ST. DAVID'S MARIST INANDA

GRADE 12 MATHEMATICS
PAPER I
SEPTEMBER 2013

SET BY: MR M JEENAH	MARKS: 150
MODERATED BY: MRS L. NAGY	TIME: 3 hours

NAME: MEMO

HIGHLIGHT YOUR TEACHERS NAME:

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INSTRUCTIONS:

- ✓ This paper consists of 24 pages and a Formula sheet on page 24. ROUGH WORK can be done on page 23. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ Answers must be rounded off to two decimal places unless otherwise stated.
- ✓ It is in your interest to show all your working details.
- ✓ Work neatly. Do **NOT** answer in pencil.
- ✓ Diagrams are not drawn to scale.

SECTION A	Q1	Q2	Q3	Q4	Q5	Q6		TOTAL
MARKS	16	11	14	18	12	6		77
SECTION B	Q7	Q8	Q9	Q10	Q11	Q12		TOTAL
MARKS	11	16	11	16	12	7		73

SECTION A

Question 1

a) Solve for x:

i) $4x^{-1} + x = -5$ (3)

$\frac{4}{x} + x = -5$

$4 + x^2 + 5x = 0$

$x^2 + 5x + 4 = 0$ ✓

$(x + 4)(x + 1) = 0$

$x = -4$ ✓ $x = -1$ ✓

ii) $2^{2x+3} + 4^x = 72.2^{3x}$ (3)

✓ $2^{2x+3} + 2^{2x} = 72 \cdot 2^{3x}$

$2^{2x}(2^3 + 1) = 72 \cdot 2^{3x}$

$2^{2x}(9) = 72 \cdot 2^{3x}$

$2^x = \frac{1}{8}$ ✓

$2^x = 2^{-3}$

$x = -3$ ✓

iii) $\frac{x^2 + 2}{4 - x} \leq 0$ (2)

CV : 4

$+$ | $-$

4

$x > 4$ ✓✓

b) Solve simultaneously for x and y in the following set of equations: (5)

$2x^2 - xy - 4y^2 = 8$

$y = 5 - 2x$

✓sub

$2x^2 - x(5-2x) - 4(5-2x)^2 = 8$

$2x^2 - 5x + 2x^2 - 4(25 - 20x + 4x^2) = 8$

$-12x^2 + 75x - 108 = 0$

$12x^2 - 75x + 108 = 0$

$$x = \frac{+75 \pm \sqrt{(-75)^2 - 4(12)(108)}}{2(12)}$$

$x = 4$ ✓a $\qquad x = \frac{9}{4}$ ✓a

$y = -3$ ✓ca $\qquad y = \frac{1}{2}$ ✓ca

c) If $\log 3 = p$ and $\log 7 = q$, calculate the value of $\log 4\frac{2}{7}$ in terms of p and q. (3)

$\log \frac{30}{7}$ ✓

$\log 30 - \log 7$

$\log(10 \times 3) - \log 7$

$\log 10 + \log 3 - \log 7$ ✓

$1 + p - q$ ✓

[16]

Question 2

a) Given $f(x) = x - 3x^2$, determine $f'(3)$ **from first principles.** (5)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

✓sub

$$= \lim_{h \to 0} \frac{(x+h) - 3(x+h)^2 - (x - 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{x + h - 3x^2 - 6xh - 3h^2 - x + 3x^2}{h}$$ ✓

✓

$$= \lim_{h \to 0} \frac{h(-6x - 3h + 1)}{h}$$

$$= -6x + 1$$ ✓

$$f'(3) = -6(3) + 1$$

$$= -18 + 1$$ ✓ca

$$= -17$$

b) Writing your answers with positive exponents, find $\frac{dy}{dx}$ if:

i) $y = \left(1 - \frac{1}{2\sqrt{x}}\right)^2$ (3)

$y = 1 - \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} + \frac{1}{4x}$

$y = 1 - x^{-1/2} + \frac{1}{4}x^{-1}$. ✓a

$\frac{dy}{dx} = \frac{1}{2}x^{-3/2} - \frac{1}{4}x^{-2}$ ✓a ✓a

$= \frac{1}{2\sqrt{x^3}} - \frac{1}{4x^2}$

ii) ✓a $yx + y = 3x^2 + 4x + 1$ (3)

$y(x+1) = (3x+1)(x+1)$

$y = \frac{(3x+1)(x+1)}{(x+1)}$

$y = 3x+1$ ✓a

$\frac{dy}{dx} = 3$. ✓a

[11]

