ST JOHN'S COLLEGE

Upper V

Mathematics – Paper II

2 August 2013 **Time: 3 hours**
Examiner: BT **Marks: 150**
Moderator: SM

Read the following instructions and information carefully:

1. This question paper consists of 11 pages. There are 14 questions in the paper. Attached to the paper is a double-sided Answer Sheet, and a double-sided Formula Sheet. **Please check that your paper is complete**.

2. You may detach the Answer Sheet and the Formula Sheet.

3. Answer all questions. Question 13 must be answered on the Answer Sheet provided at the end of this script.

4. Number your answers clearly and exactly as the questions are numbered in the question paper.

5. You may use an approved non-programmable and non-graphics calculator.

6. Where necessary, answers must be rounded off to **two decimal digits**, unless otherwise stated.

7. It is in your interest to show all your working details, to write legibly and to present your work neatly.

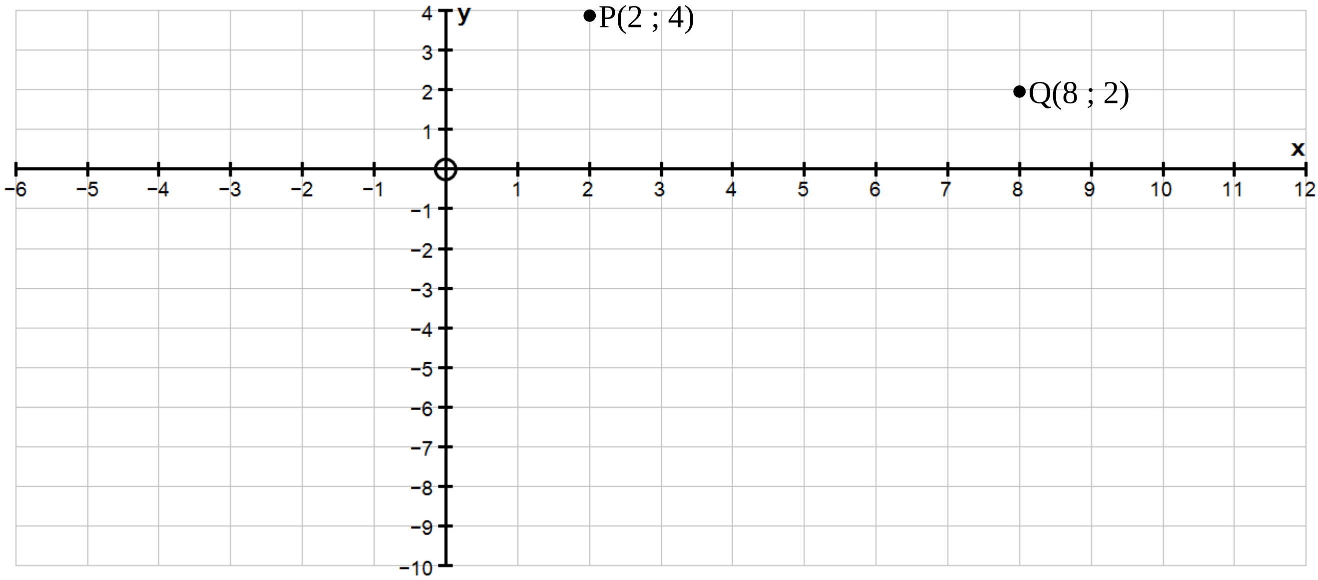
8. Diagrams are not drawn to scale unless this is stated in the question.

9. Write your name, your maths set and your teacher's name on your answer script **and** on your Answer Sheet.

10. Question papers must be handed in together with your answer book and the Answer Sheet.

QUESTION 1 [14 marks]

In the diagram below the points P(2 ; 4) and Q(8 ; 2) are shown. The point R(– 1 ; *a*) is also a point in the plane.



