SINGULI OMNES

CORNWALL HILL COLLEGE

Mathematics Department

EXAMINATIONS – 22 JULY 2013

Mathematics – Paper 1

Grade: 12

Time: 3 Hours

Marks: 150

Examiner: Mrs. S. Hickling

Moderator(s): Mrs. M. van Niekerk, Mrs. T. Knoetze and Mr. P. van Schalkwyk

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Instructions:

1. The question paper consists of 10 pages and 14 Questions. Please check that your paper is complete.
2. Read the questions carefully.
3. It is in your own interest to write legibly and to present your work neatly.
4. All the necessary working details must be clearly shown.
5. Approved calculators may be used except where otherwise stated.
6. Answers to be rounded off to **2 decimal digits** where necessary.
7. *GOOD LUCK!*

SECTION A [77 MARKS]

QUESTION 1 [31]

a) Solve for x (rounding off to 2 decimal places if necessary):

(1) $3x^2 - 9x = 2$ (4)

(2) $\sqrt{3x-2} = x - 2$ (5)

(3) $x(x - 3) \leq 28$ (4)

(4) $\log_x 3^3 + \log_x 4 - \log_x 3 = 2$ (5)

(5) $\dfrac{x}{x-2} = \dfrac{1}{x-3} - \dfrac{2}{2-x}$ (6)

b) (1) Given that $2^x . 4^{y-1} = 32$, show that $x + 2y = 7$. (3)

(2) Hence, find the values of x and y if

$3x - 2y = 5$ and $x + 2y = 7$. (4)

QUESTION 2 [12]

a) Find the derivative from first principles of $f(x) = 3x^2 - \dfrac{1}{2}$ (5)

b) Use the rules of differentiation to determine $g^{\prime}(x)$, leaving all answers with positive exponents, if:

(1) $g(x) = \dfrac{x^2}{2} + \dfrac{2}{x^2}$ (3)

(2) $g(x) = \dfrac{3x+6}{\sqrt{x}}$ (4)

QUESTION 3 **[4]**

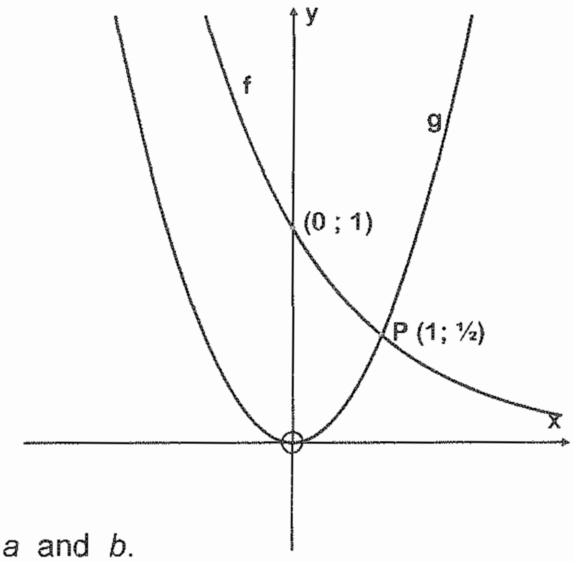
Given $f(x) = \dfrac{5}{x+1} - 2$

a) Write down the equations of the asymptotes of f. (2)

b) Write down the equations of the axes of symmetry of f. (2)

QUESTION 4 **[8]**

The diagram below shows the graphs of $f(x) = a^x$ and $g(x) = bx^2$. The point $P\left(1; \dfrac{1}{2}\right)$ is the point of intersection of f and g.



a) Calculate the values of a and b. (2)

b) Explain why the inverse of g is not a function. (1)

c) Write down two ways in which the domain of g could be restricted in order that g^{-1} is a function. (2)

d) Determine f^{-1}, the inverse of f, in the form $y = \ldots\ldots$ (1)

e) What is the defining equation of h if h is the reflection of f in the y-axis. (1)

f) What is the defining equation of k if k is the reflection of f^{-1} in the x-axis. (1)

QUESTION 5 **[9]**

a) Given: $-5 - 1 + 3 + 7 + \ldots + 439$

(1) Find T_n. (2)

(2) Determine S_n. (2)

b) The second term of a geometric sequence is $\frac{1}{10}$, the fifth term is $\frac{27}{1250}$.

