

Question: 1

$$(a) \sqrt{x+5} - x = 3$$

$$(\sqrt{x+5})^2 = (x+3)^2 \checkmark$$

$$x+5 = x^2+6x+9$$

$$0 = x^2+5x+4 \checkmark$$

$$0 = (x+1)(x+4) \checkmark$$

$$\underline{x = -1 \checkmark} \quad x \neq -4 \checkmark \quad (5)$$

$$(b) \log_3(x+1) = 2$$

$$x+1 = 3^2 \checkmark$$

$$\underline{x = 8 \checkmark} \quad (2)$$

$$(c) 4 \cdot 3^x - 3^{x+1} = 81$$

$$3^x(4-3) \checkmark = 81$$

$$3^x = 3^4 \checkmark$$

$$\underline{x = 4 \checkmark} \quad (3)$$

$$(d) 3x^2 - 7x = 5 \checkmark$$

$$3x^2 - 7x - 5 = 0 \checkmark$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-5)}}{2(3)} \checkmark$$

$$= \frac{7 \pm \sqrt{109}}{6} \checkmark$$

$$\underline{x = 2.91 \checkmark} \quad \text{or} \quad x = -0.57 \checkmark$$

$$(4)$$

Question: 2

$$(a) (5^{2x})^{-2} \cdot 20^{x+1} \cdot 125^{x+1}$$

$$= \frac{5^{-4x} \cdot 2^{1+2x} \cdot 5^{x+1} \cdot 2^{2x+2} \cdot 5^{3x+3}}{2^{1+2x}}$$

$$= 5^{-4x+x+1+3x+3} \cdot 2^{2x+2-1-2x} \checkmark$$

$$= 5^4 \cdot 2 \checkmark$$

$$= 1250 \checkmark \quad (5)$$

$$(b) \frac{x^2}{1+x}$$

$$= \frac{(1+\sqrt{3})^2}{1+(1+\sqrt{3})} \checkmark$$

$$= \frac{1+2\sqrt{3}+3}{2+\sqrt{3}} \checkmark$$

$$= \frac{4+2\sqrt{3}}{2+\sqrt{3}} \checkmark$$

$$= \frac{2(2+\sqrt{3})}{(2+\sqrt{3})} = 2 \checkmark$$

(5)

Question: 3

$$\begin{aligned} \text{(a)} \quad f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} \checkmark \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} \checkmark \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h - 1)}{h} \checkmark \\ &= 2x + (0) - 1 = 2x - 1 \checkmark \quad (5) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \lim_{x \rightarrow 1} \frac{x^2 - 1}{1 - x} \checkmark \\ = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{-(x-1)} = \lim_{x \rightarrow 1} -x - 1 = -2 \checkmark \quad (3) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (1) \quad g(x) &= 2x^3 - 7x^2 + 3x \\ &= \frac{x(x^2 - 7x + 3)}{x-3} \checkmark \\ &= \frac{x(2x-1)(x-3)}{x-3} \checkmark \\ &= 2x^2 - x \checkmark \\ g'(x) &= 4x - 1 \checkmark \quad (5) \end{aligned}$$

$$\begin{aligned} (2) \quad g(x) &= 2\sqrt{x} + \frac{1}{2x^2} \\ &= 2x^{\frac{1}{2}} + \frac{1}{2}x^{-2} \checkmark \\ g'(x) &= 2x^{-\frac{1}{2}} - x^{-3} \checkmark \\ &= \frac{1}{\sqrt{x}} - \frac{1}{x^3} \checkmark \quad (4) \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad f(3) &= -(3)^2 + 3(3) - 2 \\ &= -2 \checkmark \end{aligned}$$

$$y = -3x + c$$

$$\begin{aligned} f'(x) &= -2x + 3 \checkmark \\ f'(3) &= -2(3) + 3 = -3 \checkmark \end{aligned}$$

$$\begin{aligned} (-2) &= -3(3) + c \checkmark \\ c &= 7 \end{aligned}$$

$$y = -3x + 7 \checkmark \quad (5)$$

Question: 4

(a) $x \in \mathbb{R} \quad x \neq 2 \quad \checkmark$
or $(-\infty; 2) \cup (2; \infty) \quad \checkmark \quad (2)$