Question 3

a) Evaluate:

$$\sum_{p=1}^{20}(5p-7)$$ (2)

$S_n = \frac{n}{2}[a+l]$

$S_{20} = \frac{20}{2}[5(1)-7 + 5(20)-7]$ ✓m

$= 910$ ✓a

b) The sequence $3;9;17;27;.....$ forms a quadratic sequence.

i) Write down the next term. (1)

39 ✓a

ii) If $T_n = an^2 + bn + c$, determine the values of a, b and c. (3)

3 ; 9 ; 17 ; 27

6 8 16

2 2

$2a = 2 \rightarrow a = 1$ ✓a

$3a+b = 6 \rightarrow b = 3$ ✓a

$a+b+c = 3 \rightarrow c = -1$ ✓a

iii) Calculate the value of the first term of the sequence that is greater than 269. (4)

$T_n = n^2 + 3n - 1$ ✓m

$n^2 + 3n - 1 = 269$

$n^2 + 3n - 270 = 0$

$(n + 18)(n - 15) = 0$ ✓a

$n = 15$ ✓a

Hence: $T_{16} = (16)^2 + 3(16) - 1$

$= 303$ ✓ca

c) Three terms are in geometric progression. When 2 is subtracted from the third term, the three terms form an arithmetic progression. If the first term is 8, determine the 2nd and 3rd terms of the geometric progression if $r > 1$. (4)

$a\ ;\ ar\ ;\ ar^2$

$\downarrow$

$ar^2 - 2$

$T_2 - T_1 = T_3 - T_2$ ✓m

$ar - a = ar^2 - 2 - ar$

$8r - 8 = 8r^2 - 2 - 8r$

$8r^2 - 16r + 6 = 0$

$4r^2 - 8r + 3 = 0$ ✓a

$(2r - 1)(2r - 3) = 0$

$r = \frac{1}{2}$ $\quad r = \frac{3}{2}$ ✓a

$\therefore\ 8\ ;\ 12\ ;\ 18$ ✓ca

[14]

Question 4

a) Capitec Bank offers the following ***effective*** interest rates on a Savings Account as well as the costs involved in having this account.

Savings Account

R0 – R9 999	6.00%
R10 000 – R24 999	4.75%
R25 000 – R99 999	4.75%
R100 000 +	4.75%
Monthly administration fee	4.50

i) For a R10 000 investment, convert the effective interest rate into an annual rate compounded monthly. (3)

$$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$$

✓

$$1 + 0{,}0475 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$$

✓

$$i_{nom} = 12\left(\sqrt[12]{1{,}0475} - 1\right)$$

$$i_{nom} = 0{,}046496$$

$$r = 4{,}65\%$$ ✓

ii) Use the nominal rate to calculate how much money you will have in your account after 1 year if you invest R10 000 with Capitec Bank. Take into consideration the ***monthly*** administration fee which is deducted at the end of each month. (6) ✓m

✓a ✓a ✓a

$$A = 10\,000\left(1 + \frac{0{,}0465}{12}\right)^{12} - 4{,}5\left[\frac{\left(1 + \frac{0{,}0465}{12}\right)^{12} - 1}{\frac{0{,}0465}{12}}\right]$$

$$= 10475{,}03944 - 55{,}16587$$

$$= 10\,419{,}87$$ ✓ca.

b) Mrs Issigonis has taken out a loan of R275 000 to buy a new Mini Cooper. The loan is repayable in equal monthly instalments over 5 years at a rate of 14,75% p.a, compounded monthly.

i) Calculate her monthly repayment. (4)

$$275\,000 = x\left[\frac{1-\left(1+\frac{0,1475}{12}\right)^{-5\times 12}}{\frac{0,1475}{12}}\right] \quad ✓a \; ✓a \; ✓m$$

$$x = 6506,2 \quad ✓ca$$

ii) If she doubles her monthly repayment, how long will it take her to repay the loan? (5)

$$275\,000 = 13012,4\left[\frac{1-\left(1+\frac{0,1475}{12}\right)^{-n}}{\frac{0,1475}{12}}\right] \quad ✓ca$$