Find the n^{th} term. (5)

QUESTION 6 **[13]**

Mrs Van buys a van today for R165 000 cash. She decides that she will sell it in exactly four years' time and put the money towards the purchase of another van of the same model. She calculates that the van will depreciate on a reducing balance at 18% p.a compounded annually and she will use the money from the sale of this van towards the cost of her new van. She anticipates that the inflation rate will be 14% pa compounded annually.

a) How much will her van be worth at the end of four years? (3)

b) What will be the price of the new van of the same model be in 4 years' time? (3)

c) She decides to invest a set amount of money each year in a sinking fund to cover the balance of the purchase price, so that by the end of the four years she will have sufficient funds to buy her new van for cash. The sinking fund pays interest at the rate 12,5% pa, compounded monthly. If she invest R30 000 at the beginning of each year, starting immediately, will she have enough money to purchase the new van?(4)

d) Find the nominal interest rate p.a. compounded semi-annually for an investment at 9 % p.a. effective. (3)

SECTION B **[73 MARKS]**

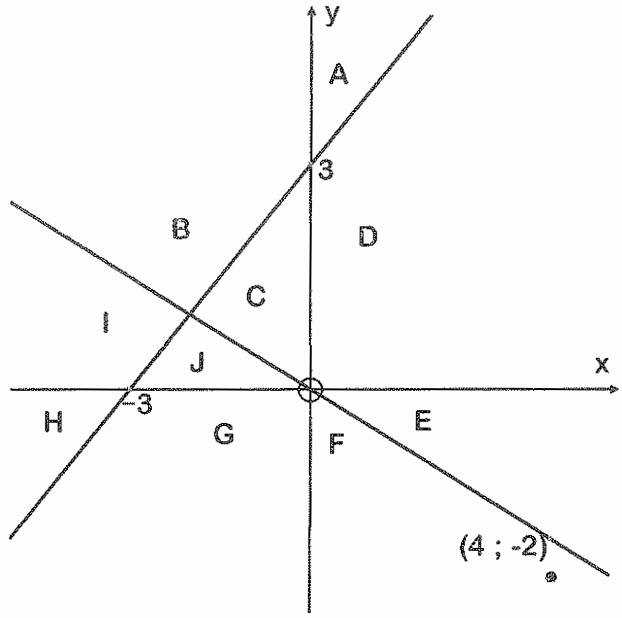
QUESTION 7 **[3]**

Given the constraints below, which of the regions A to J in the diagram, represents the following feasible region?

$y \geq 0$

$y \leq x + 3$

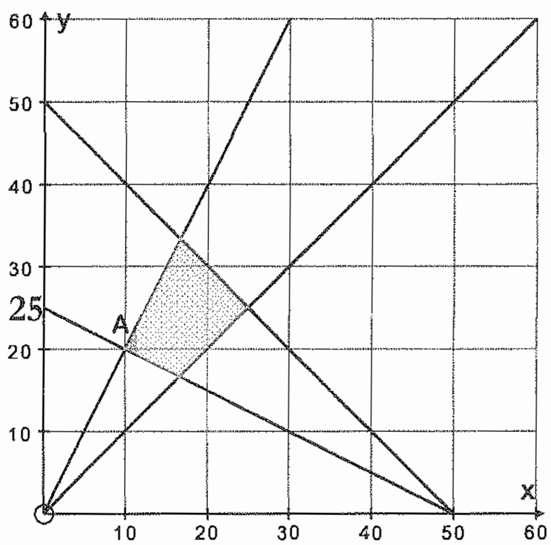
$y \leq \frac{-1}{2}x$



QUESTION 8 **[9]**

The feasible region for a linear programming problem is shown in the diagram, where $x > 0$ and $y > 0$.

