



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2015

MEMORANDUM

MARKS: 150

PUNTE: 150

**This memorandum consists of 27 pages./
Hierdie memorandum bestaan uit 27 bladsye.**

NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.
- Penalty of only 1 mark for incorrect rounding throughout the paper (Q1.2.1)

LET WEL:

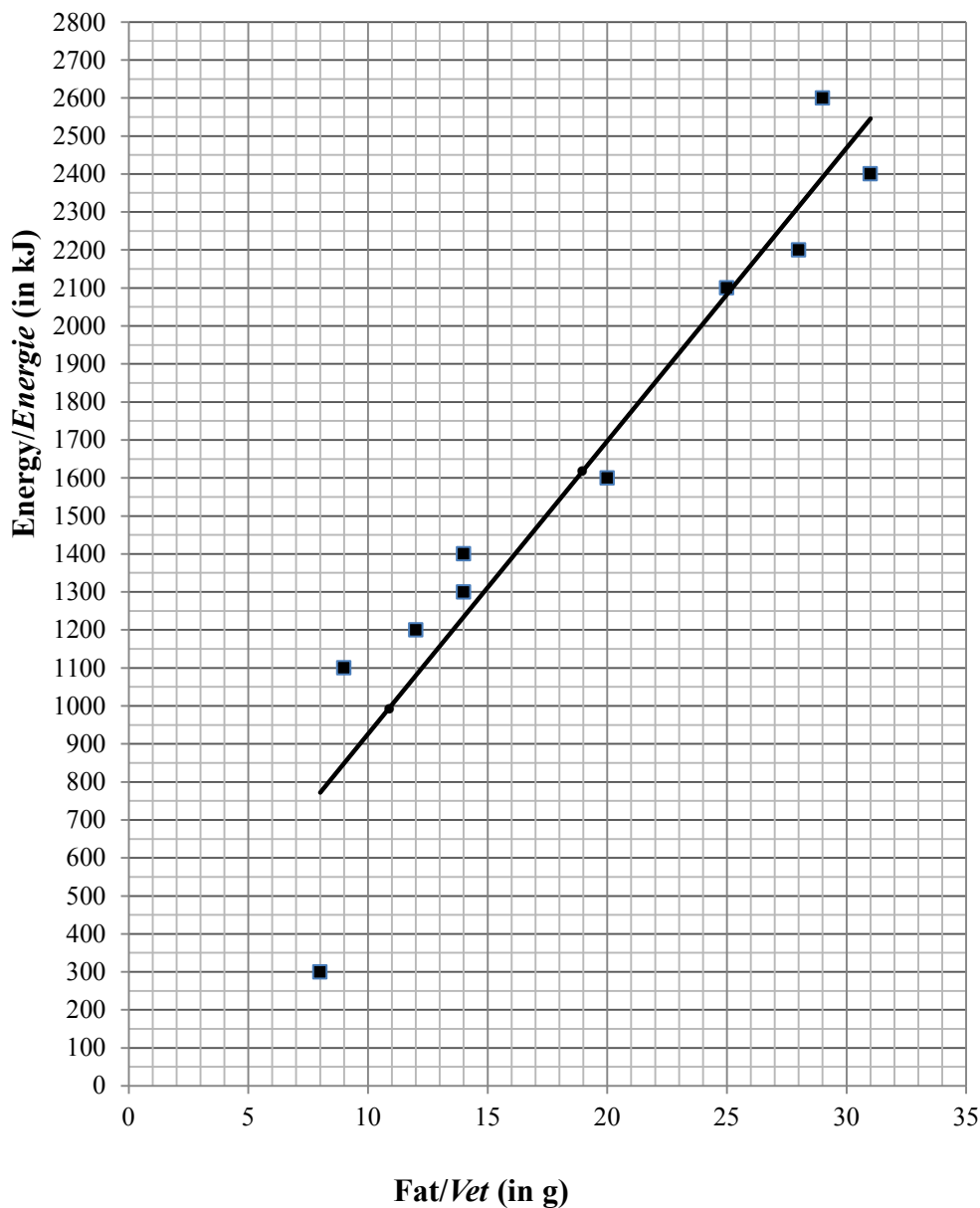
- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek het en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

QUESTION/VRAAG 1

Fat/Vet (in g)	9	14	25	8	12	31	28	14	29	20
Energy/Energie (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1

Scatter plot/Spreidiagram



1.2.2

- 1.1
no marks:
0 – 2 points correctly
- ✓ plotting
3 – 5 points correctly
- ✓✓ plotting
6 – 9 points correctly
- ✓✓✓ plotting
all 10 points correctly
- geen punte:*
0 – 2 punte korrek
- ✓ stip 3 – 5 pte korrek
- ✓✓ stip 6 – 9 pte korrek
- ✓✓✓ stip al 10 pte korrek
- (3)
- 1.2.2
✓ y – int close to (0 ; 150)
✓ one pt close to (25 ; 2100) or (20 ; 1700)
- (2)

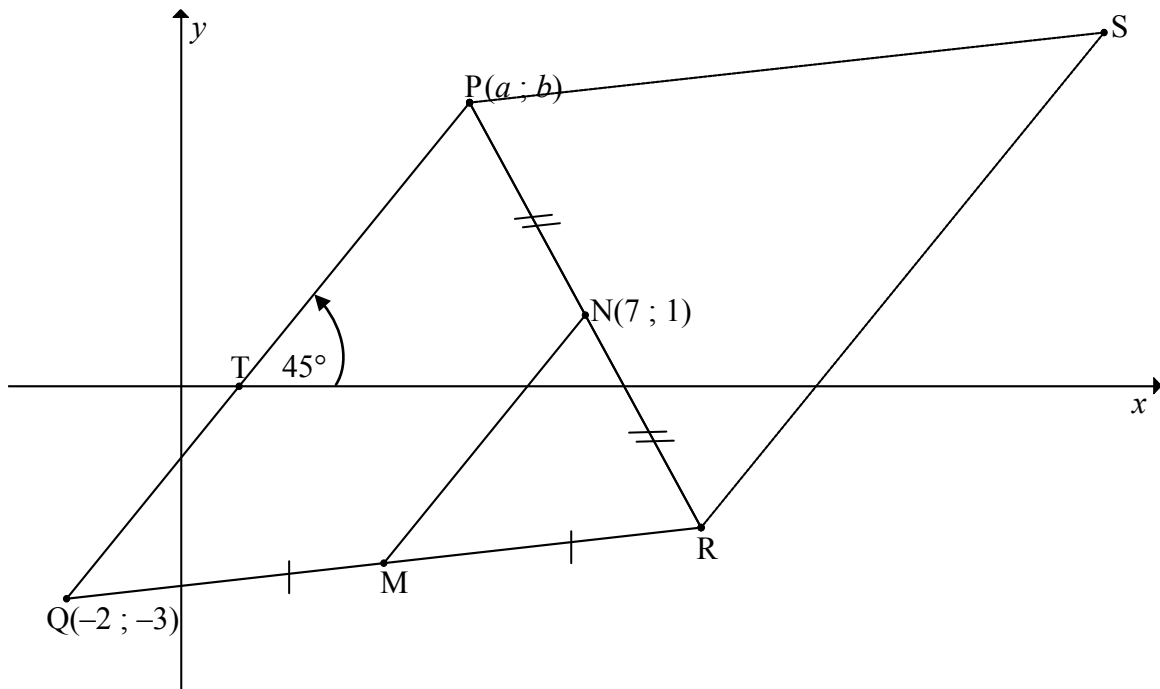
1.2.1	$\hat{y} = 154,60 + 77,13(18)$ $= 1\,542,94 \approx 1\,500 \text{ kJ}$	✓ subst ✓ answ rounded off correctly/ <i>antw korrek</i> <i>afgerond</i> (2)
1.3	(8 ; 300)	✓ answ/ <i>antw</i> (1)
1.4	$r = 0,9520... \approx 0,95$	✓✓ answ/ <i>antw</i> (2)
1.5	very strong positive relationship/ <i>baie sterk positiewe verband</i>	✓ strong/ <i>sterk</i> (1) [11]

