NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2015

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $(x-3)(x+1)=5$
$x^2-2x-8=0$
$(x-4)(x+2)=0$
$x=4$ or $x=-2$ (3)

(2) $9^{2x-1}=\frac{3^x}{3}$

$3^{2(2x-1)}=3^{x-1}$
$3^{4x-2}=3^{x-1}$

$4x-2=x-1$
$x=\frac{1}{3}$ (3)

(3) $2\sqrt{2-7x}=\sqrt{-36x}$
$\left(2\sqrt{2-7x}\right)^2=\left(\sqrt{-36x}\right)^2$
$4(2-7x)=-36x$
$8-28x=-36x$
$x=-1$ (3)

(b) $x^2+2kx+k=kx+k$
$x^2+kx=0$
$x(x+k)=0$
$\therefore x=0$ or $x=-k$
Point $(0;k)$ and $(-k;k-k^2)$ (5)

(c) (1) $x=\frac{-n\pm\sqrt{n^2-4(9)(49)}}{2(9)}$
$x=\frac{-n\pm\sqrt{n^2-1764}}{18}$ (2)

(2) $n^2-1764=0$
$\therefore n=42$ or $n=-42$ (2)

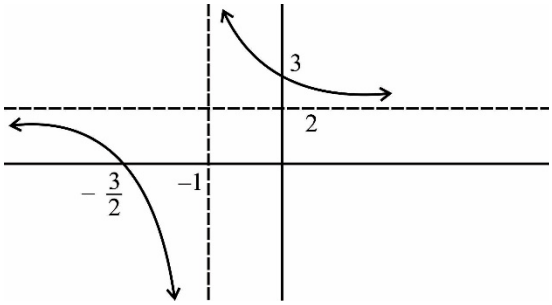
[18]

QUESTION 2

(a) (1) $x = -1$ and $y = 2$ (2)

(2) $x = -\frac{3}{2}$ and $y = 3$ (3)

(3)



(3)

(b) (1)

Let $x = 0$ $\quad 2.3^0 - 1 = 1 \quad y$-intercept $= (0;1)$

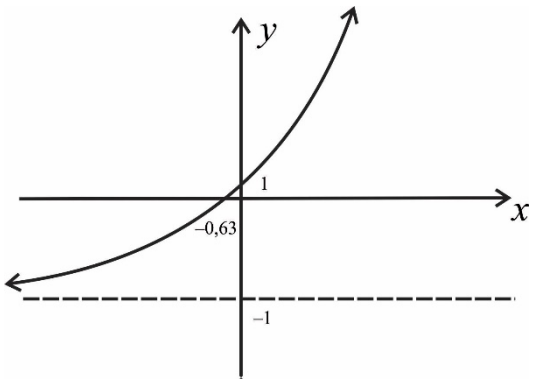
Let $y = 0$ $\quad 2.3^x - 1 = 0$

$3^x = \frac{1}{2}$

$x = -0,63$

x-int $(-0,63;0)$ (4)

(2)



(3)

[15]

QUESTION 3

(a) Deposit paid =

$$1\,800\,000 \times \frac{60}{100} = \text{R}1\,080\,000$$

Balance owed =

$$\text{OR } 180\,000 \times \frac{40}{100} = \text{R}720\,000$$

Therefore R720 000 was financed by the bank was paid by the bank.

$$720\,000 = x\left[\frac{1-\left(1+\frac{8}{1\,200}\right)^{(-10\times12)}}{\frac{8}{1\,200}}\right]$$

$$x = \text{R}8\,736 \qquad (5)$$

(b) $$A = 720\,000\left(1+\frac{8}{1\,200}\right)^{(3\times12)}$$

$$A = \text{R}914\,570$$

$$A = \text{R}914\,571$$

$$F = 8\,736\left[\frac{\left(1+\frac{8}{1\,200}\right)^{36}-1}{\frac{8}{1\,200}}\right]$$

$$F = \text{R}354\,119$$

Balance of the loan $= \text{R}914\,571 - \text{R}354\,119$

$= \text{R}560\,452$

Alternate Solution:

$$P = 8\,736\left[\frac{1-\left(1+\frac{8}{1\,200}\right)^{(-7\times12)}}{\frac{8}{1\,200}}\right]$$