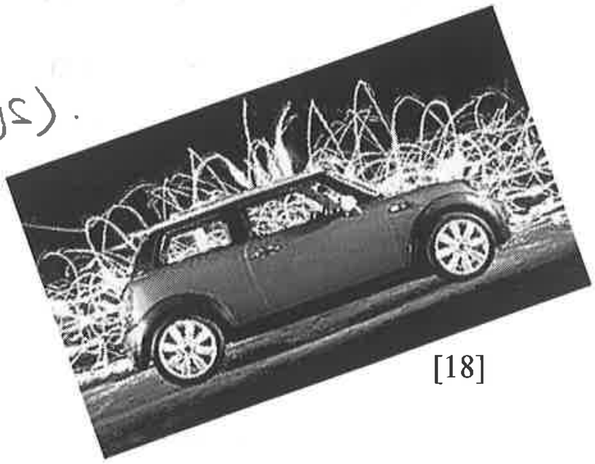
$$0,740231 = \left(\frac{4859}{4800}\right)^{-n} \quad ✓m$$

$$-n = \log_{\left(\frac{4859}{4800}\right)}(0,740231) \quad ✓m \to \text{logs}$$

$$n = 24,62 \quad ✓ca$$

25 months ✓ca

(24 months and 37days).

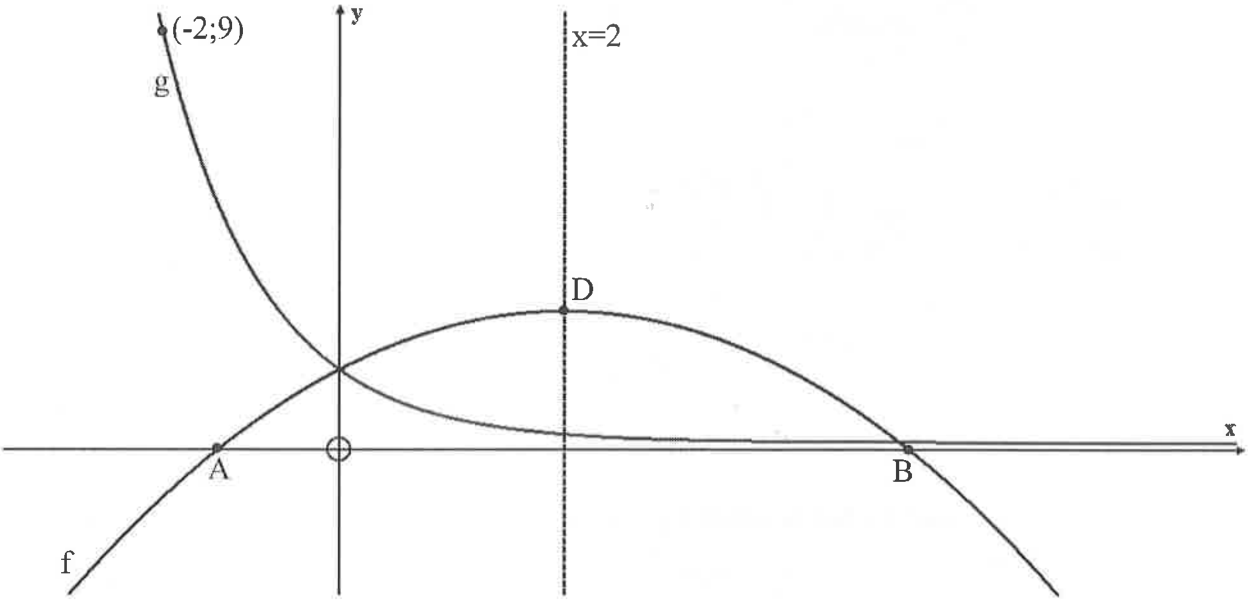


[18]

Question 5

The sketch below represents the curves of $f(x) = -\frac{1}{5}x^2 + \frac{4}{5}x + c$ and $g(x) = k^x$ if $k \neq 0$.

The point $(-2;9) \in g$, the equation of the axis of symmetry of the curve of f is $x = 2$ and the length of AB is 6 units. The curves of f and g intersect on the y-axis.



a) Determine the value of k. (3)

$(-2;9)$

$9 = k^{-2}$ ✓m

$\therefore k^2 = \frac{1}{9}$ ✓m

$k = \pm\frac{1}{3}$ ✓a

$\therefore k = \frac{1}{3}$ or $k = 3^{-1}$

b) Calculate the co-ordinates of A and B, the x-intercepts of the curve of f. (4)

$x = 2$ axis of symmetry

if AB = 6 units

$\therefore A(-1;0)$ ✓✓a $B(5;0)$ ✓✓a

c) Show that c =1. (1)

$f(0) = g(0)$

and $g(0) = 3^0$ ✓m

$g(0) = 1$

$\therefore f(0) = 1 \quad \therefore c = 1$.

d) Calculate the co-ordinates of D, the turning point of the curve of f. (2)

✓m → sub

$f(2) = -\frac{1}{5}(2)^2 + \frac{4}{5}(2) + 1$

$= \frac{9}{5}$ ✓a

$\therefore D(2; \frac{9}{5})$ or $D(2; 1\frac{4}{5})$

e) Use the graph to determine the values of p which will satisfy the following inequality:

$-\frac{1}{5}x^2 + \frac{4}{5}x + p < 0$ (2)

Shift the graph down by more than $1\frac{4}{5}$ units.