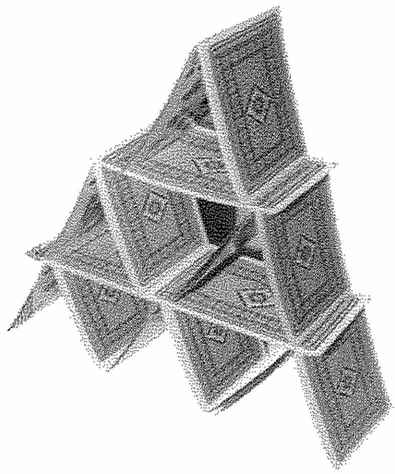
Another constraint is $y \leq 2x$.



a) Write down the further three constraints which define the region. (6)

b) An objective function for this problem is $P = sx + ty$ where s and t are positive integers. Write down the smallest values of s and t so that P is minimised at A (10 ; 20). (3)

QUESTION 9 **[9]**



This picture shows a tower of cards in a 3 storey triangular pattern.

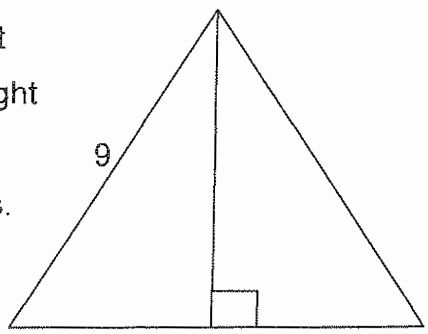
a) Find the 4th term, using the table below:

Number of storeys of the tower	1	2	3	4
Number of cards used	2	7	15	

(1)

b) Following the pattern in the picture, what is the most number of storeys for a tower built from one 52 card pack? (1)

c) Given that a playing card is 9cm long and that each triangle is equilateral, determine the height of the triangular tower four storeys high, round your answer off to three decimal places. (3)



d) How many 52 card packs are needed to build a tower 12 storeys high? (4)

QUESTION 10 **[14]**

a) Sam is training for a fun run by running every week for 26 weeks. She runs 3 km in the first week and each week after that she runs 2 km more than the previous week, until she reaches 25 km in a week. **She then continues to run 25 km each week**.

(1) How far does Sam run in the 9^{th} week? (1)

(2) In which week does she first run 25 km? (2)

(3) What is the total distance that Sam runs in 26 weeks? (3)

b) Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First Year	Second Year	Third Year	Fourth Year
Tree Height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$\frac{128}{9}$

(1) Determine **the increase** in the height of the tree during the seventeenth year. (2)

(2) Calculate the height of the tree after 10 years. (3)

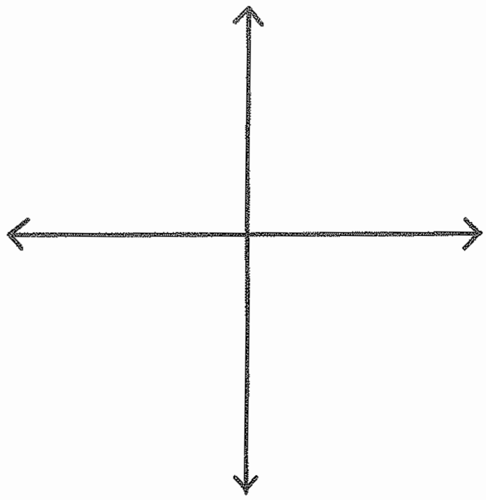
(3) Show that the tree will never reach a height of more than 312 cm. (3)

QUESTION 11 [6]

For each question below draw a sketch graph of the curve which satisfies the conditions specified.

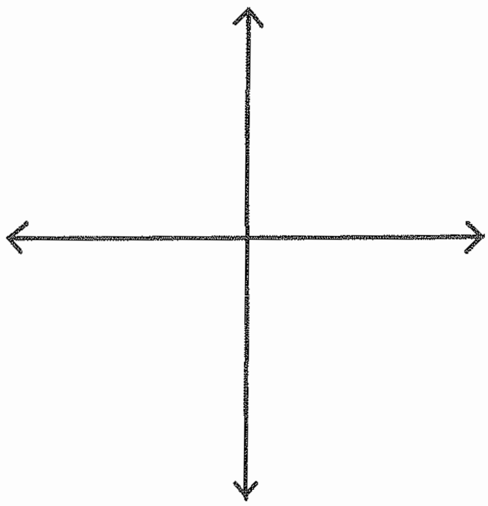
Where possible, indicate the cuts on the x - *axis*; the equation of the axis of symmetry and/or the coordinates of the turning point.