Use the given information to answer the following questions:

a) Find the value of a if $M\left(3\frac{1}{2}\,;\,-2\frac{1}{2}\right)$ is the midpoint of QR. (2)

b) Find the equation of the perpendicular bisector of QR. (4)

c) Calculate the angle of inclination to the *x*-axis of the line PQ, correct to one decimal place. (2)

d) Calculate $R\hat{P}Q$ if $a = -7$. (3)

e) Find the length of PQ in simplest surd form. (2)

f) State the coordinates of P if it is reflected in the line $y = x$. (1)

QUESTION 2 [6 marks]

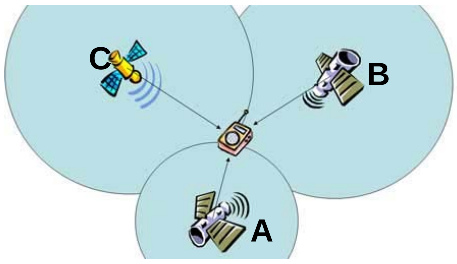
The points A(3 ; 1) , B(2 ; – 2) and C(2 ; 3) are given.

a) If CD is parallel to AB with D(– 3; *t* + 1), determine the value of *t*. (3)

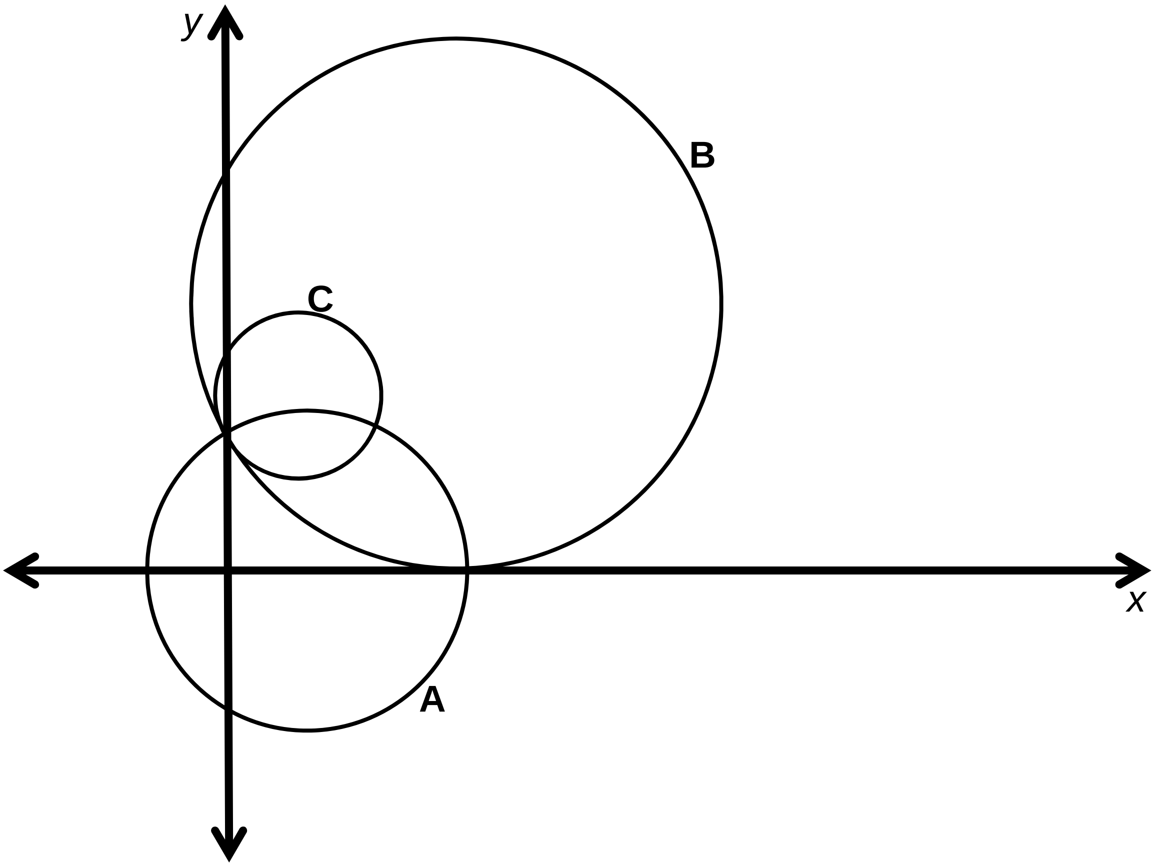
b) If A , B and E(*r* ; 4) are collinear, evaluate *r*. (3)

QUESTION 3 [17 marks]

Imagine you are standing somewhere on Earth with three satellites in the sky above you. If you know how far away you are from satellite A, then you know you must be located somewhere on the circle A. If you do the same for satellites B and C, you can work out your location by seeing where the three circles intersect. This is just what your GPS receiver does, although it uses overlapping spheres rather than circles.