$\therefore 1 - p > 1\frac{4}{5}$

$-p > 1\frac{4}{5} - 1$

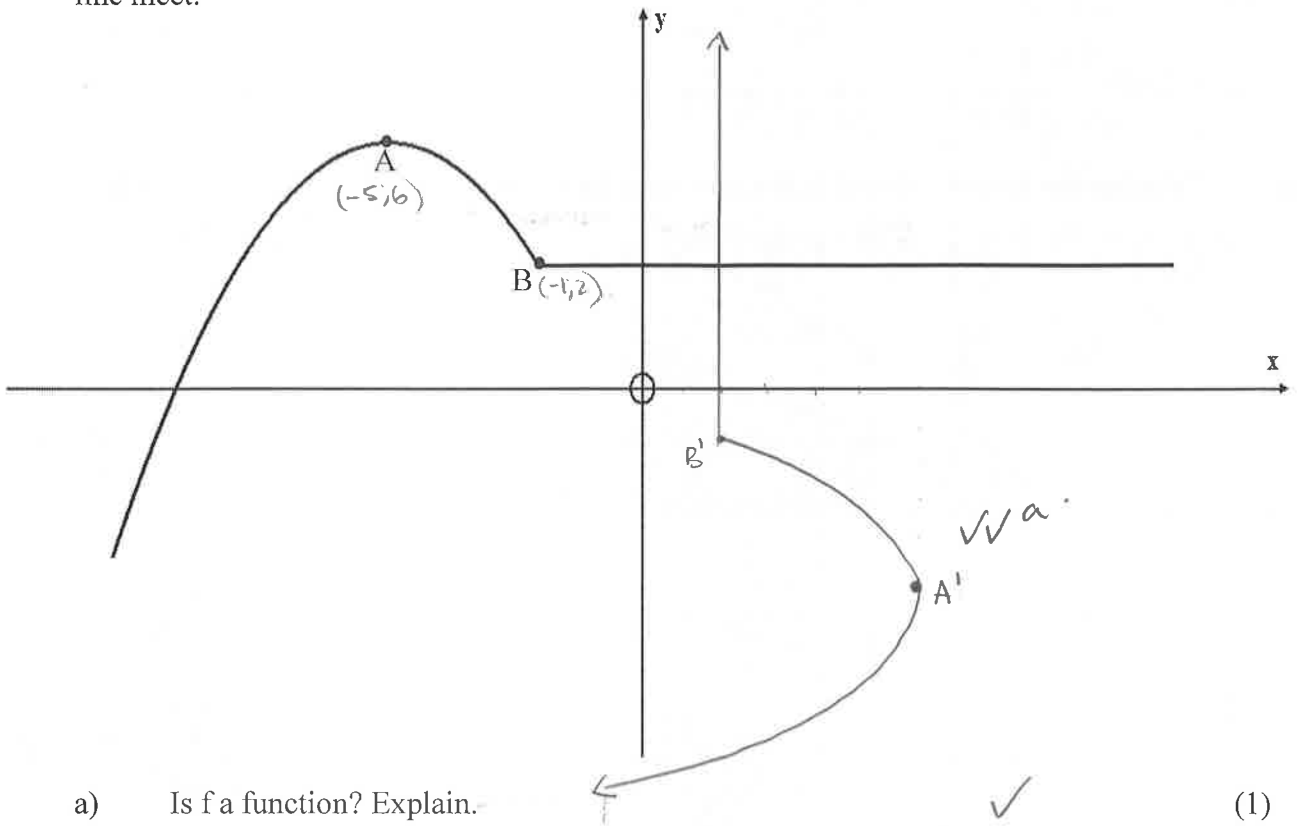
$-p > \frac{4}{5}$

$p < \frac{4}{5}$ ✓✓a

[12]

