NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2014

MATHEMATICS: PAPER I

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) Solve for x:

(1) $x^2 - 5x = -6$ (3)

(2) $(3x+1)(x-4) < 0$ (3)

(3) $\log_2(x+6) = 1$ (2)

(4) $2x + \sqrt{x+1} = 1$ (6)

(5) $12^{5+3x} = 1$ (2)

(b) Solve for x and y:

$2x - y = 8$ and $x^2 - xy + y^2 = 19$ (7)

(c) The polynomial $x^{10} - 2x^5 + c$ is divisible by $x+1$. Calculate the value of c. (3)

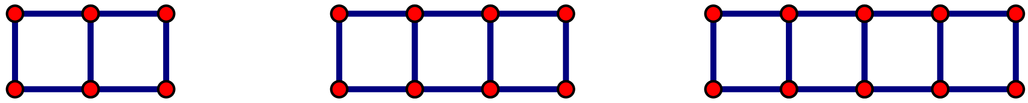
(d) Determine the slope of the tangent to the graph of $y = x^2$ at the point (–1; 1). (2)

[28]

QUESTION 2

(a) Given the sequence: 4; 7; 10; 13; 16 …
Assuming that this pattern remains consistent, write down an expression for the n^{th} term of the sequence. (2)

(b) The first three figures of a pattern comprising dots and sticks are shown.



(1) Write down the number of dots (d) and the number sticks (s) in the fourth figure. (2)

(2) Determine expressions in terms of n for each of the number of dots and the number of sticks in the n^{th} figure. (4)

(3) 100 dots are available to make a figure in the sequence. Calculate how many sticks will be needed for this figure. (4)

(c) An athlete runs 20 km on a certain Monday.
Thereafter, he increases the distance by 10% every day.

Calculate the number of kilometres he ran:

(1) on the following Saturday. (2)

(2) altogether over the 6 days. (3)

[17]

QUESTION 3

(a) Given: $f(x) = \frac{x}{2} - 1$

The graph of $y = f(x)$ is transformed by a reflection in the x-axis followed by a translation 4 units in the positive x-direction.
Determine the simplified equation of this graph. (3)

(b) Given: $g(x) = \frac{-x^2}{4}$

(1) Write down the domain and range of g. (2)

(2) Hence write down the domain of the inverse of g. (1)

(3) Determine the equation of the inverse of g in the form $y = ...$ (3)

(4) On the same set of axes, draw the graphs of $y = g(x)$ and its inverse. (4)

(c) Given: $j(x) = 2x^2 - 8x + 5$

(1) Determine q such that $j(x) = 2(x-2)^2 + q$. (2)

(2) For which values of c, $c \in \mathrm{R}$, will the equation $j(x) = c$ have real roots? (3)

[18]

QUESTION 4

(a) Given $f(x) = 6x^2 - 5x$, determine $f'(x)$ from first principles. (5)

(b) Determine $\frac{dy}{dx}$ given:

(1) $y = 3x^4 - 2\sqrt{x} + 6$ (4)

(2) $y = \frac{x^3 - 2x^{-2}}{3x}$

Write your answer with positive exponents. (5)

[14]

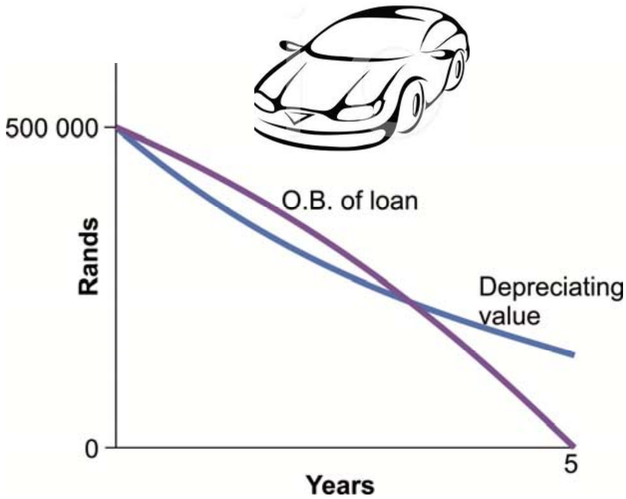
77 marks

SECTION B

QUESTION 5

Matt bought a car for R500 000 on an agreement in which he had to repay it in monthly instalments at the end of each month for 5 years, with interest charged at 18% p.a. compounded monthly.

The diagram alongside shows the reducing balance over this time period as well as the depreciating market value of the car.



(a) Calculate the annual effective interest rate of the loan. (3)

(b) Calculate Matt's monthly instalments. (4)

(c) Suppose that Matt decided to pay R12 700 each month as his repayment. Calculate the outstanding balance of the loan after 2 years. (4)

(d) At the end of the 2 years, the market value of Matt's car had reduced to R304 200. Determine the annual interest rate of depreciation on the reducing value. (3)

(e) Calculate what percentage of the original purchase price, the car's value had reduced to, at the end of the 5 years, when Matt had finished paying it off. (3)

[17]

QUESTION 6

(a) In a sample space S, the number of elements in S, $n(S) = 30$ and there are two events A and B such that $n(A) = 15$, $n(B) = 20$ with $n(A \text{ and } B) = 6$.