QUESTION/VRAAG 2

Sum of the values on uppermost faces/ Som van die waardes op boonste vlakke	Frequency/ Frekwensie
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

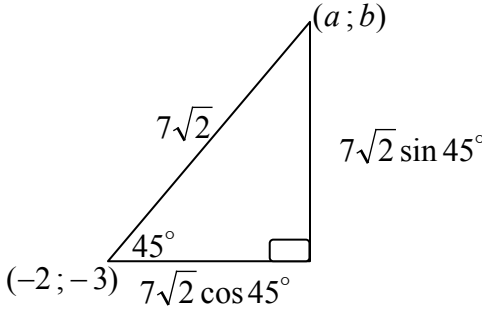
2.1	$\text{mean/gemiddelde} = \frac{2(0) + 3(3) + 4(2) + \dots + 12(1)}{30} = \frac{202}{30}$ $= 6,73$	✓ 202 ✓ answ/antw (2)
2.2	$\text{median/mediaan} = \frac{T_{15} + T_{16}}{2} = \frac{7 + 7}{2} = 7$	✓✓ answ/antw (2)
2.3	$\text{SD/SA} = 2,264\dots \approx 2,26$	✓✓ answ/antw (2)
2.4	$(6,73 - 2,26 ; 6,73 + 2,26)$ $= (4,47 ; 8,99)$ $\therefore 4 + 4 + 8 + 3 = 19 \text{ times/keer}$	✓ lower boundary ✓ upper boundary ✓ answ/antw (3) [9]

QUESTION/VRAAG 3

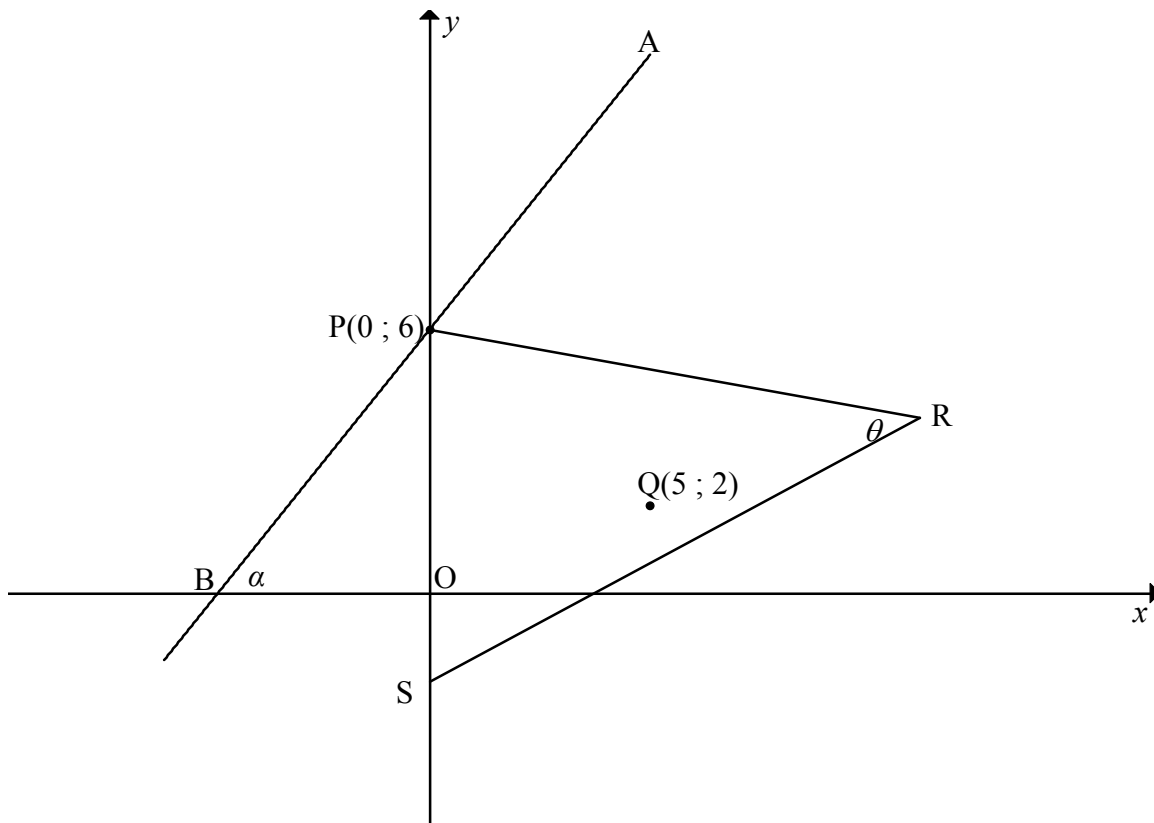


3.1	$m_{PQ} = \tan 45^\circ$ $= 1$	✓ $m = \tan 45^\circ$ ✓ answ/antw (2)
3.2	$MN \parallel QP$ [midpt theorem/midpt-stelling] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore y = x - 6$ OR/OF $MN \parallel PQ$ [midpt theorem/midpt-stelling] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore y = x - 6$	✓ S OR R ✓ m_{MN} ✓ subst m and/en $N(7; 1)$ ✓ equation/vgl (4) ✓ S OR R ✓ m_{MN} ✓ subst m and/en $N(7; 1)$ ✓ equation/vgl (4)
3.3	$MN = \frac{1}{2} PQ$ [midpoint theorem/midp stelling] $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4,95$	✓ S ✓ answ/antw (2)