Question 6

Refer to the sketch below.
A (-5;6) is the turning point of f. B(-1;2) is the point where the parabolic curve and the straight line meet.



a) Is f a function? Explain. (1)

Yes as it passes the vertical line test. ✓
(or explaing this idea)

c) Determine the range of f. (1)

$x \leq 6$ ✓a

d) Give one possible restriction on the domain that would ensure that the inverse is a function. (1)

$x \geq -5$ or $-5 \leq x \leq -1$ ✓a

e) Determine f(3). (1)

$f(3) = 2$ ✓a

f) Draw a neat sketch of the inverse of $f(x)$ on the above axes. (2)

On graph

[6]

SECTION B

Question 7

a) Given $f(x)=ax^2+bx+c$. If $f(x)\geq -8$ for all x values, $f'(1)=0$ and $(x-3)$ is a factor of $f(x)$. Determine the values of a, b and c. (6)

$y=a(x-p)^2+q$ ✓m

$y=a(x-1)^2-8$ ✓m → sub TP As Turning Point $(1;-8)$

$0=a(3-1)^2-8$ ✓m → sub point from $f(3)=0$

$0=4a-8$

$4a=8$ ✓a

$a=2$

$y=2(x-1)^2-8$

$\therefore y=2x^2-4x-6$

✓a ✓a

$a=2 \quad b=-4 \quad c=-6$

b) Given $g(x)=2x$, find a simplified expression for:

$$g(x)+g\left(\frac{1}{x}\right)+\frac{1}{g(x)}+g^{-1}(x)$$ (5)

$2x+2\left(\frac{1}{x}\right)+\frac{1}{2x}+\frac{x}{2}$

$2x+\frac{2}{x}$ ✓a $+\frac{1}{2x}$ ✓a $+\frac{x}{2}$ ✓✓a

$=\dfrac{4x^2+4+1+x^2}{2x}$

$=\dfrac{5x^2+5}{2x}$ ✓ca

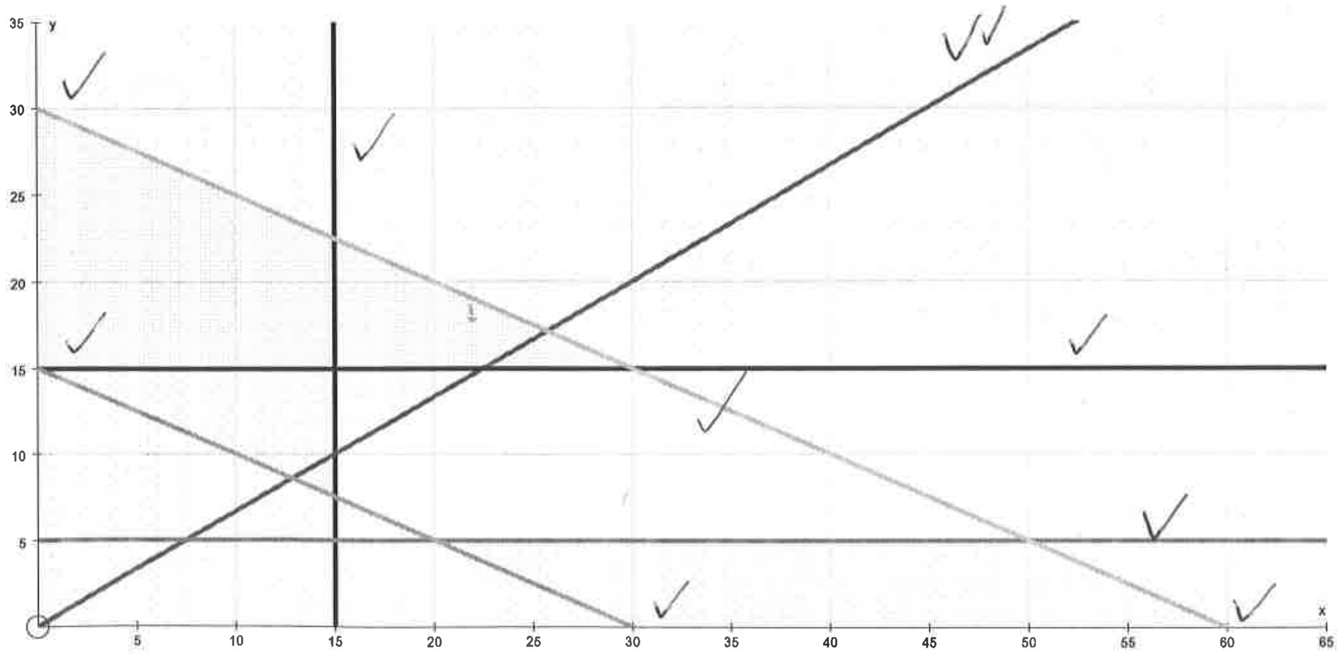
[11]