a) $f(x) = ax^2 + bx + c$; $f(x) \geq 0$ when $-5 \leq x \leq 1$ and $a < 0$



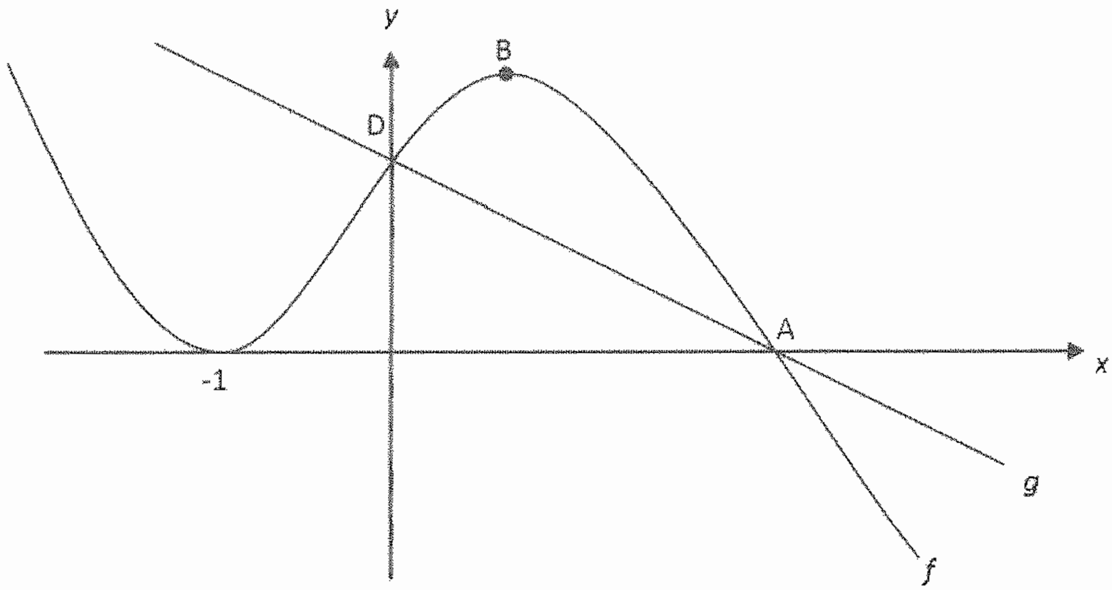
(3)

b) $h(x) = ax^2 + bx$; $h^{\prime}(-1) = 0$; $h(-1) = 5$; $h^{\prime}(x) < 0$ for all $x > -1$.



(3)

QUESTION 12 [15]



The graph represents the functions $f(x) = ax^3 + bx + 2$ and $g(x) = -x + 2$.
The x-axis is a tangent to f. D is a common y-intercept of the two graphs.

a) Determine the coordinates of point A. (2)

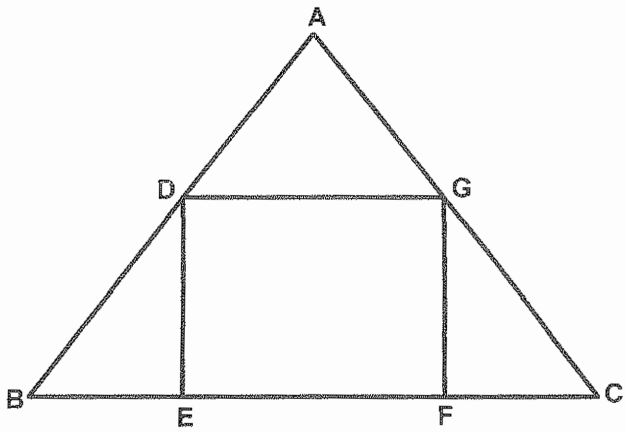
b) Show that $a = -1$ and $b = 3$. (4)

c) Find the coordinates of B, the local maximum turning point of f. (3)

d) Determine the equation of a tangent to f at point A. (4)

e) For which values of x is $f'(x) \cdot g'(x) < 0$? (2)

QUESTION 13 **[8]**



In the diagram Δ ABC is an **equilateral triangle** with sides equal to p units.