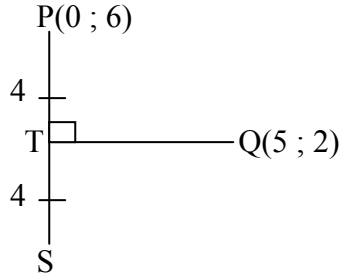
3.5	<p>QN = NS [diag of m/hoekl van m]</p> $\frac{-2 + x_s}{2} = 7 \quad \text{and/en} \quad \frac{-3 + y_s}{2} = 1$ <p>$\therefore x_s = 16$ $\therefore y_s = 5$</p> <p>OR/OF</p> <p>QN = NS [diag of m/hoekl van m]</p> <p>\therefore by inspection/deur inspeksie: S(16 ; 5)</p>	<p>✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)</p> <p>✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)</p>
3.6	<p>Equation of/Vgl van PQ: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$(1)</p> <p>From distance formula/Van afstandformule:</p> $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}$ $\therefore 98 = (a + 2)^2 + (b + 3)^2$(2) <p>Subst (1) into (2):</p> $98 = (b + 1 + 2)^2 + (b + 3)^2$ $98 = b^2 + 6b + 9 + b^2 + 6b + 9$ $0 = 2b^2 + 12b - 80$ $0 = b^2 + 6b - 40$ $\therefore 0 = (b + 10)(b - 4)$ $\therefore b = 4 \quad (\text{since } b > 0)$ <p>Subst $b = 4$ into (1):</p> $\therefore a = 4 + 1 = 5$ $\therefore P(5 ; 4)$ <p>OR/OF</p> <p>Equation of/Vgl van PQ: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$(1)</p> <p>From distance formula/Van afstandformule:</p> $7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}$ $\therefore 98 = (a + 2)^2 + (b + 3)^2$(2) <p>Subst (1) into (2):</p> $98 = (b + 1 + 2)^2 + (b + 3)^2$ $98 = 2(b + 3)^2$ $49 = (b + 3)^2$ $\pm 7 = b + 3$ $\pm 7 - 3 = b$ $\therefore b = 4 \quad (\text{since } b > 0)$ <p>Subst $b = 4$ into (1):</p> $\therefore a = 4 + 1 = 5$ $\therefore P(5 ; 4)$	<p>✓ eq of/vgl van PQ</p> <p>✓ subst Q & $7\sqrt{2}$ into/in distance formula/ afstandformule</p> <p>✓ subst eq of/vgl v. PQ</p> <p>✓ st form/st vorm</p> <p>✓ value of/waarde van b</p> <p>✓ value of/waarde van a (6)</p> <p>✓ eq of/vgl van PQ</p> <p>✓ subst Q & $7\sqrt{2}$ into/in distance formula/ afstandformule</p> <p>✓ subst eq of/vgl v. PQ</p> <p>✓ simplification/ vereenvoudig</p> <p>✓ value of/waarde van b</p> <p>✓ value of/waarde van a (6)</p>

	<p>OR/OF</p> <p>Equation of/Vgl van PQ: $y = x + c$ $-3 = -2 + c$ $y = x - 1 \quad \therefore a = b + 1 \quad \dots(1)$</p> <p>From distance formula/Van afstandformule: $7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}$ $98 = (a + 2)^2 + (a - 1 + 3)^2$ $= 2(a + 2)^2$ $\therefore a + 2 = 7 \quad (\text{since/aangesien } a > 0)$ $\therefore a = 5$ Subst $a = 4$ into (1): $\therefore b = 5 - 1 = 4$ $\therefore P(5 ; 4)$</p> <p>OR/OF</p>  <p>$a = -2 + 7\sqrt{2} \cos 45^\circ = 5$ $b = -3 + 7\sqrt{2} \sin 45^\circ = 4$</p>	<p>✓ eq of/vgl van PQ</p> <p>✓ subst Q & $7\sqrt{2}$ into/in distance formula/afstandformule</p> <p>✓ subst eq of/vgl v. PQ</p> <p>✓ simplification/vereenvoudig</p> <p>✓ value of/waarde van a</p> <p>✓ value of/waarde van b</p> <p>(6)</p> <p>✓✓✓✓</p> <p>✓</p> <p>✓</p> <p>(6) [17]</p>
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QUESTION/VRAAG 4

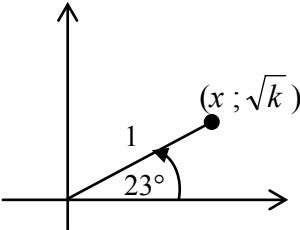


<p>4.1</p>	$(x - 5)^2 + (y - 2)^2 = r^2$ $(0 - 5)^2 + (6 - 2)^2 = r^2$ $25 + 16 = r^2$ $41 = r^2$ $\therefore (x - 5)^2 + (y - 2)^2 = 41$ <p>OR/OF</p> $PQ = \sqrt{(0 - 5)^2 + (6 - 2)^2}$ $= \sqrt{25 + 16}$ $r = \sqrt{41}$ $\therefore (x - 5)^2 + (y - 2)^2 = 41$	<ul style="list-style-type: none"> ✓ subst (5 ; 2) into circle eq/in sirkelvgl ✓ value of/waarde van r^2 ✓ equation/vgl (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/in afst. form ✓ value of/waarde van r ✓ equation/vgl (3)
<p>4.2</p>	$(0 - 5)^2 + (y - 2)^2 = 41$ $25 + (y - 2)^2 = 41$ $25 + y^2 - 4y + 4 = 41$ $y^2 - 4y - 12 = 0$ $(y - 6)(y + 2) = 0$ $y \neq 6 \text{ or / of } y = -2$ $\therefore S(0 ; -2) \text{ or } y = -2$	<ul style="list-style-type: none"> ✓ $x = 0$ ✓ st form/st. vorm ✓ answ/antw (neg value) (3)

	<p>OR/OF</p> $(0 - 5)^2 + (y - 2)^2 = 41$ $25 + (y - 2)^2 = 41$ $(y - 2)^2 = 16$ $y - 2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6 \text{ or / of } y = -2$ <p>$\therefore S(0 ; -2)$</p> <p>OR/OF</p> <p>Draw/Trek QT \perp PS PT = TS [line from centre \perp to chord/ lyn van midpt \perp koord]</p> $PT = y_P - y_Q = 6 - 2 = 4$ $y_Q - y_S = 4$ $y_S = 2 - 4 = -2$ <p>$\therefore S(0 ; -2)$</p> 	<p>$\checkmark x = 0$</p> <p>\checkmark square form/ kwadraatvorm</p> <p>\checkmark answ/antw (neg value)</p> <p>(3)</p> <p>$\checkmark x = 0$ $\checkmark\checkmark y = -2$</p> <p>(3)</p>
<p>4.3</p>	$m_{PQ} = \frac{6 - 2}{0 - 5}$ $= -\frac{4}{5}$ <p>$m_{PQ} \times m_{APB} = -1$ [tan/raakl \perp radius]</p> $\therefore m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	<p>\checkmark subst (0 ; 6) & (5 ; 2) into grad form/in grad. formule</p> <p>$\checkmark m_{PQ}$</p> <p>$\checkmark m_{APB}$</p> <p>\checkmark equation/vgl</p> <p>(4)</p>
<p>4.4</p>	$\tan \alpha = \frac{5}{4}$ $\therefore \alpha = 51,34^\circ$ <p>OR/OF</p> <p>B(4,8 ; 0)</p> $\therefore \tan \alpha = \frac{6}{4,8}$ $\therefore \alpha = 51,34^\circ$	<p>$\checkmark \tan \alpha = m_{APB}$</p> <p>$\checkmark$ answ/antw</p> <p>(2)</p> <p>$\checkmark \tan \alpha = \frac{6}{4,8}$</p> <p>$\checkmark$ answ/antw</p> <p>(2)</p>