Balance of the loan: $P = \text{R}560\,495$ (5)

(c) Amount paid towards the loan after 3 years $= \text{R}720\,000 - \text{R}560\,452$
$= \text{R}159\,548$
Amount paid through monthly installments $= \text{R}8\,736 \times 36$
$= \text{R}314\,496$
Interest paid $= \text{R}314\,496 - \text{R}159\,548$
$= \text{R}154\,948$
Therefore percentage of total paid to bank as interest charges over 3 years $\frac{154\,948}{314\,496} \times 100 = 49\%$

Alternative:

Using the Balance of loan of $\text{R}560\,495$ (done using Pv formula)
Amount paid towards the loan after 3 years $= \text{R}720\,000 - \text{R}560\,495$
$= \text{R}159\,505$
Amount paid through monthly installments $= \text{R}8\,736 \times 36$
$= \text{R}314\,496$
Difference $= \text{R}314\,496 - \text{R}159\,505$
$= \text{R}154\,991$
Therefore percentage of total paid to bank as interest charges over
3 years $= 49\%$ (4)
[14]

QUESTION 4

(a) $f(x)=\frac{7}{x}$

$$f'(x)=\lim_{h\to 0}\frac{\frac{7}{x+h}-\frac{7}{x}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{7x-7x-7h}{x(x+h)}\times\frac{1}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-7h}{x(x+h)}\times\frac{1}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{-7}{x(x+h)}$$

$$f'(x)=\frac{-7}{x^2} \qquad (4)$$

(b) $D_x\left(\frac{14\pi}{x^{-1}}-3\sqrt[3]{x^2}\right)$

$$D_x\left(14\pi x-3x^{\frac{2}{3}}\right)$$

$$=14\pi-2x^{-\frac{1}{3}}$$

$$=14\pi-\frac{2}{x^{\frac{1}{3}}}$$

$$=14\pi-\frac{2}{\sqrt[3]{x}} \qquad (4)$$

[8]

QUESTION 5

(a) $18 = (6) + (4)(d)$

$d = 3$

$S_{38} = \frac{38}{2}\left(2(6) + (37)(3)\right)$

$S_{38} = 2337$ (4)

(b) (1) $\frac{1\,000}{3} + \frac{1\,000}{3^2} + \frac{1\,000}{3^3}$

$= \frac{13\,000}{27}$ (3)

(2) $S_\infty = \frac{\frac{1\,000}{3}}{1 - \frac{1}{3}}$

$S_\infty = 500$

$\therefore$ Fraction $= \frac{500}{1\,000} = \frac{1}{2}$ (3)

[10]

Alternative:

(1) Day 1: $\frac{1}{3}$ of $1000 = \frac{1000}{3}$ (as decimals: 333,3)

Day 2: $\frac{1000}{3} + \left(\frac{1}{3} \text{ of } \frac{1000}{3}\right) = \frac{4000}{9}$ (as decimal: 444,4)

Day 3: $\frac{4000}{9} + \left(\frac{1}{3} \text{ of } \frac{4000}{9}\right) = \frac{16000}{27}$ (as decimal: 592,6)

Total area blocked = $\frac{16000}{27} \approx 592,6$ sq units

(2) $r = \frac{4}{3}$

Since $r > 1$, the series diverges

Eventually, the entire screen would be blocked out.