In the diagram given below, circle A has centre (3 ; 0) and a radius of 5 units.



a) State the equation of circle A. (2)

b) Give the coordinates of the positive *x*-intercept of circle A. (2)

c) If the centre of circle B is (7 ; 8) and it shares a *y*-intercept with circle A, give the equation of circle B. (4)

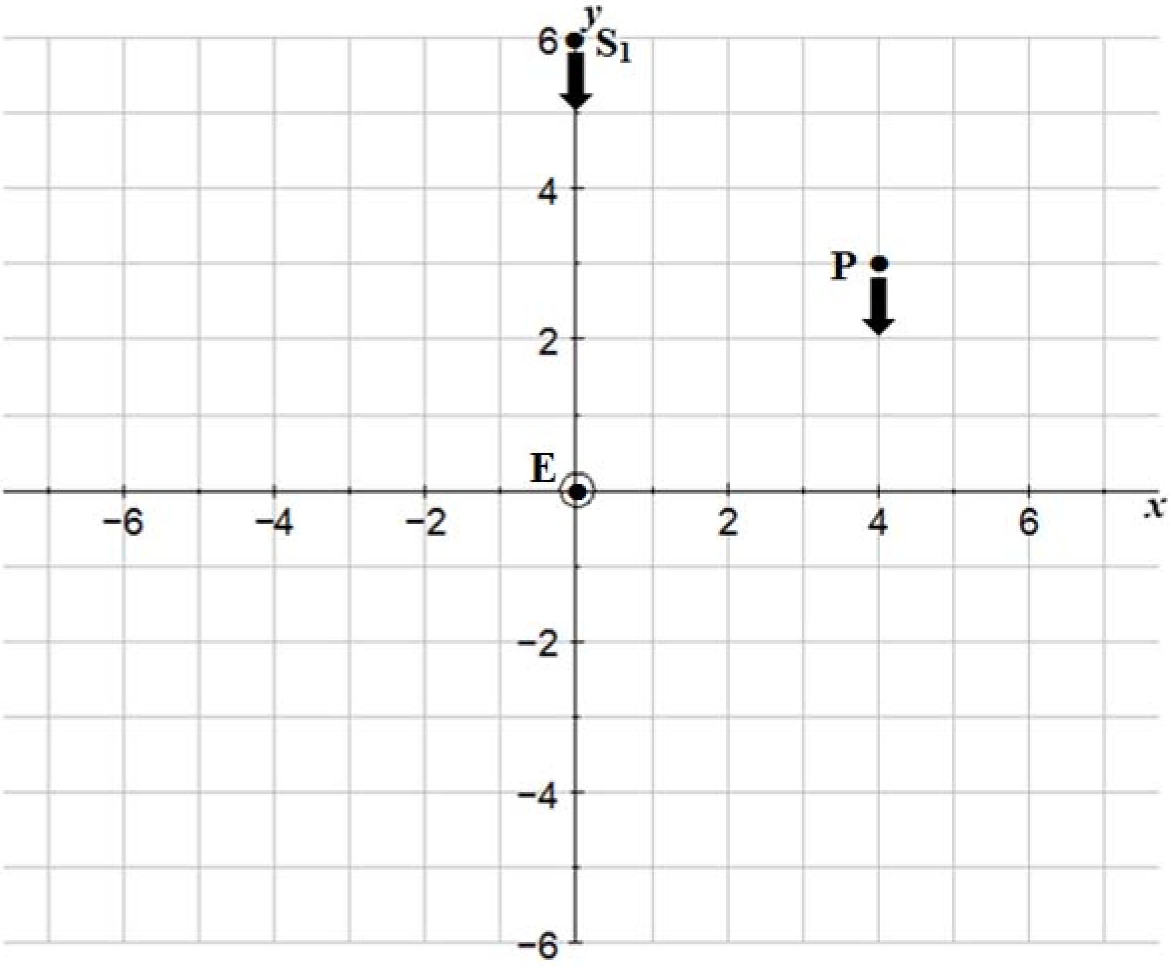
d) Circle C has equation $x^2 - 2x + y^2 - 12y + 32 = 0$. Find the centre of circle C and give its radius. (4)