<p>4.5</p>	<p> $\theta = \hat{B}\hat{P}\hat{S}$ [tan-chord th/raakl-koordst.] $= 90^\circ - \alpha$ [\angle sum in Δ/\angle som van Δ] $= 90^\circ - 51,34^\circ$ $= 38,66^\circ$ </p> <p>OR/OF</p> <p> $PS = 8$ $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos\hat{P}\hat{Q}\hat{S}$ $64 = 41 + 41 - 2.41.\cos\hat{P}\hat{Q}\hat{S}$ $\cos\hat{P}\hat{Q}\hat{S} = \frac{18}{82}$ $\hat{P}\hat{Q}\hat{S} = 77,32^\circ$ $\theta = \frac{1}{2}\hat{P}\hat{Q}\hat{S}$ [\angle at centre = $2 \times \angle$ circumf] $= 38,66^\circ$ </p>	<p> \checkmark S \checkmark R \checkmark $90^\circ - \alpha$ \checkmark answ/antw (4) </p> <p>\checkmark correct subst into cosine rule</p> <p>\checkmark $\hat{P}\hat{Q}\hat{S} = 77,32^\circ$</p> <p>$\checkmark$ R \checkmark answ/antw (4)</p>
<p>4.6</p>	<p> $\text{Area } \Delta PQS = \frac{1}{2} PS \times \text{height/hoogte}$ $= \frac{1}{2} (8)(5)$ $= 20 \text{ sq units/vk eenh}$ </p> <p>OR/OF</p> <p> $\hat{P}\hat{Q}\hat{S} = 2 \times 38,66^\circ$ [\angle at centre = $2 \times \angle$ at circum/ <i>midpts $\angle = 2$ omtreks \angle</i>] $= 77,32^\circ$ $\text{Area } \Delta PQS = \frac{1}{2} PQ.QS.\sin\hat{P}\hat{Q}\hat{S}$ $= \frac{1}{2} \cdot \sqrt{41} \cdot \sqrt{41} \cdot \sin 77,32^\circ$ $= 20 \text{ sq units/vk eenh}$ </p>	<p> \checkmark area formula/e: ΔPQS \checkmark $PS = 8$ \checkmark $\perp h = 5$ \checkmark answ/antw (4) </p> <p>\checkmark size of/grootte v $\hat{P}\hat{Q}\hat{S}$</p> <p>\checkmark area rule/reël: ΔPQS \checkmark subst correctly/ <i>subst korrek</i> \checkmark answ/antw (4)</p> <p>[20]</p>

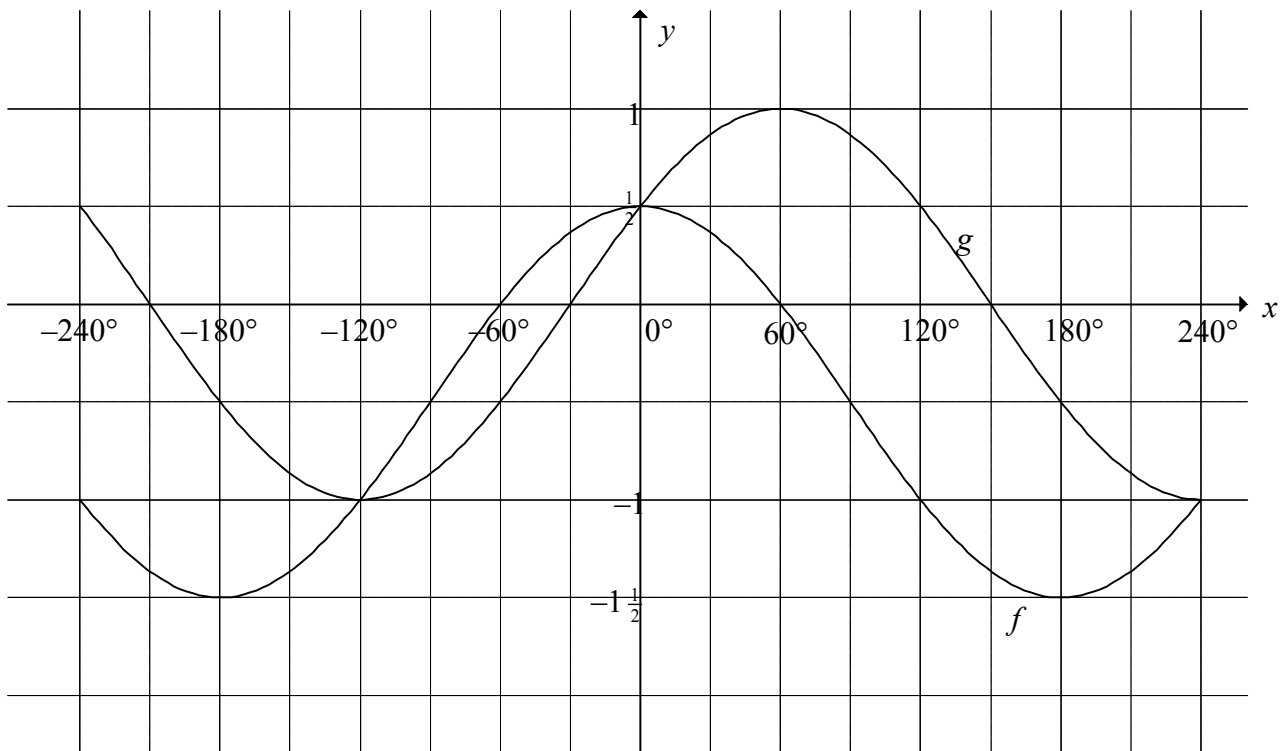
QUESTION/VRAAG 5

5.1.1	$\sin 203^\circ$ $= -\sin 23^\circ$ $= -\sqrt{k}$	✓ reduction/ <i>reduksie</i> ✓ answ ito/antw itv k (2)
5.1.2	$\cos^2 23^\circ = 1 - \sin^2 23^\circ$ $= 1 - k$ $\cos 23^\circ = \sqrt{1 - k}$ <p>OR/OF</p> $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1 - k}$ $\cos 23^\circ = \frac{\sqrt{1 - k}}{1} = \sqrt{1 - k}$ 	✓ identity/identiteit ✓ cos ² 23° ito/itv k ✓ answ/antw (3)
5.1.3	$\tan (-23^\circ) = -\tan 23^\circ$ $= -\frac{\sin 23^\circ}{\cos 23^\circ}$ $= -\frac{\sqrt{k}}{\sqrt{1 - k}} = -\sqrt{\frac{k}{1 - k}}$ <p>OR/OF</p> $\tan (-23^\circ) = -\tan 23^\circ$ $= -\frac{\sqrt{k}}{\sqrt{1 - k}} = -\sqrt{\frac{k}{1 - k}}$	✓ reduction/ <i>reduksie</i> ✓ answ ito/antw itv k (2)
5.2	$\frac{4 \cos x \cdot (-\sin x)}{\sin(30^\circ - x + x)}$ $= \frac{-4 \sin x \cdot \cos x}{\sin 30^\circ}$ $= \frac{-4 \sin x \cdot \cos x}{\frac{1}{2}}$ $= -8 \sin x \cdot \cos x$ $= -4(2 \sin x \cdot \cos x)$ $= -4 \sin 2x$	✓ cos x ✓ - sin x ✓ sin (α + β) ✓ $\frac{1}{2}$ ✓ double sine form / <i>dubbel sin form</i> ✓ answ/antw (6)

