



St John's College

Upper V

Mathematics Paper 1

July 2013

Examiner: GE

Moderator: DG

MEMO

Time: 3 hrs

Marks: 150

Question 1 19 marks

a) $2x - 1 = 9$ ✓
 $\therefore x = 5$ ✓ (2)

b) $x(x-2)(x+2) > 0$ ✓

$\therefore x > 2$ or $-2 < x < 0$ ✓ (4)

c) $\left(\frac{3}{2}\right)^{4m-2} = \left(\frac{3}{2}\right)^{12-3m}$ ✓
 $\therefore 4m + 3m = 12 + 2$ ✓ (4)
 $\therefore m = 2$ ✓

d) $1 - \frac{4}{x} = 3$; $x > 0$ ✓
 $x - 4 = 3x$ ✓ (4)
 $\therefore x = -2$ ✓
 $\therefore$ no solution

e) $xy = 16$; $x + y = 10$ ✓
 $x(10 - x) = 16$ ✓
 $x^2 - 10x + 16 = 0$ (5)
 $(x - 8)(x - 2) = 0$ ✓
 $\therefore x = 8$ or $x = 2$ ✓
 $\therefore y = 2$ or $y = 8$ ✓

Question 2 18 marks

a) $f(t) = 4 - 12t + 9t^2$ ✓
 $\therefore f'(t) = -12 + 18t$ ✓ (4)
 $\therefore f'(2) = 24$ ✓

b) (i) $f'(x) = 3ax^2$ ✓
(ii) $\therefore a(2)^3 + q = 3$ ✓
 $\therefore 8a + q = 3$ ✓
 $12 = 3a(2)^2$ ✓
 $\therefore a = 1$ ✓ (5)
 $\therefore q = -5$ ✓

(iii) $y - 3 = -\frac{1}{12}(x - 2)$ ✓
 $\therefore 12y = -x + 38$ ✓ (3)

c) $f(x) = 2x^3 - 7$ ✓
 $\therefore f'(x) = 6x^2$ ✓ (5)

Question 3 18 marks

a) $T_n = \frac{2n-1}{n^2}$ ✓ (3)

b) $ar^6 = \frac{32}{9}$ ✓

$ar = 18$ ✓

$\therefore r^5 = \frac{16}{81}$

$\therefore r = \pm \frac{2}{3}$ ✓ (4)

$\therefore a = \pm 27$

c) (i) $27 - y = y - 3$ ✓
 $\therefore y = 15$ ✓ (3)

(ii) $\frac{27}{y} = \frac{y}{3}$ ✓
 $\therefore y = \pm 9$ ✓ (3)

d) $\frac{4}{27}$; $-\frac{8}{81}$; $\frac{16}{243}$; ... ✓

$\therefore S_\infty = \frac{\frac{4}{27}}{1 - (-\frac{2}{3})}$ ✓ (5)
 $= \frac{4}{45}$ ✓

Question 4 16 marks

a) (i) $F_{28} = 30000 \left(1 + \frac{0,08}{12}\right)^{28}$ ✓ (3)
 $= R36134,37$ ✓

(ii) $F_{36} = 36134,37 \left(1 + \frac{0,095}{12}\right)^8$ ✓ (2)
 $= R38487,30$

(iii) $38487,30 = 30000(1+i)^3$ ✓
 $\therefore i = 0,0865...$ (4)
rate = 8,66% ✓

Question 4 contd...

b) (i) $15000 \left[\frac{(1+0,03)^{17} - 1}{0,03} \right]$ ✓
 $= R326423,82$ ✓ (4)

(ii) $15000(1+0,03)^8$ ✓
 $= R19001,55$ (3)

Question 5 4 marks

$T = 2 + \log_2 \left(1 + \frac{2D}{2}\right)$ ✓
 $\therefore T - 2 = \log_2(1+D)$ ✓ (4)
 $D = 2^{T-2} - 1$ ✓