e) Find the equation of the tangent to circle C at the point of intersection of the three circles. (3)

f) Write down the equation of the circle D which is tangential to circle A at $(-2;0)$ and passing through $(12;0)$. (2)

QUESTION 4 [8 marks]

Edwin is configuring a computer game. The aim is for P to shoot and eliminate the enemy at point E. The arrow indicates the direction in which P will shoot.



a) Starting at point P(4 ; 3), with E(0 ; 0), describe one transformation that will position P at S_1, in line for a shot at E. (2)

b) Position S_2 is a reflection of S_1 in the x-axis. State the rule that will transform S_1 to position S_2 in the form $(x; y) \to ...$ (1)

c) Describe (in words) a single transformation that will move S_1 to a position on the positive x-axis, with the arrow pointing to E. (1)

d) State a rule (in the form $(x; y) \to ...$) for the transformation in c) above. (1)

e) State a rule (in the form $(x; y) \to ...$) that will rotate the point S_1 through an angle of 150^0 anti-clockwise about the origin, to point S_3, and calculate the coordinates of S_3 in simplest surd form. (3)

QUESTION 5 [13 marks]

(This question is to be done without a calculator.)

a) Simplify: $\dfrac{\tan^2(180^0 - A).\cos(-A)}{\sin(A - 180^0).\cos(90^0 - A)}$ (6)

b) Evaluate: $\dfrac{\cos(-250^0).\cos 120^0}{\sin 20^0} + \dfrac{1}{\cos 157{,}5^0.\sin 22{,}5^0}$

Give your answer in simplest surd form. (7)

QUESTION 6 [8 marks]

If $\sin 21^0 = t$ evaluate the following in terms of t :

a) $\cos 249^0$ (2)

b) $\tan 69^0$ (3)

c) $\cos 42^0$ (3)

QUESTION 7 [7 marks]

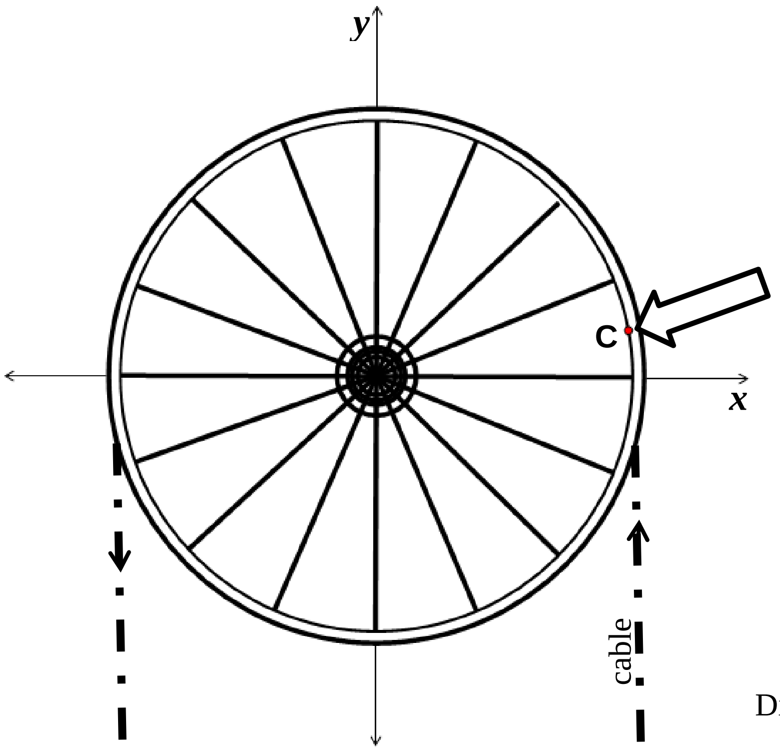
a) Show without a calculator that $2\sin(x - 45^0) = \sqrt{2}(\sin x - \cos x)$ (3)

b) Hence, or otherwise, deduce the minimum value of $\sqrt{2}(\sin x - \cos x)$ (1)

c) Evaluate $\sqrt{2}(\sin 15^0 - \cos 15^0)$ without a calculator. (Show all working). (3)