	<p>OR/OF</p> $\frac{4 \cos x \cdot (-\sin x)}{(\sin 30^\circ \cos x - \cos 30^\circ \sin x) \cos x + (\cos 30^\circ \cos x + \sin 30^\circ \sin x) \sin x}$ $= \frac{-4 \sin x \cdot \cos x}{\left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x\right) \cos x + \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x\right) \sin x}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} (\cos^2 x + \sin^2 x)}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} (1)}$ $= -8 \cos x \sin x$ $= -4(2 \sin x \cos x)$ $= -4 \sin 2x$	<p>✓ $\cos x$ ✓ $-\sin x$</p> <p>✓</p> <p>$\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x$</p> <p>✓ $\frac{1}{2}$</p> <p>✓ double sine form / <i>dubbel sin form</i></p> <p>✓ <i>answ/antw</i></p> <p>(6)</p>
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5.3	$\cos 2x - 7 \cos x - 3 = 0$ $2 \cos^2 x - 1 - 7 \cos x - 3 = 0$ $2 \cos^2 x - 7 \cos x - 4 = 0$ $(2 \cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2} \text{ or/of } \cos x = 4 \text{ (no solution)}$ $\therefore x = 120^\circ + n.360^\circ \text{ or/of } x = 240^\circ + n.360^\circ ; n \in \mathbb{Z}$ <p>OR/OF</p> $\therefore x = \pm 120^\circ + n.360^\circ ; n \in \mathbb{Z}$	<ul style="list-style-type: none"> ✓ expansion/ uitbreiding ✓ $2 \cos^2 x - 7 \cos x - 4 = 0$ ✓ factors/faktore ✓ $\cos x = -\frac{1}{2}$ ✓ 120° & 240° ✓ $+ n.360^\circ$ OR/OF ✓ $\pm 120^\circ$ ✓ $+ n.360^\circ$ <p style="text-align: right;">(6)</p>
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ $= 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	<ul style="list-style-type: none"> ✓ expansion of/ uitbreiding van $\sin(2\theta + \theta)$ ✓ expansions of $\sin 2\theta$ AND $\cos 2\theta$ ✓ $1 - \sin^2 \theta$ ✓ subst ✓ answ/antw <p style="text-align: right;">(5) [24]</p>

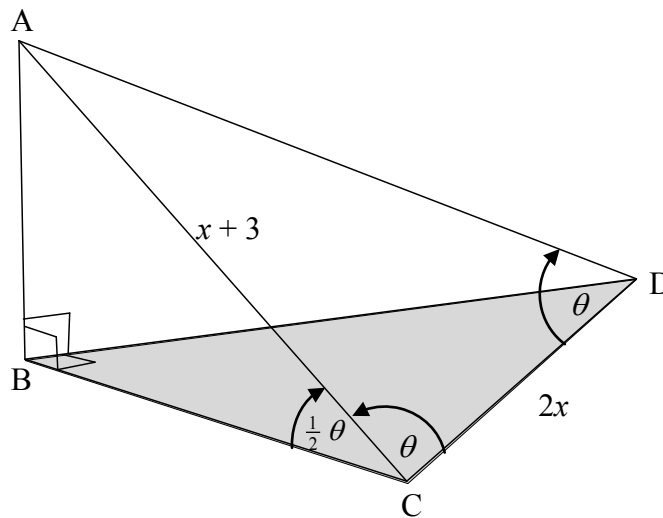
QUESTION/VRAAG 6



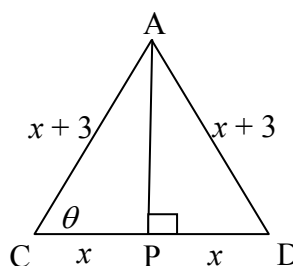
<p>6.1</p>	<p>$f(x) = \cos x - \frac{1}{2}$ and/en $g(x) = \sin(x + 30^\circ)$ $\therefore p = 30^\circ$ and/en $q = -\frac{1}{2}$</p> <p>OR/OF</p> <p>$\sin(60^\circ + p) = 1$ and/en $\cos 0^\circ + q = \frac{1}{2}$ $\therefore p = 30^\circ$ and/en $\therefore q = -\frac{1}{2}$</p>	<p>✓ $f(x) = \cos x - \frac{1}{2}$ ✓ $g(x) = \sin(x + 30^\circ)$ ✓ value of/waarde v p ✓ value of/waarde v q (4)</p> <p>✓ $\sin(60^\circ + p) = 1$ ✓ $\cos 0^\circ + q = \frac{1}{2}$ ✓ value of/waarde v p ✓ value of/waarde v q (4)</p>
<p>6.2</p>	<p>$x \in (-120^\circ ; 0^\circ)$ OR/OF $-120^\circ < x < 0^\circ$</p>	<p>✓ critical values/ kritiese waardes ✓ correct interval/ korrekte interval (2)</p>

6.3	<p>The graph of g has to shift 60° to the left and then be reflected about the x-axis./<i>Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word.</i></p> <p>OR/OF</p> <p>The graph of g must be reflected about the x-axis and then be shifted 60° to the left./<i>Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word.</i></p> <p>OR/OF</p> <p>The graph of g has to shift 120° to the right./<i>Die grafiek van g moet 120° na regs geskuif word.</i></p> <p>OR/OF</p> <p>The graph of g has to shift 240° to the left./<i>Die grafiek van g moet met 240° na links geskuif word</i></p>	<p>✓ 60° left/<i>links</i> ✓ reflection about x-axis/<i>refleksie om x-as</i> (2)</p> <p>✓ reflection about x-axis/<i>refleksie om x-as</i> ✓ 60° left/<i>links</i> (2)</p> <p>✓ ✓ 120° right/<i>regs</i> (2)</p> <p>✓ ✓ 240° left/<i>links</i> (2)</p> <p>[8]</p>
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QUESTION/VRAAG 7

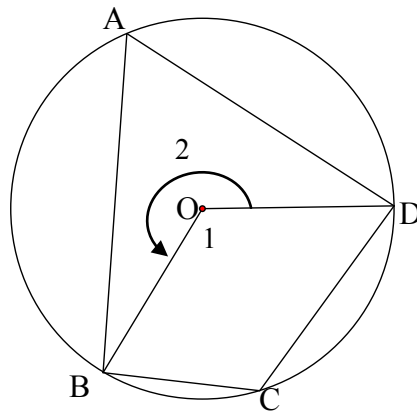


7.1	$\hat{C}AD = 180^\circ - 2\theta$ [\angle s sum of Δ / \angle e som van Δ]	✓ answ/antw (1)
7.2	$\frac{\sin \theta}{x+3} = \frac{\sin(180^\circ - 2\theta)}{2x}$ $\frac{\sin \theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin \theta}{x+3} = \frac{2 \sin \theta \cdot \cos \theta}{2x}$ $\cos \theta = \frac{2x \sin \theta}{2(x+3) \sin \theta}$ $\cos \theta = \frac{x}{x+3}$ <p>OR/OF $AD = x + 3$ [sides opp = \angles/sye to = \anglee] $AC^2 = AD^2 + CD^2 - 2AD \cdot CD \cdot \cos \theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3) \cdot \cos \theta$ $0 = 4x^2 - 4x(x+3) \cos \theta$ $\cos \theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$</p> <p>OR/OF Draw/Trek $AP \perp CD$</p> $\cos \theta = \frac{x}{x+3}$	✓ correct subst into sine rule/korrekte subst in sin-reël ✓ $\sin 2\theta$ ✓ $2 \sin \theta \cdot \cos \theta$ ✓ $\cos \theta$ as subject/as onderwerp (4) ✓ $AD = x + 3$ ✓ correct subst into cosine rule/korrekte subst in cos-reël ✓ simplification/vereenvoudiging ✓ $\cos \theta$ as subject/as onderwerp (4) ✓ ✓ constr/konstr ✓ ✓ sketch shown/toon skets (4)