(b) $\frac{-4}{x-2} + 3 = -x + 2 \quad \checkmark$

$$\frac{-4}{x-2} = -x - 1$$

$$-4 = -x^2 - x + 2x + 2 \quad \checkmark$$

$$x^2 + x - 6 = 0$$

$$(x+2)(x-3) = 0 \quad \checkmark$$

$$x = -2 \quad x = 3$$

$$y = 4$$

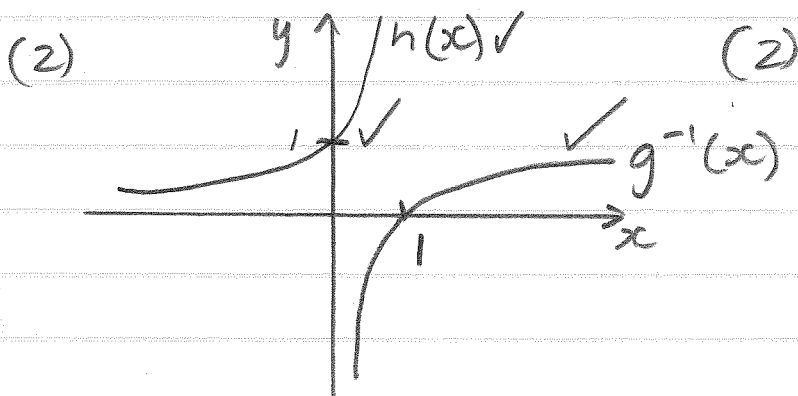
$$y = -1$$

$$(-2; 4) \quad \checkmark \quad (3; -1) \quad \checkmark \quad (5)$$

(c) $(-\infty; -2) \quad \checkmark \quad \cup \quad (2; 3) \quad \checkmark \quad (2)$

Question: 5

(a) (1) $h(x) = 4^x \quad \checkmark \quad (1)$



(b) $g^{-1}(x) = \log_4 x \quad \checkmark \quad \text{sketch} \quad \checkmark \quad (2)$

(c) $x = -3 \quad \checkmark \quad (1)$

Question: 6

$$(a) (1) r = -\frac{2}{3} \checkmark$$

$$a = 4 \checkmark$$

$$T_n = 4 \cdot \left(-\frac{2}{3}\right)^{n-1} \quad (3)$$

$$(2) S_\infty = \frac{a}{1-r}$$

$$= \frac{4}{1 - \left(-\frac{2}{3}\right)} = \frac{4}{\frac{5}{3}}$$

$$= \frac{12}{5} \checkmark \quad (2)$$

$$(b) (1) S_6 = 3(6)^2 + 2 \checkmark$$

$$= 110 \checkmark \quad (2)$$

$$(2) S_5 = 3(5)^2 + 2$$

$$= 77 \checkmark$$

$$T_6 = S_6 - S_5 \checkmark$$

$$= 110 - 77$$

$$= 33 \checkmark \quad (3)$$

$$(c) (1) 16 \text{ girls} \checkmark \quad (1)$$

$$(2) \begin{array}{cccc} 1 & 2 & 4 & 8 \\ \downarrow & \downarrow & \downarrow & \\ \times 2 & \times 2 & \times 2 & \end{array}$$