QUESTION 8 [10 marks]

The wheel in the headgear of a mineshaft rotates anti-clockwise as the lift cage descends deeper into the mine. A coupling (indicated at point C by the arrow in the diagram) is at an angle of 15^0 from the *x*-axis (i.e. $X\hat{O}C = 15^0$).



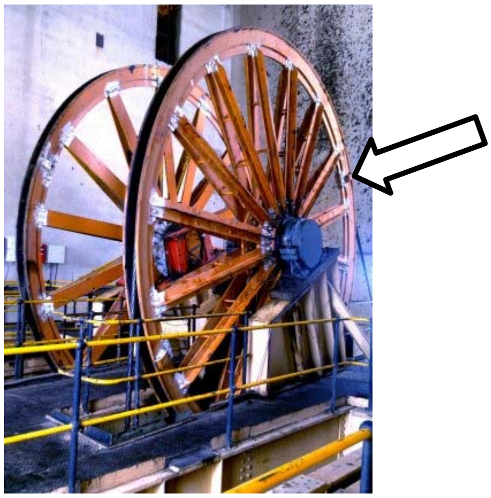


Diagram NOT drawn to scale.

a) If the wheel has a radius of 2m, calculate the co-ordinates of C correct to two decimal places. (3)

C rotates through 18 full revolutions and comes to rest at the point C', which has coordinates $(-1\,;-\sqrt{3})$.

b) Calculate the magnitude of the angle through which C is rotated. (4)

c) How far down the mineshaft has the cage descended when the coupling finally comes to rest at C' ? (3)

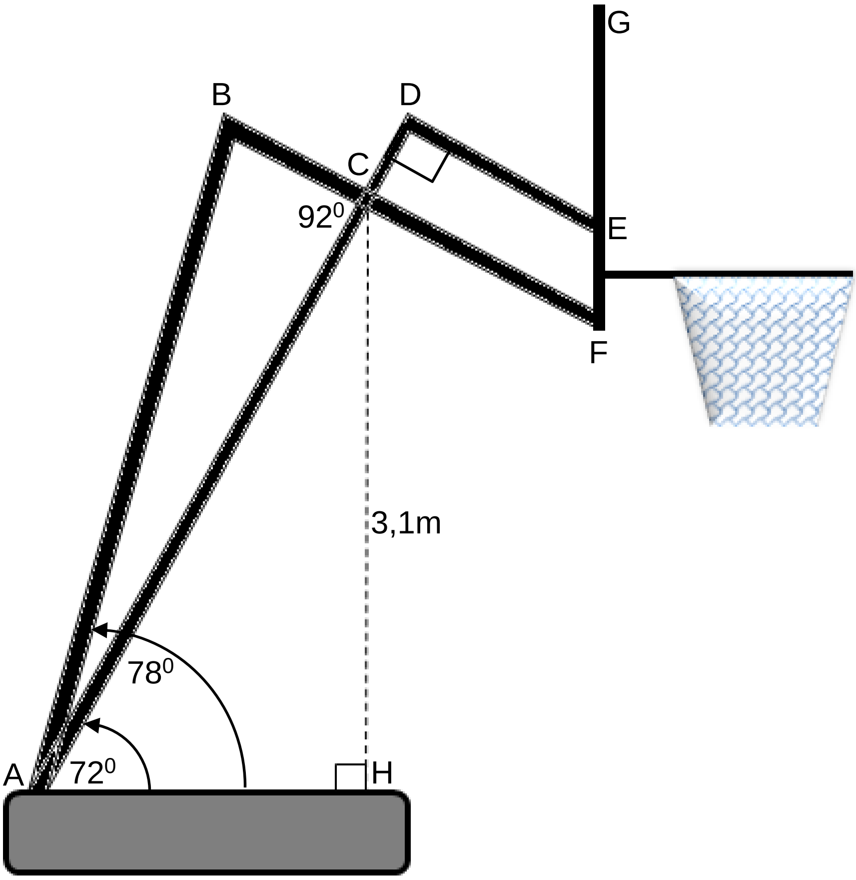
QUESTION 9 [15 marks]