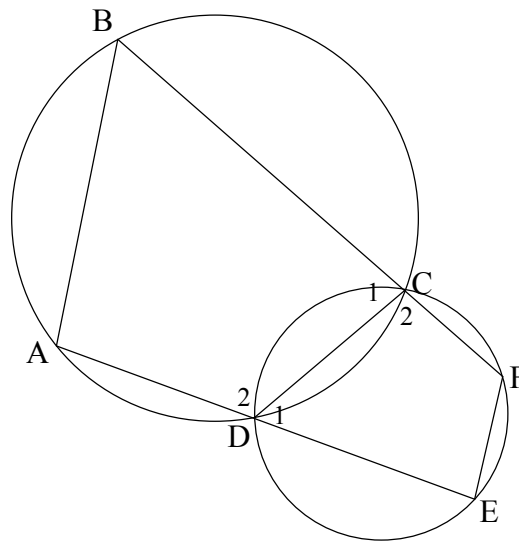
7.3	$\cos \theta = \frac{2}{5}$ $\therefore \theta = 66,42^\circ$ <p>In $\triangle ABC$:</p> $\sin \frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33,21^\circ = \frac{AB}{5}$ $\therefore AB = 5 \sin 33,21^\circ$ $= 2,74$ <p>OR/OF</p> $\sin \frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5 \sin \frac{\theta}{2}$ <p>but/maar:</p> $\cos \theta = \frac{2}{5}$ $1 - 2 \sin^2 \frac{\theta}{2} = \frac{2}{5}$ $\sin^2 \frac{\theta}{2} = \frac{3}{10}$ $\sin \frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5 \sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2,74$	$\checkmark \cos \theta = \frac{2}{5}$ $\checkmark \text{ size of/grootte v } \theta$ $\checkmark \text{ correct ratio/}$ korrekte verh $\checkmark \text{ subst correctly/}$ korrek $\checkmark \text{ answ/antw}$ <p style="text-align: right;">(5)</p> $\checkmark AB = 5 \sin \frac{\theta}{2}$ $\checkmark \text{ equation/vgl}$ $\checkmark \text{ simplification/}$ vereenvoudiging $\checkmark \text{ value of/waarde v}$ $\sin \frac{\theta}{2}$ $\checkmark \text{ answ/antw}$ <p style="text-align: right;">(5)</p> <p style="text-align: right;">[10]</p>
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QUESTION/VRAAG 8



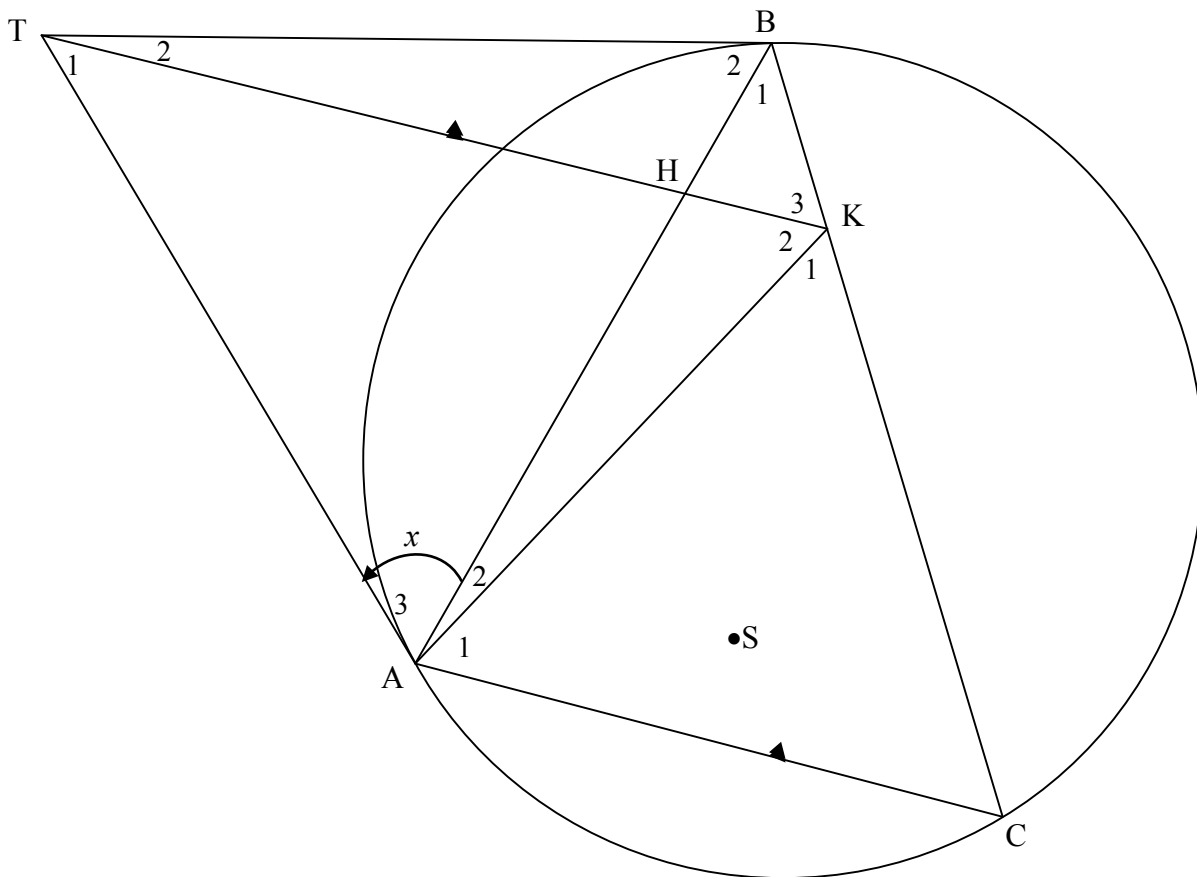
8.1.1	twice or double /twee keer of dubbel	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [\angle at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times$ omtreks \angle] $\hat{O}_2 = 2\hat{C}$ [\angle at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times$ omtreks \angle] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [\angle s in a rev/ \angle e in omw of om 'n pt] $2\hat{A} + 2\hat{C} = 360^\circ$ $\therefore \hat{A} + \hat{C} = 180^\circ$ OR/OF Let/Gestel $\hat{O}_1 = 2x$ $\hat{A} = x$ [\angle at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times$ omtreks \angle] $\hat{O}_2 = 360^\circ - 2x$ [\angle s in a rev/ \angle e in omw of om 'n pt] $\hat{C} = 180^\circ - x$ [\angle at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times$ omtreks \angle] $\therefore \hat{A} + \hat{C} = 180^\circ$	✓ S ✓ S ✓ S ✓ S ✓ S ✓ S (3)