$$T_n = 2^{n-1} \checkmark$$

$$\therefore 10 \text{ days} \checkmark$$

$$2^{n-1} = 420 \checkmark$$

$$n-1 = \log_2 420$$

$$n-1 = 8,714$$

$$n = 9,714$$

$$(3)$$

Question: 7

$$200x + 150y \leq 2400 \checkmark$$

$$100x + 150y \leq 1800 \checkmark$$

$$x \geq 3 \checkmark$$

$$y \geq 4 \checkmark$$

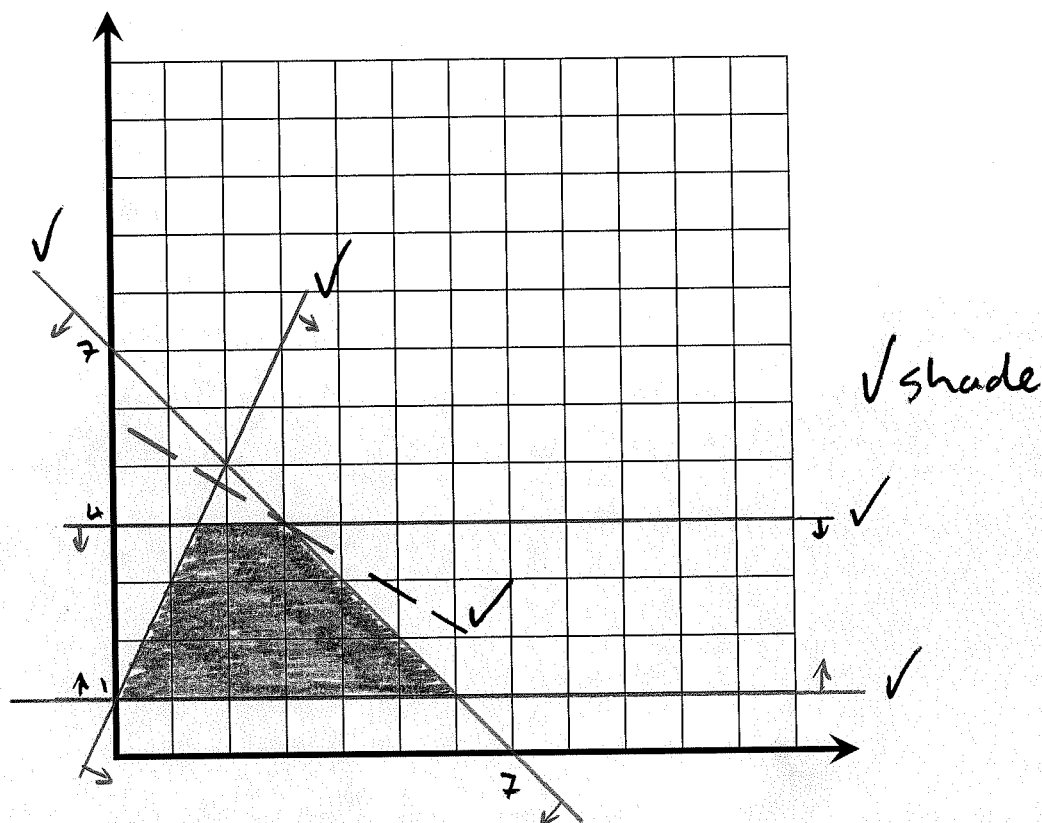
$$x, y \in \mathbb{N}$$

$$(4)$$

Question: 8

- (a) Sketch the following constraints on the axes provided, showing the feasible Region. $1 \leq y \leq 4$ $y \leq -x + 7$ $y \leq 2x + 1$

(5)



- (b) If $P = 3x + 4y$, determine the x and y values which will give a maximum value for P . (Show all working)

$$4y = -3x + P \quad \checkmark$$

$$y = -\frac{3}{4}x + \frac{P}{4} \quad \checkmark$$

$$4 = -x + 7 \quad \checkmark$$

$$x = 3 \quad \checkmark$$

$$y = 4 \quad \checkmark$$

(5)

[10]

Question: 9

$$(a) A = 18000(1 + 0,05)^{16} \\ = R392917,43 \quad (3)$$

$$(b) 400000 = x \left[\frac{\left(1 + \frac{0,085}{12}\right)^{192} - 1}{\frac{0,085}{12}} \right] \\ x = R984,63 \quad (5)$$

$$(c)(1) 400000 = 5900 \left[\frac{1 - \left(1 + \frac{0,06}{12}\right)^{-n}}{\frac{0,06}{12}} \right] \quad \checkmark \checkmark$$

$$\frac{4000}{59} \cdot \frac{0,06}{12} - 1 = -\left(1 + \frac{0,06}{12}\right)^{-n}$$

$$-\frac{39}{59} = -\left(\frac{201}{200}\right)^{-n} \quad \checkmark$$

$$-n = \log_{\frac{201}{200}} \frac{39}{59} \quad \checkmark$$

$$-n = -83,002 \quad \checkmark \quad (6)$$

$n = 83$ payments (84 payments)

$$(2) P = 5900 \left[\frac{1 - \left(1 + \frac{0,06}{12}\right)^{-59}}{\frac{0,06}{12}} \right] \quad \checkmark \quad \text{or } B0 = 400000 \left(1 + \frac{0,06}{12}\right)^{24} \\ - 5900 \left[\frac{\left(1 + \frac{0,06}{12}\right)^{24} - 1}{\frac{0,06}{12}} \right] \\ = R300806,71 \quad \checkmark \quad (4)$$