a) i) Prove that $\dfrac{1-\sin 2x}{\sin x-\cos x}=\sin x-\cos x$ (4)

ii) For what values of $x \in \mathbb{R}$ is the identity above not defined? (3)

b) Find the general solution for θ if $\tan^2\theta+\dfrac{3}{\cos\theta}+3=0$ (8)

QUESTION 10 [5 marks]

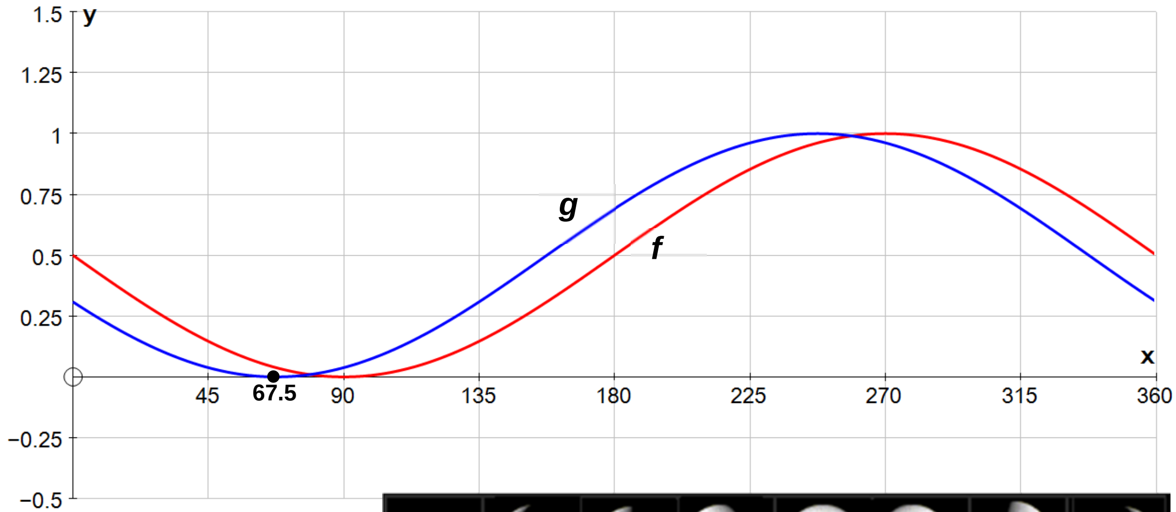


A portable metal tubing support for a basketball net is to be constructed for an indoor court with dimensions as shown in the diagram.

a) How far along the tube AD (from A) must the hole be drilled for the bolt to be inserted at C? (i.e. calculate the length of AC.) (2)

b) If $B\hat{C}A=92^0$ find the length of tubing needed for AB. (3)

QUESTION 11 [11 marks]



July 2013	90^0		180^0		270^0		360^0	

The phases of the moon can be approximated using trigonometric functions as models. The function f has equation $f(x) = a\sin x + b$, and this can be used to estimate the dates for the moon phases for July 2013 in the southern hemisphere.

a) Give the values of a and b. (2)

b) The model assumes a 28 day moon cycle, and on 1 July 2013 we saw the last quarter (half-moon, waning) for June 2013. On what date in July did we see the new moon? (2)