8.2



8.2	$\hat{A} = \hat{C}_2$ $\hat{E} = 180^\circ - \hat{C}_2$ $\therefore \hat{E} = 180^\circ - \hat{A}$ $\therefore EF \parallel AB$ OR/OF $\hat{B} = \hat{D}_1$ $\hat{F} = 180^\circ - \hat{D}_1$ $\therefore \hat{F} = 180^\circ - \hat{B}$ $\therefore EF \parallel AB$	[ext \angle of cyclic quad/ <i>buite</i> \angle v <i>kdvh</i>] [opp \angle s of cyclic quad/ <i>tos</i> \angle e v <i>kdvh</i>] [co-interior \angle s 180° / <i>ko-binne</i> \angle e 180°] [ext \angle of cyclic quad/ <i>buite</i> \angle v <i>kdvh</i>] [opp \angle s of cyclic quad/ <i>tos</i> \angle e v <i>kdvh</i>] [co-interior \angle s 180° / <i>ko-binne</i> \angle e 180°]	✓ S ✓ R ✓ S ✓ R ✓ R (5) ✓ S ✓ R ✓ S ✓ R ✓ R (5) [9]
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QUESTION/VRAAG 9

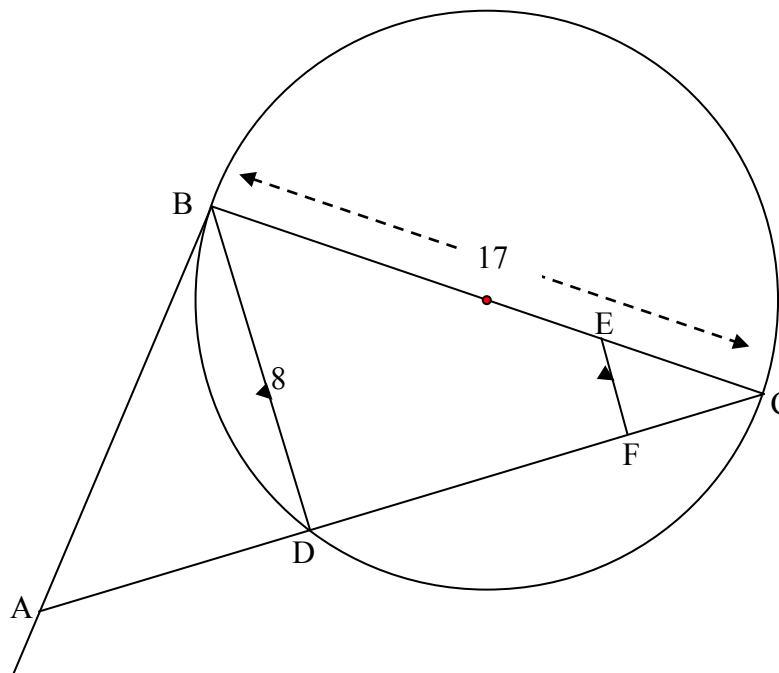


9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp \angle s/ooreenk $\angle e$; CA KT] [tan-chord th/raakl-koordst]	\checkmark S \checkmark R \checkmark S \checkmark R	(4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ \therefore AKBT is cyc quad	[proved/bewys in 9.1] [line (BT) subtends equal \angle s/ lyn (BT) onderspan gelyke $\angle e$] OR/OF [converse \angle s in same segment/ omgek $\angle e$ in dies segment]	\checkmark S \checkmark R	(2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ \therefore TK bisects/halveer $A\hat{K}B$ OR/OF $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/raakl-koordst] [\angle s in the same segm/ $\angle e$ in dies segm]	\checkmark S \checkmark R \checkmark S \checkmark R \checkmark S \checkmark R \checkmark S \checkmark R	(4)

	$\therefore = \hat{K}_3$ [proven in 9.1] \therefore TK bisects/halveer $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/bewys] \therefore TA tangent [converse tan chord theorem OR \angle between line and chord/ <i>omgekeerde raakl-kdst OF \angle tussen lyn en koord</i>]	\checkmark S \checkmark R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/konsiklies] $A\hat{T}B = 180^\circ - 2x$ [A,T,B & K concyclic/konsiklies] \therefore points A, S, B and T are also concyclic/ <i>punte A, S, B en T is ook konsiklies</i> [opp \angle s of quad = 180° /tos \angle e van vierhoek = 180°] OR/OF A, S K and B are concyclic. A, K, B and T are concyclic. \therefore A, S, B and T are concyclic. OR/OF The circle passing through points A, K and B contains the point S on the circumference (A, S, K and B concyclic)./ <i>Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).</i> The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./ <i>Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).</i> \therefore points A, S, B and T are also concyclic/ <i>punte A, S, B en T is konsiklies</i>	\checkmark S (both/beide statements/bewerings) \checkmark R (2) \checkmark S \checkmark S (2) \checkmark S \checkmark S (2)

[14]

QUESTION/VRAAG 10



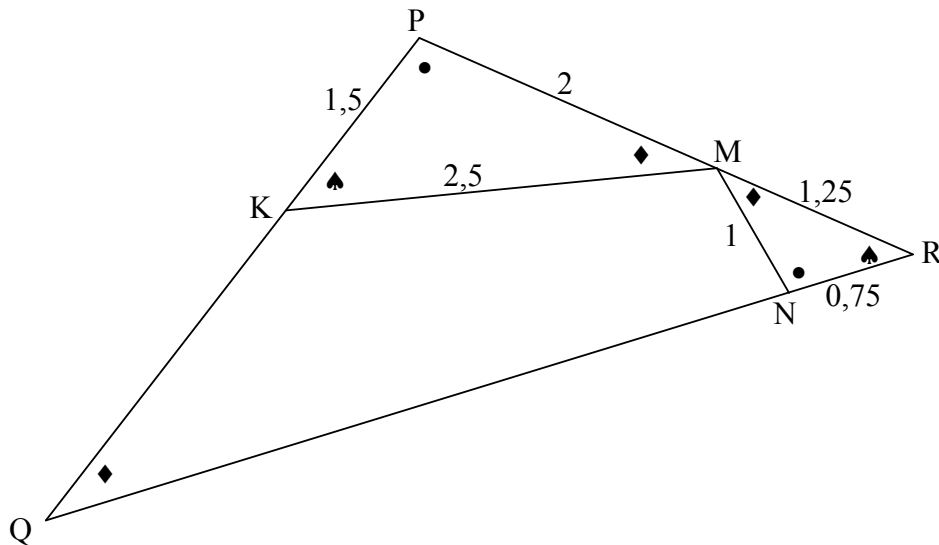
<p>10.1</p>	<p>$\hat{BDC} = 90^\circ$ $DC^2 = 17^2 - 8^2$ $= 225$ $\therefore DC = 15$</p>	<p>[\angle in semi circle/\angle in halfsirkel] [Th of/stelling v Pythagoras]</p>	<p>✓ S ✓ using/gebruik Pyth korrek/ correctly ✓ answ/antw (3)</p>
<p>10.2.1</p>	<p>$\frac{CF}{CD} = \frac{CE}{CB}$ $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3,75$</p>	<p>[line one side of Δ/lyn een sy van Δ] OR/OF $\Delta CEF \sim \Delta CBD$</p>	<p>✓ S/R ✓ subst correctly/ korrek ✓ answ/antw (3)</p>
<p>10.2.2</p>	<p>$\hat{BDC} = 90^\circ$ $\hat{EFC} = \hat{BDC}$ $\hat{ABC} = 90^\circ$ In ΔBAC and/en ΔFEC: $\hat{ABC} = \hat{EFC}$ [proven/bewys] $\hat{C} = \hat{C}$ [common/gemeen] $\therefore \Delta BAC \sim \Delta FEC$ [$\angle\angle\angle$] OR/OF $\hat{BDC} = 90^\circ$ $\hat{EFC} = \hat{BDC}$ $\hat{ABC} = 90^\circ$ In ΔBAC and/en ΔFEC: $\hat{ABC} = \hat{EFC}$ [proven/bewys] $\hat{C} = \hat{C}$ [common/gemeen]</p>	<p>[\angle in semi circle/\angle in halfsirkel] [corresp \angles/ooreenk \anglee; EF BD] [tan \perp diameter/raakl \perp middellyn] [proven/bewys] [common/gemeen] [$\angle\angle\angle$] [\angle in semi circle/\angle in halfsirkel] [corresp \angles/ooreenk \anglee; EF BD] [tan \perp diameter/raakl \perp middellyn] [proven/bewys] [common/gemeen]</p>	<p>✓ S/R ✓ S ✓ R ✓ S ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S</p>