Question 8

A new brand *Locomotion* sells x ties and y shirts per day subject to the following constraints:

- The number of ties must be at least 15.
- The number of shirts must be at most in the ratio 2:3 to the number of ties.
- $5 \leq y \leq 15$
- $y \leq \frac{-1}{2}x + 30$
- $2y + x \geq 30$

a) Represent the constraints graphically and shade the feasible region on the axes below.
(10)



b) The profit on a tie is R60 and on a shirt is R240.

i) Determine how many shirts and ties must be sold per day to obtain a maximum profit. (4)

$P = 60x + 240y$

$\therefore y = \frac{-60}{240}x + \frac{P}{240}$

$\therefore m = -\frac{1}{4} \quad \frac{\Delta y}{\Delta x}$

✓✓a ✓✓a

$\therefore x = 30 \quad y = 15$ (using search line)

ii) Hence, calculate the maximum profit. (2)

✓m

$P = 60(30) + 240(15)$

$P = 5400$ ✓ca

[16]

Question 9

Alex decides to include both swimming and running in his exercise plan.
On day 1, Alex swims 150 m and runs 600 m. Each day he will increase the distance he swims by 50 m and the distance he runs will increase by 2,5% of the distance he ran on the previous day.

a) Determine, in terms of n, the distance that Alex

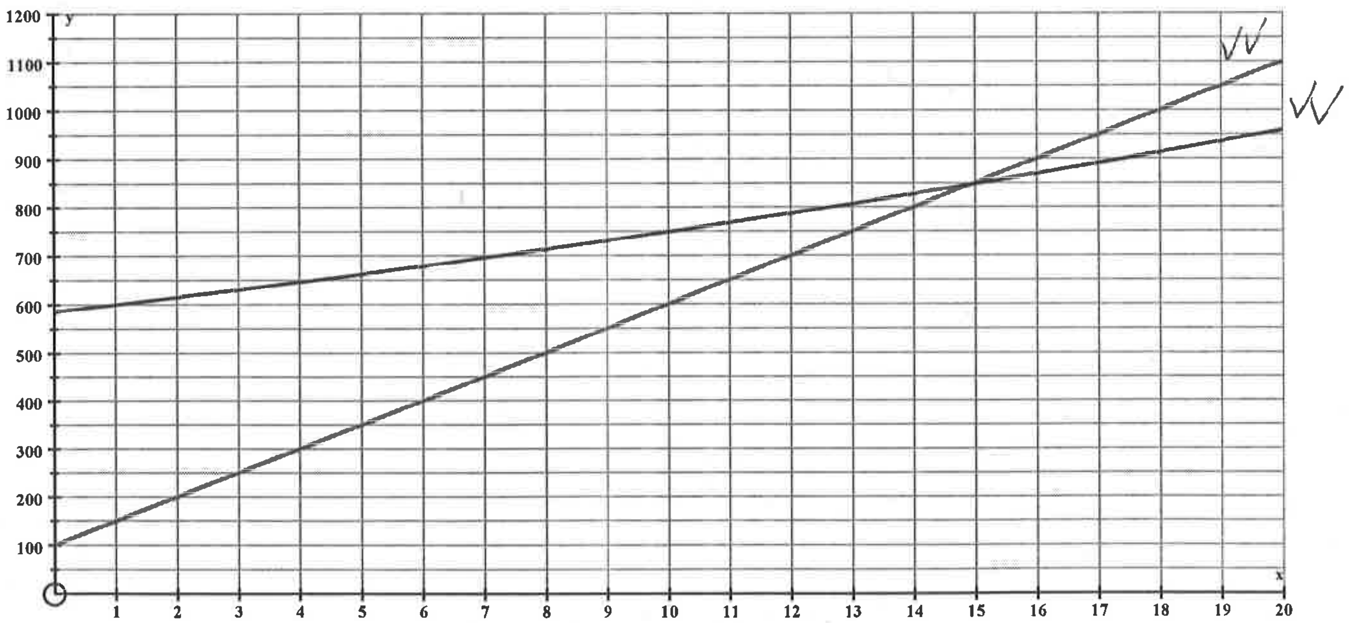
i) swims on the nth day of his exercise plan. (3)

$$T_n = 100 + 50n$$

ii) runs on the nth day of his exercise plan. (3)

$$T_n = 600(1{,}025)^{n-1}$$

b) On the set of axes below, plot points for each of the exercise types. You may join the points to illustrate the trend. (4)



c) **Use your graphs** to determine the first day on which the distance Alex swims will be greater than the distance she runs. (1)

From graph 16th day √a or 15th day.