DEFG is a rectangle with BE = FC = x units

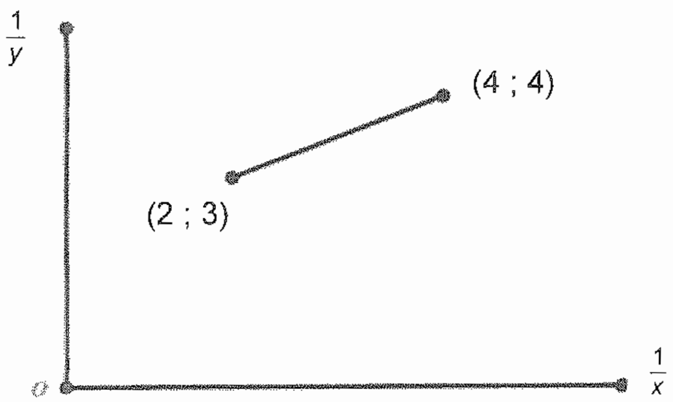
a) Show that the area of the rectangle is $A = \sqrt{3}\,x\,(p - 2x)$ units^2. (4)

b) Determine, **in terms of** p, the maximum area of the rectangle. (4)

QUESTION 14 **[9]**

a) Find the value of a, $a > 0$, where $f(x) = ax + 3$ and $f(f(2)) - 3a = 4$. (4)

b) The diagram below shows part of a straight line obtained by plotting $\frac{1}{y}$ against $\frac{1}{x}$, together with the coordinates of two of the points on the line.
Express y in terms of x. (5)



Gr 12 - July 2013 (Paper 1)

Question 1 [31]

a) (1) $3x^2 - 9x = 2$

$3x^2 - 9x - 2 = 0$

$x = \dfrac{9 \pm \sqrt{105}}{6}$

$= 3,21 \quad \text{or} \quad -0,21$ (4)

(2) $\sqrt{3x-2} = x - 2$

$3x - 2 = x^2 - 4x + 4$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x = 6 \quad \text{or} \quad x = 1$ (5)
NA.

(3) $x(x-3) \leq 28$

$x^2 - 3x - 28 \leq 0$

$(x-7)(x+4) \leq 0$

$+ \quad -4 \quad - \quad 7 \quad +$

$\therefore -4 \leq x \leq 7$ (4)

(4) $\log_x 3^3 + \log_x 4 - \log_x 3 = 2$

$\log_x (3^3 \times 4 \div 3) = 2 \log_x x$

$\log_x 36 = \log_x x^2$

$\therefore x^2 = 36$

$x = \pm 6$ (5)

but $x > 0 \quad \therefore x = 6.$

(5) $\frac{x}{x-2} = \frac{1}{x-3} - \frac{2}{2-x}$

$\frac{x}{x-2} = \frac{1}{x-3} + \frac{2}{x-2}$

LCD: $(x-2)(x-3)$ Rest: $x \neq 2, 3$

$x(x-3) = x-2 + 2(x-3)$

$x^2 - 3x = x - 2 + 2x - 6$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4$ or $x = 2$

NA.

(6)

b) (1) $2^x \cdot 4^{y-1} = 32$

$2^x \times 2^{2y-2} = 2^5$

$\therefore x + 2y - 2 = 5$

$x + 2y = 7$

(3)

(2) $3x - 2y = 5$ ——— ①

$x + 2y = 7$ ——— ②

From ②: $x = 7 - 2y$

Subs into ①: $3(7-2y) - 2y = 5$

$21 - 6y - 2y = 5$

$16 = 8y$

$2 = y$

$\therefore x = 7 - 2(2)$

$= 3$

$\therefore (3; 2)$

(4)