QUESTION 6

(a) (1) (iii) Roots are real and equal (1)

(2) (i) Roots are non-real (1)

(3) (ii) Roots are real and unequal (1)

(b) (1) $y = a(x-3)(x-7)$
$6 = a(6-3)(6-7)$
$a = -2$
$y = -2(x-3)(x-7)$
$y = -2x^2 + 20x - 42$ (4)

(2) $y = -2x + c$
$0 = -2(3) + c$
$c = 6$
$y = -2x + 6$ (3)

(3) C and D have x-co-ord of 5.
y- co-ord C: $y = -2(5)^2 + 20(5) - 42 \quad \therefore y = 8$
$C\ (5\ ;\ 8)$
y-co-ord D: $y = -2(5) + 6 \quad \therefore y = -4$
$D(5\ ;-4)$
Length $CD = 12$ units (4)

Alternative:

Vertical Dist between graphs = $-2x^2 + 20x - 42 - (-2x + 6)$
= $-2x^2 + 22x - 48$
Sub. Axis of Symm: $x = 5$
CD = $-2(5)^2 + 22(5) - 48$
= 12

(c) $y = -\frac{1}{50}(x^2 - 100) \quad \therefore x = -\frac{1}{50}(y^2 - 100)$
$y^2 = 100 - 50x$
$y = \sqrt{100 - 50x}\ ;\ 0 \le x \le 2$ (3)

[17]

82 marks

SECTION B

QUESTION 7

(a) $f'(x) = 9x^2 + 2bx + c$

$f'(1) = 9 + 2b + c$

$12 = 9 + 2b + c$

$f''(x) = 18x + 2b$

$f''(1) = 18 + 2b = -24$

$\therefore b = -21$

$12 = 9 + 2(-21) + c$

$c = 45$

$\therefore f(x) = 3x^3 - 21x^2 + 45x - 27$ (7)

(b) $f''(x) < 0$

$18x - 42 < 0$

$x < \frac{7}{3}$ (3)

(c) $f'(x) = 9x^2 - 42x + 45$

$f'(2) = 9(2)^2 - 42(2) + 45$

$f'(2) = -3$

$y = -3x + c$

$3 = -3(2) + c$

$c = 9$

$y = -3x + 9$

$\therefore MN = -3p + 9$ (5)

(d) $f'(x) = 9x^2 - 42x + 45$

$f'(x) = 0$ when $0 = 9x^2 - 42x + 45$

$\therefore (3x^2 - 14x + 15) = 0$

$\therefore (3x - 5)(x - 3) = 0$

$x = 3$ or $x = \frac{5}{3}$

$f(x) = 0$ when $0 = 3x^3 - 21x^2 + 45x - 27$

$\therefore (x - 3)(3x^2 - 12x + 9) = 0$

$\therefore 3(x - 3)^2(x - 1) = 0$

$x = 1$ or $x = 3$

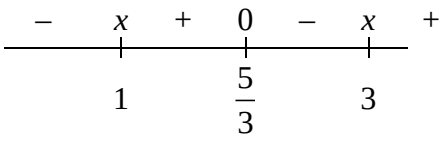
$\therefore x < 1$ or $\frac{5}{3} \leq x < 3$

Alternative:

$$\frac{9x^2 - 42x + 45}{3x^3 - 21x^2 + 45x - 27} \leq 0$$

$$\therefore \frac{3(3x - 5)(x - 3)}{3(x - 3)^2(x - 1)} \leq 0$$

$$\therefore \frac{3x - 5}{(x - 3)(x - 1)} \leq 0$$



$\therefore x < 1$ or $\frac{5}{3} \leq x < 3$

(8)

[23]

QUESTION 8

(a) $10-3y \quad ; \quad 7 \quad ; \quad 15 \quad ; \quad 8y+1$

$-3+3y \qquad 8 \qquad 8y-14 \rightarrow$ first level difference

$11-3y \qquad 8y-22 \rightarrow$ second level difference

$\therefore 11-3y=8y-22$

$\therefore y=3$ (4)

(b) (1) $a=250$

$r=4$

$$Sn=\frac{a\left(r^n-1\right)}{r-1}$$

$$21\,845\,250=\frac{250\left(4^n-1\right)}{3}$$

$4^n=262\,144$

$n=9$

$\therefore$ Level 9 (5)

(2) $r=4 \quad a=1$

$T_n=4^{n-1}$

$\therefore T_{n+1}-T_n=4^n-4^{n-1}$

$\therefore 4^n-4^{n-1}=6\times 2^{17}$

$$\therefore 4^n\left(1-\frac{1}{4}\right)=6\times 2^{17}$$

$\therefore 4^n=4^{10}$

$\therefore n=10$

$\therefore$ between the 10^{th} and 11^{th} levels. (7)