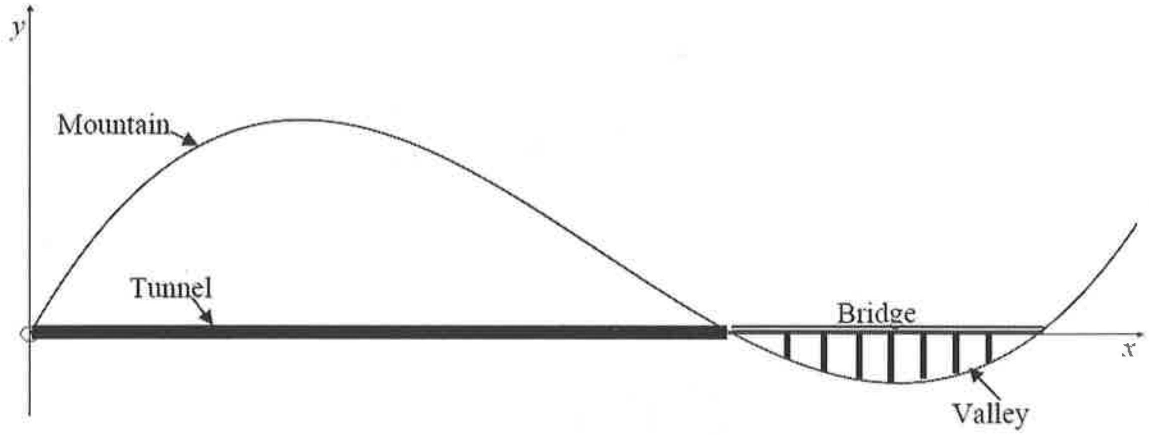
[11]

Question 10

Tender Construction have been assigned to build a tunnel and a bridge in the Drakensberg. They have determined that the cross-section can be modelled by a function:

$y = 2x^3 - 17x^2 + 35x \qquad 0 \le x \le 5$

where x is the distance (in hundreds of metres) from a point where the tunnel will start and y is the height (in hundreds of metres) above the proposed bridge.



a) Determine the length of

i) the tunnel (5)

$x(2x^2-17x+35)=0$ ✓m

$x(2x-7)(x-5)=0$ ✓✓a

$x=0 \quad x=\frac{7}{2} \quad x=5$ ✓m

$\therefore \frac{7}{2}\times 100$ ✓a

$= 350\text{m}$

ii) the bridge (2)

✓ca

$\left(5-\frac{7}{2}\right)\times 100$

$=150\text{ m}$ ✓ca

b) Calculate the height (above the tunnel) of the mountain at its peak. (to the nearest meter) (6)

$$\frac{dy}{dx} = 6x^2 - 34x + 35 = 0 \checkmark m$$

$\checkmark a$

$$x = \frac{34 \pm \sqrt{(-34)^2 - 4(6)(35)}}{2(6)}$$

$\checkmark ca$

$x = 4,31$ $\quad$ $x = 1,32$

$\checkmark m$

$\therefore y = 2(1,32)^3 - 17(1,32)^2 + 35(1,32)$

$y = 21,18828$ $\checkmark ca$

$\therefore$ 2119m high $\checkmark ca$

c) Calculate the deepest drop from the bridge to the valley below. (to the nearest meter) (3)

$\checkmark m$

$y = 2(4,31)^3 - 17(4,31)^2 + 35(4,31)$

$y = -48,17914$ $\checkmark ca$

$\checkmark ca$

$\therefore$ 482m deep

[16]

Question 11

The population of a suburban area is given by $P(t) = -t^3 + 5t^2 - 7t + 4$ thousand, where t is measured in years.