(1) Draw a Venn diagram to represent this situation. (3)

(2) Write down the value of $n(A \text{ or } B)$. (1)

(3) An element is randomly selected from S.

(i) Write down the probability that the element is in both events A and B. That is, P(A and B). (1)

(ii) Showing all working, determine whether the events A and B are independent. (3)

(b) Steve needs to set up a format for passwords onto his website.
He has decided on having letters from the alphabet (of 26 letters), followed by digits 0 to 9. Letters and digits can be repeated.

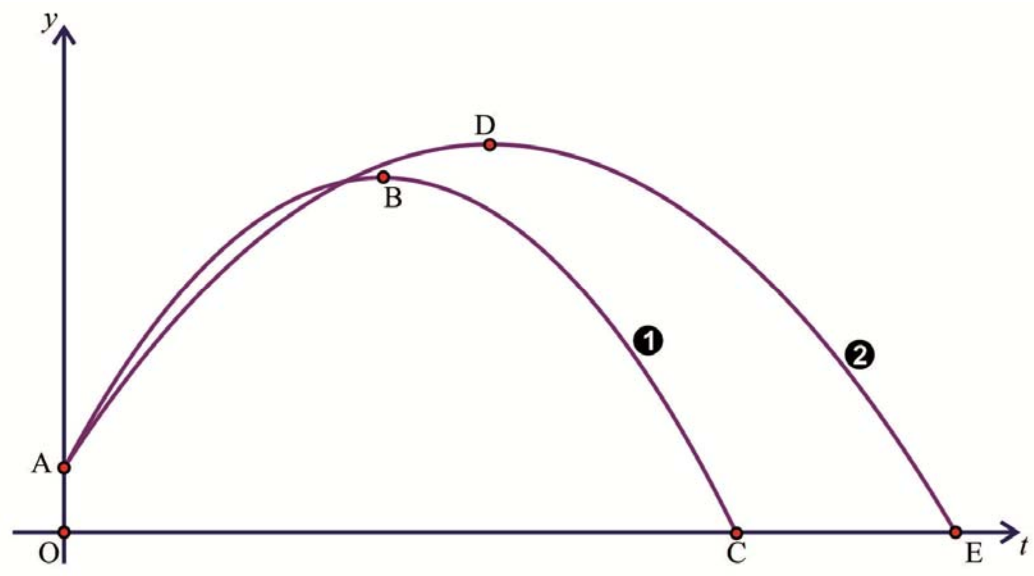
(1) Calculate the number of passwords that can be created using 2 letters followed by 2 digits. (2)

(2) Steve thinks that he will need to cater for 3 million different passwords.
He will stick with 2 letters but will need more digits.
Determine the least number of the digits he will need. (4)

[14]

QUESTION 7

At a New Year's fireworks celebration, rockets were shot off from a bridge. Each rocket displayed an explosion of lights when it reached the highest point along its parabolic path. Refer to the figure showing the path of two of these rockets.



The parabola ❶ is modelled by $f(t) = -5t^2 + 30t + 10$
where t is the number of seconds after the rocket is shot off and $f(t)$ is the number of metres that the rocket is above the water in the harbour.

(a) Write down the height of the bridge. (1)

(b) Calculate how long rocket ❶ is in the air before landing in the water at C. (3)

(c) Determine how high above the water rocket ❶ reaches. (4)

(d) Parabola ❷ reaches its maximum height of 60 metres after 4 seconds.
Determine the equation for its path, written in the form

$$h(t) = at^2 + bt + c$$ (6)

[14]

QUESTION 8

(a) Airlines have various restrictions on the size of bags that can be taken into the cabin of a plane. Some require that the largest rectangular box shape has sum of the length, breadth and height equal to 115 cm with length 55 cm.
Using these dimensions and the breadth as *x* cm,

(1) Show that the largest volume can be represented by $V = -55x^2 + 3\,300x$. (4)

(2) Hence determine the maximum volume of the bag. (6)

(b) Given: $f(x) = ax^3 + bx^2 + cx$

$f'(-1{,}5) = f'(5) = 0$ with $f(x)$ increasing for $-1{,}5 < x < 5$.

(1) Use this information to draw a rough sketch of $y = f(x)$. (5)

(2) Find the values of *x* for which:

(i) $f'(x) < 0$. (2)

(ii) f is concave upwards. (3)

[20]

QUESTION 9

Given a convergent geometric series with first term *a* and $S_\infty = p,\ p > 0$.

(a) Show that $a \in (0\,;2p)$. (5)

(b) Determine the value of the constant ratio when $a = \frac{p}{4}$. (3)

[8]

73 marks

Total: 150 marks