Question 6 15 marks

a) $x \geq -5$ ✓
 $y \leq 5$ ✓
 $3y + x \geq 1$ ✓ (6)
 $y \leq -x + 7$ ✓

b) $m = -\frac{1}{4}$ ✓
 $\therefore$ max at (2; 5) ✓ (3)
 $\therefore P_{\max} = 4(5) + 2$
 $= 22$ ✓

c) (i) mineither (0; 1) or (1; 0) ✓
 $\therefore (1; 0)$ gives min ✓ (3)
 $\therefore P = 2(0) + 1$
 $= 1$ ✓

OR if 0 excluded then min at (1; 1) and $P = 3$

(ii) $-1 < \frac{-k}{3} < 0$ ✓ (3)
 $\therefore 0 < k < 3$ ✓

Question 7

17 marks

a) $p = -3$ $q = -5$

$$-3 = \frac{a}{-2+3} - 5 \quad (4)$$

$$a = 2$$

b) $y + 5 = 1(x + 3)$

$$y = x - 2$$

$$y + 5 = -1(x + 3) \quad (4)$$

$$y = -x - 8$$

c) On diagram sheet (2)

d) $x \leq -4$; $-3 < x \leq -2$ (3)

e) $f(-2) = -3$ (2)

$$g(-3) = -5$$

f) $h(x) = -2x + 1$ (2)

Question 8

12 marks

a) $(2013 - 2011) + (2007 - 2001) + (1993 - 1983) + \dots$

$$= 2 + 6 + 10 + 14 + \dots$$

$$\therefore S_{25} = \frac{25}{2} [2(2) + 24(4)] \quad (5)$$

$$= 1250$$

b) (i) $T_1 = S_1 = 2(1)^3 - 3(1)^2 = -1$

$$T_2 = S_2 - S_1 = 2(2)^3 - 3(2)^2 - (-1) = 5 \quad (3)$$

$$T_3 = S_3 - S_2 = 2(3)^3 - 3(3)^2 - 4 = 23$$

b) $T_n = 6n^2 - 12n + 5$ (4)

Question 9

9 marks

a) $y = 2(x^2 - 2x + 3)$

$$\therefore y = 2[(x-1)^2 - 1 + 3] \quad (4)$$

$$\therefore y = 2(x-1)^2 + 4$$

b) min value of parabola = 4

$$\text{max value of function} = \frac{1}{\sqrt{4}} = \frac{1}{2} \quad (2)$$

c) $f'(x) = 2x^2 - 4x + 6$

$$= 2(x-1)^2 + 4$$

$$\therefore \text{always +ve} \quad (3)$$

$$\therefore f(x) \text{ always increasing}$$

Question 10

11 marks

a) $60 - 2x$ (1)

$$b) 60 \times 40 - \frac{40 \times (60 - 2x)}{2} - \frac{2x(40 - x)}{2} - \frac{60x}{2}$$

$$= 2400 - 1200 + 40x - 40x + x^2 - 30x$$

$$= 1200 - 30x + x^2 \quad (5)$$

c) $A' = -30 + 2x = 0$

$$\therefore x = 15$$

$$\therefore A_{\min} = 975m^2 \quad (5)$$

Question 11

11 marks

a) 1m (1)

b) $\frac{dy}{dx} = -\frac{1}{2}x + p = -\frac{1}{2}$

$$-\frac{1}{2}(4) + p = -\frac{1}{2} \quad (4)$$

$$\therefore p = 1.5$$

c) (i) $0 = x^2 - 8x + 15$

$$0 = (x-5)(x-3) \quad (3)$$

$$\therefore x = 3m$$

(ii) $\frac{dy}{dx} = -6x + 24$

$$= -6(3) + 24 \quad (3)$$

$$= 6$$

$$\tan^{-1}(6) = 80.5^\circ$$

Question 7(c)