	$\hat{B}AC = \hat{F}EC \quad [\angle \text{ sum in } \Delta / \angle \text{ som van } \Delta]$ $\therefore \Delta BAC \parallel \Delta FEC$	<p>✓ S</p> <p>(5)</p>
<p>10.2.3</p>	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC} \quad [\Delta BAC \parallel \Delta FEC]$ $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27 \text{ or/of } 19\frac{4}{15}$ <p>OR/OF</p> $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27 \text{ or/of } 19\frac{4}{15}$ <p>OR/OF</p> $\Delta BCA \parallel \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27 \text{ or/of } 19\frac{4}{15}$ <p>OR/OF</p> $\hat{C} = \hat{A}BD \quad [\text{tan-chord theorem/rkl-kdstelling}]$ $\frac{AD}{8} = \tan \hat{A}BD$ $= \tan \hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27 \text{ or/of } 19\frac{4}{15}$	<p>✓ length of/lengte v EC</p> <p>✓ S</p> <p>✓ subst correctly/ korrek</p> <p>✓ answ/antw</p> <p>(4)</p> <p>✓✓ correct ratios/ korrekte verh's</p> <p>✓ subst correctly/ korrek</p> <p>✓ answ/antw</p> <p>(4)</p> <p>✓ S OR Pyth th</p> <p>✓ correct ratio</p> <p>✓ subst</p> <p>✓ answ/antw</p> <p>(4)</p> <p>✓ S</p> <p>✓ correct ratio</p> <p>✓ subst</p> <p>✓ answ/antw</p> <p>(4)</p>

10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° OR converse \angle in semi circle] <i>AC is middellyn van die sirkel wat deur die punte A, B en C gaan</i> [koord onderspan 90° OF omgek \angle in halfsirkel] $\therefore \text{radius} = \frac{1}{2} \times 19,27 = 9,63$ or/of $9\frac{19}{30}$ or/of $\frac{1}{2} AC$	✓ S/R ✓ answ/antw (2) [17]
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QUESTION/VRAAG 11

11.1	equiangular or similar/ <i>gelykhoekig of gelykvormig</i>	✓ answ/antw (1)
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11.2.1	$\frac{KP}{RN} = \frac{1,5}{0,75} = 2 ; \frac{PM}{NM} = \frac{2}{1} = 2 ; \frac{KM}{RM} = \frac{2,5}{1,25} = 2$ $\therefore \frac{KP}{RN} = \frac{PM}{NM} = \frac{KM}{RM}$ <p>$\therefore \Delta KPM \parallel \Delta RNM$ [Sides of Δ in prop/sye v Δ eweredig]</p> <p>OR/OF</p> $\frac{RN}{KP} = \frac{0,75}{1,5} = \frac{1}{2} ; \frac{NM}{PM} = \frac{1}{2} ; \frac{RM}{KM} = \frac{1,25}{2,5} = \frac{1}{2}$ $\therefore \frac{RN}{KP} = \frac{NM}{PM} = \frac{RM}{KM}$ <p>$\therefore \Delta KPM \parallel \Delta RNM$ [Sides of Δ in prop/sye v Δ eweredig]</p> <p>OR/OF</p> <p>In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore \hat{MNR} = 90^\circ$ [converse Pyth theorem]</p> <p>In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{PKM} = 90^\circ$ [converse Pyth theorem]</p> $\cos \hat{PKM} = \frac{1,5}{2,5} = \frac{3}{5} \text{ and } \cos \hat{R} = \frac{0,75}{1,25} = \frac{3}{5}$ <p>$\therefore \hat{PKM} = \hat{R}$</p> <p>In ΔKPM and ΔRNM $\hat{PKM} = \hat{R}$ [proved] $\hat{P} = \hat{MNR}$ [proved] $\therefore \Delta KPM \parallel \Delta RNM$ [\angle; \angle; \angle OR 3rd \angle]</p>	<p>✓✓✓ all 3 statements/ <i>al 3 bewerings</i> (3)</p> <p>✓✓✓ all 3 statements/ <i>al 3 bewerings</i> (3)</p> <p>✓ $\hat{P} = \hat{MNR}$</p> <p>✓ $\hat{PKM} = \hat{R}$</p> <p>✓ [\angle; \angle; \angle OR 3rd \angle] (3)</p>
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11.2.2	$\hat{P}\hat{K}M = \hat{R}$ [ΔKPM ΔRNM] ✓ S $\therefore \hat{P}$ is common/ <i>gemeen</i> $\therefore \Delta RPQ \Delta KPM$ [∠∠∠] ✓ ΔRPQ ΔKPM $\frac{RP}{KP} = \frac{RQ}{KM}$ [ΔRPQ ΔKPM] ✓ S $\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}$ ✓ subst correctly/ <i>korrek</i> $\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42 \text{ or } 5 \frac{5}{12}$ ✓ $RQ = 5 \frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4 \frac{2}{3}$ ✓ $NQ = \text{answ/antw}$ (6)
	OR/OF $\hat{R}\hat{N}M = \hat{P}$ [ΔKPM ΔRNM] ✓ S $\therefore \hat{R}$ is common/ <i>gemeen</i> $\therefore \Delta RNM \Delta RPQ$ [∠∠∠] ✓ ΔRNM ΔRPQ $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ΔRPQ] ✓ S $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ ✓ subst correctly/ <i>korrek</i> $\therefore RQ = 5,42 \text{ or } 5 \frac{5}{12}$ ✓ $RQ = 5 \frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4 \frac{2}{3}$ ✓ $NQ = \text{answ/antw}$ (6)
	OR/OF In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ ✓ S $\therefore \hat{M}\hat{N}R = 90^\circ$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90^\circ$ [converse Pyth theorem] In ΔMNR and ΔQPR ∠R is common $\hat{M}\hat{N}R = \hat{P} = 90^\circ$ $\therefore \Delta MNR \Delta QPR$ [∠∠∠] ✓ ΔMNR ΔQPR $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ΔRPQ] ✓ S $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ ✓ subst correctly/ <i>korrek</i> $\therefore RQ = 5,42 \text{ or } 5 \frac{5}{12}$ ✓ $RQ = 5 \frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4 \frac{2}{3}$ ✓ $NQ = \text{answ/antw}$ (6)
	[10]

TOTAL/TOTAAL:

149