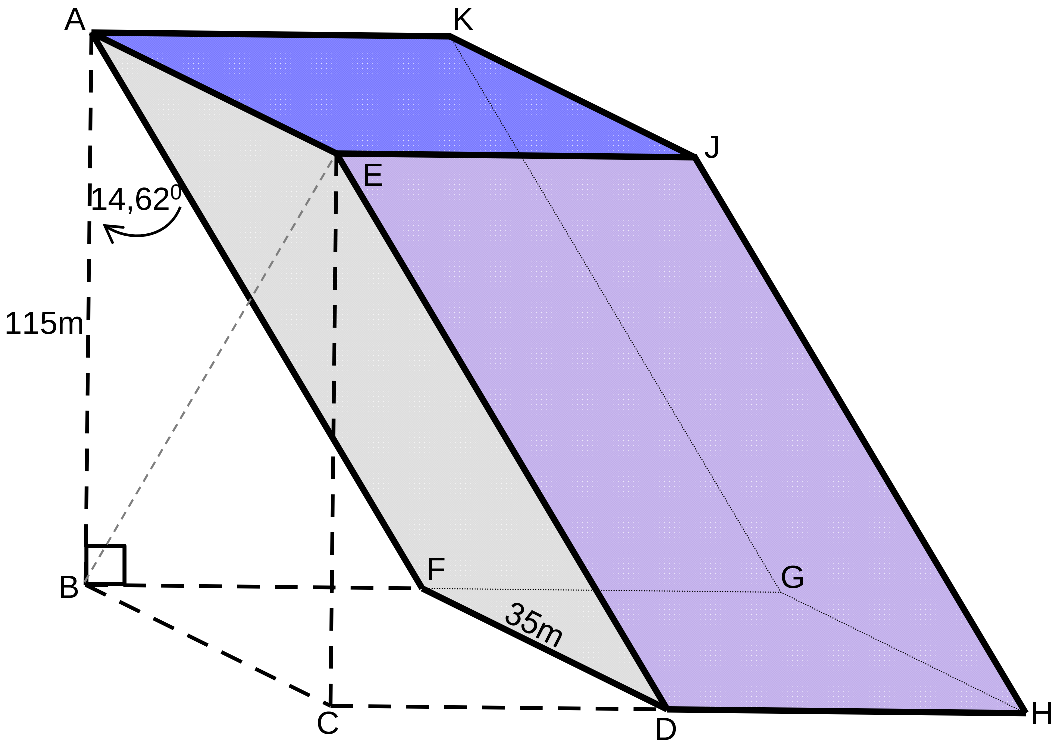
c) The graph of g shows the moon phases for August 2013. On what date in August can we expect to see the new moon? (2)

d) State the equation of g in the form $g(x) = m\cos(x - p) + q$. (3)

e) Give the equation of the reflection of f in the y-axis. (2)

QUESTION 12 [9 marks]

The Puerta de Europa (gate of Europe) can be found in Madrid. Completed in 1996, the twin towers were the first inclined skyscrapers in the world and consist of 25 stories having a vertical height of 115m. The angle of lean from the vertical is $14{,}62^0$, and each building stands on a square base of sides 35m long. The three levels of below-ground parking garages and substantial concrete reinforcing prevent the buildings from toppling over.



a) Calculate the volume of one of the towers. (2)

b) Calculate the length of the overhang (i.e. the length of BF). (2)

c) Calculate the length of BE. (1)

d) Hence calculate the size of $B\hat{E}D$. (4)

QUESTION 13 [15 marks]

(This question is to be answered on the Answer Sheet provided.)

The Irene Running Club announced the following figures for their members who ran and finished the 2013 Comrades Marathon.

No. of hours	Frequency	Cumulative frequency	Mid-point of interval
6 – 7	0		
7 – 8	2		
8 – 9	3		
9 – 10	19		
10 – 11	36		
11 - 12	40		

a) What was the total number of Irene runners who finished the Comrades? (1)

b) Fill in the cumulative frequency column on your answer sheet, and use this to sketch an ogive (cumulative frequency curve) on the provided axes. (4)

c) Use your graph to find an estimate of the median for this set of data, and show where you read off this value using the letter M on your graph. (2)

d) Use your graph to find estimates of the 1st and 3rd quartiles. Use Q_1 and Q_3 to show where these values are read off your graph. (2)

e) Calculate the mean running time for the Irene Comrades finishers. (2)

f) Calculate the standard deviation for these runners. (2)

g) Explain what the standard deviation tells us about these runners. (2)

QUESTION 14 [12 marks]

A franchise operator receives the following percentage profit figures from each of 12 franchises for the month of July 2013:

63	41	28	68
59	52	48	59
61	59	59	60

"Here's your lemonade and here's some descriptive literature about my franchising opportunities ."

a) Find the five number summary for this set of data and sketch a box and whisker plot. (5)

b) Explain what the box and whisker plot tells us about the data. (2)

c) A 13th operator submits his July figure late. His percentage profit was p and, when added to the data set p did not alter the five number summary in any way. Write an inequality for all possible values of p. (2)

d) The 13th operator then notifies them that p was incorrect. He submits a new percentage profit of q for July 2013. When q is included in the figures instead of p, it changes the mean to 52. Calculate the value of q. (3)

End

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Answer Sheet

Name ____________________________ Set _______ Teacher __________

QUESTION 13

a) ____

b) and c)

No. of hours	**Frequency**	**Cumulative frequency**	**Mid-point of interval**
6 – 7	0		
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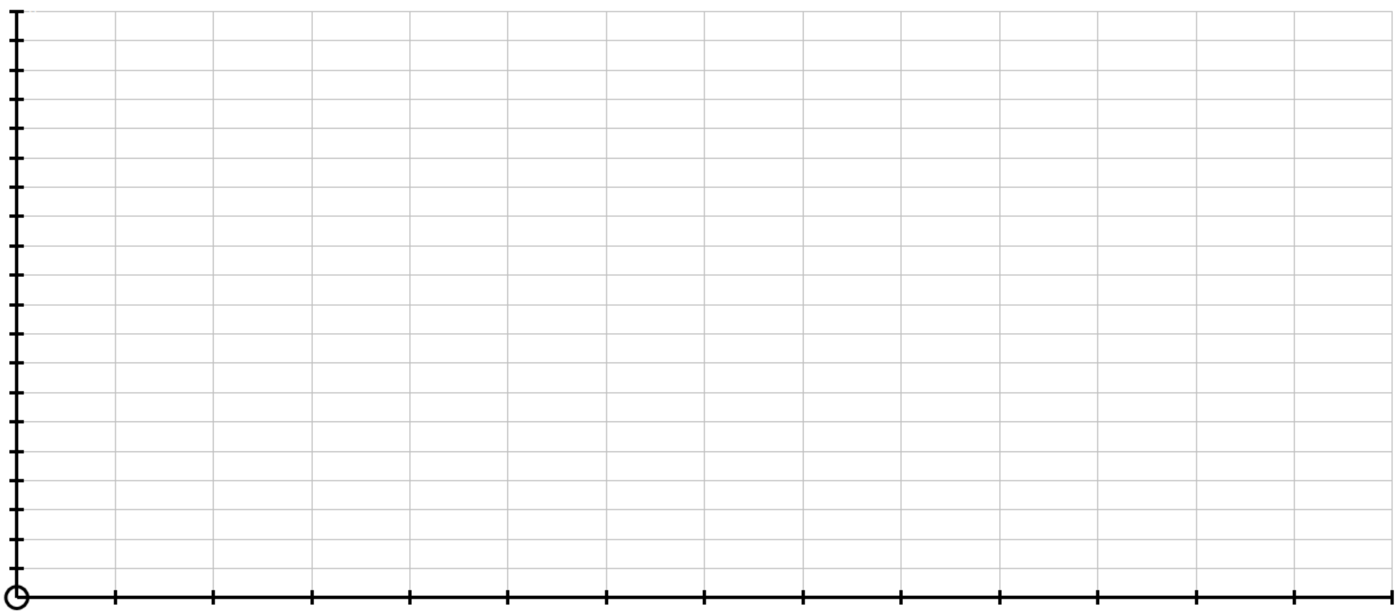

d)

e)

f)

g)

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \; ; \qquad S_n = \frac{a(r^n - 1)}{r-1} \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r}; \; -1 < r < 1, \; r \neq 0$$

$$T_n = an^2 + bn + c$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$In \ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x \ ; \ y) = ((x_A\cos\alpha - y_A\sin\alpha) \ ; \ (y_A\cos\alpha + x_A\sin\alpha))$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$\text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$s.d = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \ or \ B) = P(A) + P(B) - P(A \ and \ B)$$