

$= \frac{1}{2} \left[-\cos x - \frac{\cos 7x}{7} \right] dx.$ ✓ ✓ ✓

$= -\frac{1}{2} \cos x + \frac{\cos 7x}{14} + C$ ✓ 10.

b) $\int \frac{4x}{\sqrt{4 - x^2}} \, dx.$

$=$ let $4 - x^2 = u$ ✓

$-2x = \frac{du}{dx}$

$-2x \, dx = du$ ✓

subst.

$-2 \int \frac{1}{\sqrt{u}} \, du$ ✓

$= -2 \int u^{-1/2} \, du$ ✓

$= -2 \times \frac{2}{1} u^{1/2}$ ✓ ✓

$= -4 u^{1/2}$ ✓ 8.

$= -4 \sqrt{4 - x^2}$ ✓ $+C$ ✓

c. $= \int_{-1}^{2} x^{-3} + 1 \, dx.$

$= \left[\frac{x^{-2}}{-2} + x \right]_{-1}^{2}$

$= \left[-\frac{1}{2x^2} + x \right]_{-1}^{2}$

$= \frac{7}{4} - \left(-\frac{5}{2}\right) = \frac{15}{8} - \left(-\frac{5}{2}\right)$

$= \frac{13}{4} = \frac{27}{8}$ 8.

$= 3,375.$

Q 9

$$\frac{x+2}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \quad \checkmark$$

$$\therefore \quad x+2 = Ax(x+1) + B(x+1) + x^2C$$

$$\therefore \quad x+2 = Ax^2 + Ax + Bx + B + x^2C$$

$$x+2 = (A+C)x^2 + (A+B)x + B \quad \checkmark$$

$$\therefore \quad B = 2 \quad \checkmark$$
$$A + B = 1 \quad \checkmark$$
$$A + C = 0 \quad \checkmark$$

$$\therefore \quad A = -1 \quad \checkmark$$
$$\therefore \quad C = 1 \quad \checkmark$$

$$\frac{x+2}{x^2(x+1)} = \frac{-1}{x} + \frac{2}{x^2} + \frac{1}{x+1}$$

7

Q10

a. $x^{1/3} = \frac{1}{4}x$. ✓

$4 = x^{2/3}$ ✓

$4^{3/2} = x$. ✓

$(2^2)^{3/2} = x$.

$\pm 8 = x$ and $x = 0$. ✓ ✓ ✓

$x = -8$ N/V

$\therefore A(0;0)$ and $B(8;2)$. ✓ ✓ ✓ ✓ 10

b. $x = y^3$ ✓

$x = 4y$ ✓ 4

c. $V = \pi \int_0^2 (y^3)^2 dy - \pi \int_0^2 (4y)^2 dy$. ✓ ✓ ✓ ✓

$= \pi \int_0^2 y^6 dy - \pi \int_0^2 16y^2 dy$. ✓ ✓

$= \pi \left[\frac{y^7}{7}\right]_0^2 - \pi \left[\frac{16}{3}y^3\right]_0^2$ ✓ ✓ ✓

$= \frac{128}{7}\pi - \frac{128}{3}\pi$ ✓ ✓

$= -\frac{512}{21}\pi$ ✓

$\therefore$ Volume $= \frac{512}{21}\pi$ units^3 $\simeq 24,38$ units^3 ✓ 16

30

FINANCE

Q1. $i = \frac{11\%}{4}$ $\quad 1+i = \frac{411}{400}$

1.1 $20\,000\,P_1(1+i)^{4\times 7} = 20\,000$ ✓

For 11 year old: $P_2(1+i)^{4\times 14} = 25\,000$ ✓

For 10 year old: $P_3(1+i)^{4\times 4} = 20\,000$ ✓

$P_4(1+i)^{4\times 11} = 25\,000$.

$P_1 = 9357,045479$

$P_2 = 5472,143756$

$P_3 = 12\,957,148477$

$P_4 = 7577,73589$

7

She will have to invest R35364,41 ✓ →D.

1.2 $35364,41(1+i)^{14} = 90\,000$ ✓ ✓

$(1+i)^{14} = 2,5449$

$1+i = 1,06899$

$i = 0,06899$

$\therefore 6,89\%$ p.a. ✓ 3

1.3 $85\,000 = 120\,000(1-i)^4$ ✓ ✓ ✓

$\sqrt[4]{\frac{17}{24}} = 1-i$ ✓

$i = 0,082598$ ✓

$\therefore i = 8,26\%$ 5

2010 Jan

2011 Jan

2012 Jan

2013 Jan

15

Q2

2.1 $A = 85000\left(1 + \frac{21\%}{12}\right)^{3}$

$= 89541,05$

$P_V = 89541,05$

$89541,05 = 2000\left(\frac{1-\left(1+\frac{21\%}{12}\right)^{-n}}{21\%/12}\right)$

$0,21651 = \left(1+\frac{21\%}{12}\right)^{-n}$

$\log_{\left(1+\frac{21\%}{12}\right)} 0,21651 = -n$

$-88,1966 = -n.$

$n = 88,19$

$\therefore$ 89 months.

14

2.2 $89541,05 = 2000\left(\frac{1-\left(1+\frac{21\%}{12}\right)^{-88}}{21\%/12}\right) + y\left(1+\frac{21\%}{12}\right)^{-89}$

$84,5611 = y\left(1+\frac{21\%}{12}\right)^{-89}$

$y = 396,04$

8

2.3

2.3.1

$$BO = 85000\left(1+\frac{21\%}{12}\right)^{3\times 12} - \frac{2000\left(\left(1+\frac{21\%}{12}\right)^{36}-1\right)}{21\%/12}$$

$BO = 70420,79$

with payment.

10

new $BO = 10420,79$

2.3.2

$$10420,79 = x\left(\frac{1-\left(1+\frac{21\%}{12}\right)^{-(89-36)}}{\frac{21\%}{12}}\right)$$

$x = 307,30$

5

37

1.1 $P = 20000\left(1+\frac{0,11}{4}\right)^{-16} + 20000\left(1+\frac{0,11}{4}\right)^{-28} + 25000\left(1+\frac{0,11}{4}\right)^{-44}$
$+ 25000\left(1+\frac{0,11}{4}\right)^{-56}$.

$= R\,35364,41$

1.2 $\left(1+\frac{0,11}{4}\right)^{4} = 1+i$

$i = 11,46\%$. ? ✓ 3

1.3 $85000 = 120000(1-i)^{3}$

$i = 10,86\%$. ✓ 5

2.1 35000. monthly payments 2000 21%.

$85000\left(1+\frac{0,21}{12}\right)^{2} = 85001,03$

$35001,0125 = \frac{2000(1-(1+i)^{-n})}{i}$

$1543,01804 = 2000\left(1-\left(1+\frac{0,21}{12}\right)^{-n}\right)$

$0,771 = 1-\left(1+\frac{0,21}{12}\right)^{-n}$

$(1,0175)^{-n} = 0,2289$

$-n = -84,72$

$n = 85$ months. +2

87 months