$3x - 2y = 5$ ——— ①

$x + 2y = 7$ ——— ②

$4x = 12$

$x = 3$

$x + 2y = 7$

$3 + 2y = 7$

$2y = 4$

$y = 2.$

Question 2 [12]

a) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - \frac{1}{2} - (3x^2 - \frac{1}{2})}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(6x + 3h)}{h}$$

$$= \lim_{h\to 0} 6x + 3h$$

$$= 6x.$$ (5)

b) (1) $$g(x) = \frac{x^2}{2} + \frac{2}{x^2}$$

$$= \frac{x^2}{2} + 2x^{-2}$$

$$g'(x) = x - 4x^{-3}$$

$$= x - \frac{4}{x^3}$$ (3)

(2) $$g(x) = \frac{3x + 6}{\sqrt{x}}$$

$$= \frac{3x}{x^{1/2}} + \frac{6}{x^{1/2}}$$

$$= 3x^{1/2} + 6x^{-1/2}$$

$$g'(x) = \frac{3}{2}x^{-1/2} - 3x^{-3/2}$$

$$= \frac{3}{2x^{1/2}} - \frac{3}{x^{3/2}}$$ (4)

$$= \frac{3}{2\sqrt{x}} - \frac{3}{\sqrt{x^3}}$$

Question 3 [4]

a) $x = -1$
$y = -2$

b) $y = x - 1$
$y = -x - 3$

Question 4 [8]

a) Subs $(1; \frac{1}{2})$ in:

$$y = a^x$$
$$\tfrac{1}{2} = a^1$$
$$\therefore y = (\tfrac{1}{2})^x$$

$$y = bx^2$$
$$\tfrac{1}{2} = b(1)^2$$
$$\tfrac{1}{2} = b$$
$$y = \tfrac{1}{2}x^2$$

(2)

b) g is a many-to-one function and inverse would be one-to-many which is not a function. (1)

c) $x \geq 0$ or $x \leq 0$ (2)

d) $y = \log_{\frac{1}{2}} x$ (1)

e) $y = (\tfrac{1}{2})^{-x}$
$= 2^x$ (1)

f) $y = -\log_{\frac{1}{2}} x$ (1)
$y = \log_2 x$

Question 5 [9]

a) $-5 - 1 + 3 + 7 + \ldots + 439$

(1) $T_n = a + (n-1)d$
$= -5 + (n-1)(4)$
$= 4n - 9$

(2)

(2) $439 = 4n - 9$
$448 = 4n$
$112 = n$

$S_n = \frac{n}{2}(a + l)$
$S_{112} = \frac{112}{2}(-5 + 439)$
$= 24304.$

(2)

b) $T_2 = ar = \frac{1}{10}$ $\qquad$ $T_5 = ar^4 = \frac{27}{1250}$

$$\frac{ar^4}{ar} = \frac{27/1250}{1/10}$$

$$r^3 = \frac{27}{125}$$

$$\therefore \quad r = \frac{3}{5}$$

$$a\left(\frac{3}{5}\right) = \frac{1}{10}$$

$$a = \frac{1}{6}$$

(5)

$$\therefore \quad T_n = \frac{1}{6}\left(\frac{3}{5}\right)^{n-1}$$

Question 6 [13]

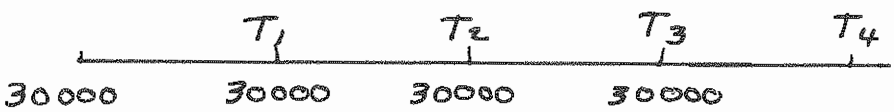
a) $A = 165\,000\,(1 - 0{,}18)^4$

$= 74\,600{,}09$ (3)

b) $A = 165\,000\,(1 + 0{,}14)^4$

$= 278\,678{,}43$ (3)

c) $R278\,678{,}43 - R74\,600{,}09 = R204\,078{,}34$



$$A = 30000\left(1 + \frac{0{,}125}{12}\right)^{12\times4} + 30000\left(1 + \frac{0{,}125}{12}\right)^{12\times3} + 30000\left(1 + \frac{0{,}125}{12}\right)^{12\times2} + 30000\left(1 + \frac{0{,}125}{12}\right)^{12}$$