Question: 10

$$(a) g(0) = 2 \quad \checkmark$$

$$f(x) = (x+1)^2(x-2) \quad \checkmark \checkmark \\ = (x^2 + 2x + 1)(x-2) \quad \checkmark \\ = x^3 + 2x^2 + x - 2x^2 - 4x - 2 \\ = x^3 - 3x - 2 \quad \checkmark$$

(5)

$$\therefore a = -3 \quad \text{and} \quad b = -2$$

$$(b) f'(x) = 3x^2 - 3 \quad \checkmark$$

$$0 = 3(x^2 - 1) \quad \checkmark$$

$$0 = (x-1)(x+1) \quad \checkmark \\ x = 1 \quad \text{or} \quad x = -1 \quad \checkmark$$

$$f(1) = (1)^3 - 3(1) - 2$$

$$= -4$$

$$R(1; -4) \quad \checkmark \quad (4)$$

$$(c) x \in (-1; 1) \checkmark \checkmark (2)$$

$$(d) L_k = x - 2 - (x^3 - 3x - 2) \checkmark$$

$$L_k = -x^3 + 4x$$

$$\frac{dL_k}{dx} = -3x^2 + 4 \checkmark$$

$$-3x^2 + 4 = 0 \checkmark$$

$$x^2 = \frac{4}{3}$$

$$x = \pm \frac{2}{\sqrt{3}} \checkmark \text{ or } \pm \sqrt{\frac{4}{3}}$$

$$L_k = -\left(\frac{4}{3}\right)^3 + 4\left(\frac{4}{3}\right) - 2$$

$$= \frac{26}{27} \checkmark$$

(5)

Question: 11

$$(a) y = a(x-4)(x+4) \checkmark$$

$$8 = -16a$$

$$a = -\frac{1}{2} \checkmark$$

$$(b) y = -\frac{1}{2}(3)^2 + 8 \checkmark$$

$$= 3,5 \text{ m} \checkmark (2)$$

$$y = -\frac{1}{2}x^2 + 8 \checkmark (3)$$

Question: 12

$$P' = 12t^2 + 6t - 36 \checkmark$$

$$12t^2 + 6t - 36 = 0 \checkmark$$

$$2t^2 + t - 6 = 0$$

$$(2t-3)(t+2) = 0 \checkmark$$

$$t = \frac{3}{2} \quad t \neq -2$$

After $1\frac{1}{2}$ hours after taking them $\checkmark$ (4)

Question: 13

(a)(i) $1 \ 5 \ 11 \ 19$
 $\begin{array}{ccc} \frown & \frown & \frown \\ 4 & 6 & 8 \\ \frown & \frown & \\ 2 & 2 & \end{array}$

$$\begin{aligned} 2a &= 2 & 3a+b &= 4 & a+b+c &= 1 \\ a &= 1 \checkmark & b &= 1 \checkmark & 1+1+c &= 1 \\ & & & & c &= -1 \checkmark \end{aligned}$$

$$T_n = n^2 + n - 1 \checkmark \quad (4)$$

(2) $n^2 + n - 1 = 505 \checkmark$
 $n^2 + n - 506 = 0$
 $(n-22)(n+23) = 0 \checkmark$
 $n = 22 \checkmark \quad n \neq -23$

$$\underline{n=22} \checkmark \quad (4)$$

(b) $2S_8 = 8T_2 \checkmark$

$$\begin{aligned} T_9 &= T_6 + 12 \checkmark \\ a + 8d &= a + 5d + 12 \checkmark \\ 3d &= 12 \\ \underline{d=4} \checkmark \end{aligned}$$

$$\begin{aligned} 8(2a + 7(4)) &= 8(a + (4)) \checkmark \\ 16a + 224 &= 8a + 32 \\ 8a &= -192 \end{aligned}$$

$$\underline{a = -24} \checkmark$$

$$-24; -20; -16 \checkmark \quad (7)$$

(c) $\frac{m}{2} [2(2m-1) + (m-1)(2)] = 280 \checkmark$

$$m(4m-2+2m-2) = 560$$

$$6m^2 - 4m = 560 \checkmark$$

$$6m^2 - 4m - 560 = 0$$

$$3m^2 - 2m - 280 = 0$$

$$(3m+28)(m-10) = 0 \checkmark$$

$$m \neq -\frac{28}{3} \quad \text{or} \quad \underline{m=10} \checkmark$$

(6)