[16]

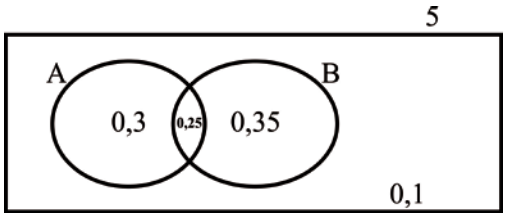
QUESTION 9

(a) For an odd product, the two numbers must be odd
First draw: 4 odd out of 7cards
Second Draw: 3 odd out of 6

$$\text{P(Odd Product)} = \frac{4}{7} \times \frac{3}{6}$$

$$\text{P(Odd Product)} = \frac{2}{7} \approx 0,3 \quad (4)$$

(b) (1)



(3)

(2) $P(A \text{ and } B') = 0,55 - 0,25 = 0,3$ (2)

(3) $P(A \text{ or } B') = 1 - 0,35 = 0,65$ (1)

(c) (1) Number of different orders possible $= 6! = 720$ (1)

(2) If Rome, Madrid and Florence are grouped, different orders of objects $= 4! = 24$ (1)

(3) P(Rome, Madrid and Florence grouped) $= \frac{3! \times 4!}{6!}$

$$= \frac{144}{720}$$

$$= 0,2 \quad (3)$$

[15]

QUESTION 10

(a) $\left(5\sqrt{3}\right)^2 = x^2 + p^2$ pythag

$p^2 = 75 - x^2$

$V = \frac{1}{3}\pi(75 - x^2)(x)$

$V = 25\pi x - \frac{1}{3}\pi x^3$ $\qquad V' = 0$

$V' = 25\pi - \pi x^2$

$0 = 25\pi - \pi x^2$

$x = \pm 5$

$x = 5$ (7)

Alternative: If cone is inverted:

An inverted cone sharing the same base as the hemisphere will have a maximum volume since its radius will be at its maximum.
Hence the maximum height of the cone will be equal to the radius of the hemisphere.
Therefore: Radius of cone will be $5\sqrt{3}$
Hence: Height of the Cone will also be $5\sqrt{3}$

(b) Busi and Khanya will meet 25 metres from the winning line.
Busi crosses the finishing line first.
Khanya will be $25 - \frac{3}{4}(25)$ metres from the winning line.
Khanya will be $6{,}25$ metres from the winning line. (7)

Note:

Calculating the 93,75 or 6,25 and implying that Busi was ahead.

If all calculations done correctly, but stated Khanya was ahead.

When working with where Busi was when Khanya corssed the line: 8,33 m ahead.

If stating that Busi's speed was *x* km/h therefore Khanya's speed is $\frac{3}{4}$ *x*. Busi runs 125 m.

Just stating Busi wins.

Just stating Busi runs 125 m.

Alternative:

If Busi ran 100 m in x seconds

Then Khanya ran 75 m in x seconds

$\therefore$ Khanya would run 100 m in: $\left(\frac{x}{75}\times 100\right)$ seconds

When Busi started 25 m behind the starting line:

She ran: 125 m in: $\left(\frac{x}{100}\times 125\right)$ seconds

$\therefore$ in $\left(\frac{x}{75}\times 100\right)$ seconds, Khanya would run:

$$\left(\frac{100}{\frac{x}{75}\times 100}\right)\times\left(\frac{x}{100}\times 125\right)$$

$= 93,75$ m

Alternative:

Busi and Khanya will meet 25 metres from the winning line.
Busi crosses the winning line first

Khanya will be $25-\frac{3}{4}(25)$ metres from the winning line.

Khanya will be 6,25 metres from the winning line.

Alternative:

Busi crosses the winning line first

Khanya would have covered $\frac{3}{4}(125) = 93,75$ m when Busi crossed

Khanya will be 6,25 metres from the winning line.

[14]

68 marks

Total: 150 marks