$= 165\,342{,}51$

No she will not have enough money. (4)

d) $1 + 0{,}09 = \left(1 + \frac{i}{2}\right)^2$

$\sqrt{1{,}09} = 1 + \frac{i}{2}$

$2\left(\sqrt{1{,}09} - 1\right) = i$

$0{,}08806\ldots = i$ $\qquad$ (3)

$\therefore \quad r = 8{,}81\,\%$

Question 7 [3]

J

Question 8 [9]

a) $x + y \leq 50$ $\qquad y \geq -\frac{1}{2}x + 25$ $\qquad y \geq x$ (6)

b) $P = sx + ty$

$ty = -sx + P$

$y = -\frac{sx}{t} + \frac{P}{t}$

$\therefore s = 1 \qquad t = 2.$ (3)

Question 9 [9]

a) $T_4 = 26$ (1)

b) 5 (1)

c) $\text{height}^2 = 9^2 - (4{,}5)^2$

$\text{height} = \frac{9\sqrt{3}}{2}$

$\therefore$ height of 4 storeys $= \frac{9\sqrt{3}}{2} \times 4$ (3)

$= 18\sqrt{3}$

$= 31{,}177$ cm

d) 2 7 15 26

5 8 11

3 3

$2a = 3$

$a = \frac{3}{2}$

$3a + b = 5$

$b = 5 - 3(\frac{3}{2})$

$= \frac{1}{2}$

$a + b + c = 2$

$c = 2 - \frac{3}{2} - \frac{1}{2}$

$c = 0$

$\therefore \quad T_n = \frac{3}{2}n^2 + \frac{1}{2}n$

$T_{12} = \frac{3}{2}(12)^2 + \frac{1}{2}(12)$

$= 222$

(4)

$\therefore \quad \frac{222}{52} = 4,269$

$\therefore$ 5 packs of cards needed.

Question 10 [14]

a) (1) $T_9 = 3 + 8(2)$

$= 19$ km

(1)

(2) $25 = 3 + (n-1)2$

$24 = 2n$

$12 = n$

(2)

(3) $S_{12} = \frac{12}{2}[2(3) + 11(2)]$

$= 168$ km

$\therefore$ 168 km for 12 weeks

then 25×14 for next 14 weeks

(3)

$\therefore$ She runs 518 km in 26 weeks.

b) (1) $T_{17} = (18)(8/9)^{16}$

$= 2,73$

(2)

(2) $S_{10} = \dfrac{18[(8/9)^{10} - 1]}{8/9 - 1}$

$= 112,11$

(3)

$\therefore$ after 10 years the tree is $150 + 112,11 = 262,11$ cm.

(3) $S_\infty = \dfrac{18}{1 - 8/9}$

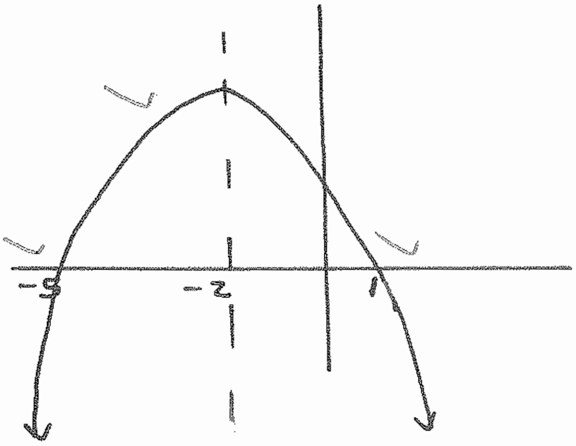
$= 162$

(3)

$\therefore$ Total height is $150 + 162 = 312$ cm.