a) Determine mathematically if the population is decreasing or increasing after 6 months. (3)

$$P'(t) = -3t^2 + 10t - 7 \quad \checkmark a$$

$$P'\left(\tfrac{1}{2}\right) = -3\left(\tfrac{1}{2}\right)^2 + 10\left(\tfrac{1}{2}\right) - 7 \quad \checkmark a$$

$$= -\frac{11}{4} \quad \checkmark ca$$

b) Calculate

i) when the population begins to increase. (3)

$$-3t^2 + 10t - 7 = 0 \quad \checkmark a$$

$$3t^2 - 10t + 7 = 0 \quad \checkmark a$$

$$(3t - 7)(t - 1) = 0$$

$$3t = 7 \qquad t = 1$$

$$t = \frac{7}{3} \qquad \checkmark ca$$

$\therefore$ After 1 year

ii) when will the population be decreasing at a rate of 7 thousand per year? (3)

$-3t^2 + 10t - 7 = -7$ ✓m

$-3t^2 + 10t = 0$ ✓a

$t(-3t + 10) = 0$

$t = 0 \qquad t = \frac{10}{3}$ ✓a

iii) at what time does the population increase at the fastest rate. (3)

$p''(t) = -6t + 10$ ✓a

$-6t + 10 = 0$ ✓m

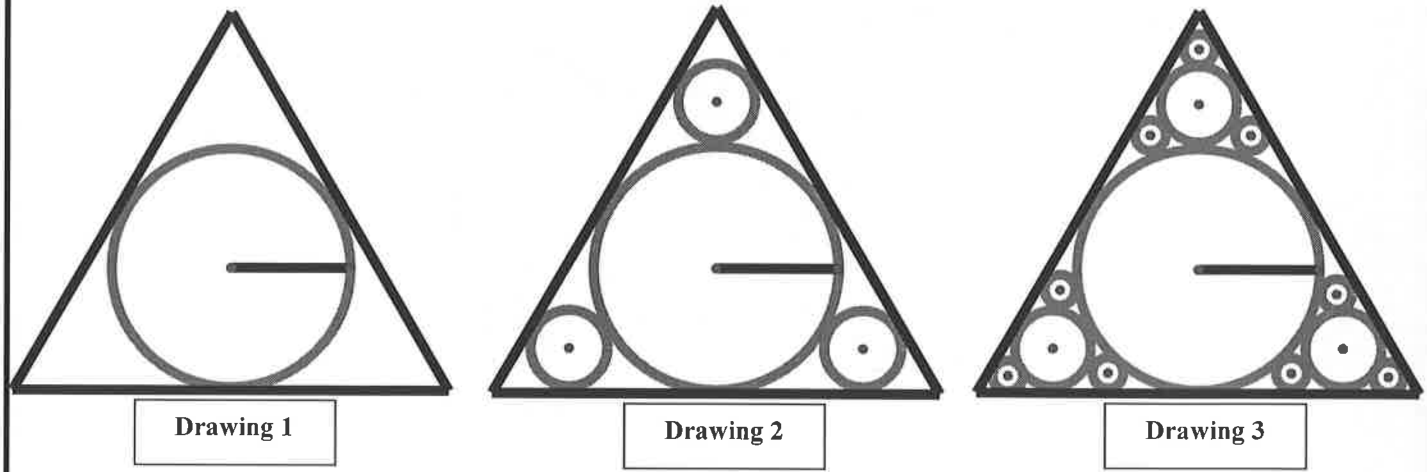
$t = \frac{-10}{-6}$ ✓ca

$\therefore t = \frac{5}{3}$ years.

[12]

Question 12

Peter argues that he can calculate the area of the following triangle using circles. The first circle he draws has a radius of 2 units, the next three congruent circles have radii of half the length of the previous one and the next nine congruent circles have radii of half the length of the previous three. This pattern continues indefinitely.



a) Complete the table below and determine the values of **(i)**,**(ii)** and **(iii)**. (3)

Drawing number	1	2	3	4
Number of additional circle/s drawn	1	3	9	**(ii)**
Total Area of the additional circle/s drawn	4π	3π	**(i)**	**(iii)**

$i = \frac{9}{4}\pi$ ✓a $ii = 27$ ✓a $iii)$ $\frac{27\pi}{16}$ ✓a

b) Hence calculate, the area of the triangle. (4)

$S_\infty = \frac{a}{1-r}$ ✓m

$= \frac{4\pi}{1-\frac{3}{4}}$ ✓a ✓ca

$= \frac{4\pi}{\frac{1}{4}} = 16\pi$ ✓ca

[7]

Rough Work

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\ ;\ y) \to (x\cos\theta + y\sin\theta\ ;\ y\cos\theta - x\sin\theta)$ $\quad$ $(x\ ;\ y) \to (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$