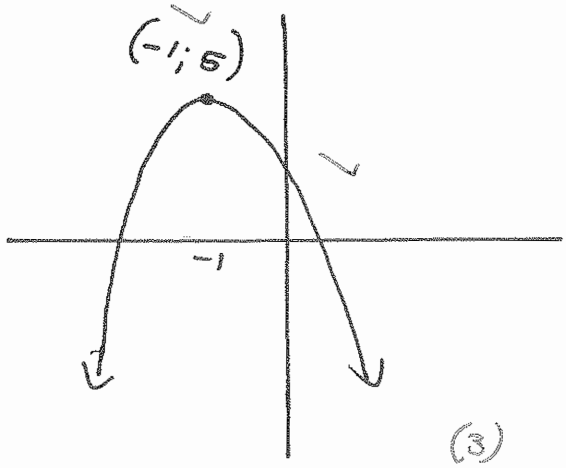
Question 11 [6]

a)



(3)

b)



(3)

Question 12 [15]

a) $-x + 2 = 0$

$x = 2$ (2)

$\therefore A(2; 0)$

b) $y = a(x - x_1)(x - x_2)(x - x_3)$

$y = a(x+1)(x+1)(x-2)$

<u>Subs (0; 2)</u>

$2 = a(0+1)(0+1)(0-2)$

$2 = -2a$

$-1 = a$

$\therefore y = -1(x+1)(x+1)(x-2)$

$y = -1(x-2)(x^2 + 2x + 1)$

$y = -1(x^3 + 2x^2 + x - 2x^2 - 4x - 2)$

$y = -x^3 + 3x + 2$ (4)

$\therefore a = -1 \qquad b = 3.$

c) $y = -1x^3 + 3x + 2$

$\therefore -3x^2 + 3 = 0$

$x^2 - 1 = 0$

$x = \pm 1$

$y = -1(1)^3 + 3(1) + 2$

$= 4$ (3)

$\therefore B(1; 4)$

d) $A(2; 0)$ $\quad m = -3x^2 + 3$

$= -3(2)^2 + 3$

$= -9$

$y - 0 = -9(x - 2)$ (4)

$y = -9x + 18.$

e) $x \in (-1; 1)$ $\quad -1 < x < 1$

(2)

Question 13 [8]

a) FG: $\tan C = \frac{FG}{FC}$ $\quad EF: p - 2x$

$\tan 60^\circ = \frac{FG}{x}$

$\therefore FG = \sqrt{3}x$

(4)

$\therefore$ Area $= \sqrt{3}x(p - 2x)$

b) $A = \sqrt{3}px - 2\sqrt{3}x^2$

$\frac{dA}{dx}$: $\sqrt{3}p - 4\sqrt{3}x = 0$

$p - 4x = 0$

$4x = p$

$x = p/4.$

$$\therefore \text{Area} = \sqrt{3}\,p\left(\frac{p}{4}\right) - 2\sqrt{3}\left(\frac{p}{4}\right)^2$$
$$= \frac{\sqrt{3}}{4}p^2 - \frac{\sqrt{3}}{8}p^2$$
$$= \frac{\sqrt{3}}{8}p^2$$

(4)

Question 14 [9]

a)
$$f(x) = ax + 3$$
$$f(2) = 2a + 3$$

$$f(f(2)) = f(2a+3) = a(2a+3) + 3$$
$$= 2a^2 + 3a + 3$$

$$\therefore f(f(2)) - 3a = 4$$
$$2a^2 + 3a + 3 - 3a = 4$$
$$2a^2 = 1$$
$$a^2 = \tfrac{1}{2}$$
$$a = \pm\sqrt{\tfrac{1}{2}}.$$

(4)

$$\therefore a = \sqrt{\tfrac{1}{2}}$$

b) Let $\frac{1}{y} = b$ and $\frac{1}{x} = a$

$$m = \frac{4-3}{4-2} = \frac{1}{2}$$

$$b - b_1 = m(a - a_1)$$
$$b - 3 = \tfrac{1}{2}(a - 2)$$
$$b = \tfrac{1}{2}a + 2$$
$$\therefore \frac{1}{y} = \tfrac{1}{2}\left(\tfrac{1}{x}\right) + 2$$
$$\frac{1}{y} = \frac{1}{2x} + 2$$
$$2x = y + 4xy$$
$$2x = y(1 + 4x)$$

(5)

$$\therefore y = \frac{2x}{1 + 4x}$$
