



Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2015

MEMORANDUM

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 150

This memorandum consists of 11 pages.

QUESTION 1

1.1.1	$x(x-5) = 0$ $x = 0$ or $x = 5$	✓✓ Answers	2
1.1.2	$4x^2 - 5x = 3$ $4x^2 - 5x - 3 = 0$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2(4)}$ $x = 1,69$ or $x = -0,44$	✓ equation in std. form ✓ substitution ✓✓ answers	4
1.1.3	$2^x(3x+1) < 0$ $2^x > 0$ for all $x \in R$ $\therefore 3x+1 < 0$ $x < -\frac{1}{3}$	✓ $2^x > 0$ and $x \in R$ ✓ $3x+1 < 0$ ✓ answer	3
1.1.4	$x - 3x^{\frac{1}{2}} = 4$ $x - 4 = 3\sqrt{x}$ $9x = x^2 - 8x + 16$ $x^2 - 17x + 16 = 0$ $(x-1)(x-16) = 0$ $x = 1$ or $x = 16$ <i>n/a</i> OR $x - 3x^{\frac{1}{2}} - 4 = 0$ $\left(x^{\frac{1}{2}} - 4\right)\left(x^{\frac{1}{2}} + 1\right) = 0$ $x^{\frac{1}{2}} = 4$ or $x^{\frac{1}{2}} = -1$ $x = 16$ no solution	✓ isolating $3\sqrt{x}$ or $3x^{\frac{1}{2}}$ ✓ squaring both sides ✓ trinomial ✓ factors ✓ both answers ✓ selection ✓ standard form ✓ factors ✓ $x^{\frac{1}{2}} = 4$ ✓ $x^{\frac{1}{2}} = -1$ ✓ answer ✓ rejecting	6
1.2	$\frac{\sqrt{9^{2028}}}{\frac{\sqrt{9^{2030}} - \sqrt{9^{2026}}}{\sqrt{9^{2028}}}}$ $\frac{\sqrt{9^{2028}}}{\sqrt{9^{2026}}(\sqrt{9^4} - 1)}$ $= \frac{\sqrt{9^2}}{80}$ $= \frac{9}{80}$	✓ factorising denominator ✓ simplifying ✓ answer	3

1.3	$2^{3x+1} = 4^y$ and $x^2 + 2y = 29$ $3x + 1 = 2y \rightarrow (1)$ $x^2 + 2y = 29 \rightarrow (2)$ Subst. (1) into (2) $x^2 + 3x + 1 = 29$ $x^2 + 3x - 28 = 0$ $(x + 7)(x - 4) = 0$ $x = -7$ or $x = 4$ $y = -10$ or $y = \frac{13}{2}$	✓ for $2y$ as subject ✓ substitution of $2y$ ✓ std. form ✓ factors ✓ x values ✓ y values	6
			[24]

QUESTION 2

2.1.1	$3 ; 2 ; 6 ; 2 ; 9 ; 2 ; \dots$ $12 ; 2$	✓✓ answers	2
2.1.2	$a = 3, d = 3, n = 50$ $S_{50} = \frac{50}{2}[3 + 150]$ or $\frac{50}{2}[2(3) + 3(50 - 1)]$ $= 3825$ $a = 2$ and is a constant sequence $\therefore S_{50} = 50 \times 2$ $= 100$ $S_{100} = 3825 + 100$ $= 3925$	✓ for a, d and n values ✓ for substitution into formula ✓ 3825 ✓ 100 ✓ answer	5
2.2	$S_n = a + ar + ar^2 + \dots + ar^{n-1} \rightarrow (1)$ $rS_n = ar + ar^2 + \dots + ar^n \rightarrow (2)$ $(2) - (1) :$ $rS_n - S_n = ar^n - a$ $S_n(r - 1) = a(r^n - 1)$ $\therefore S_n = \frac{a(r^n - 1)}{r - 1}$	✓ for equation (1) ✓ for equation (2) ✓ subtraction on LHS and RHS ✓ factorising	4
			[11]

	$T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= 5 + (n-1)(2) + \frac{(n-1)(n-2)}{2}(4)$ $= 5 + 2n - 2 + 2n^2 - 6n + 4$ $= 2n^2 - 4n + 7$	<p>✓ formula</p> <p>✓ substituting first and second difference values</p> <p>✓ simplifying</p> <p>✓ general term</p>	4
4.2	<p>First Difference n^{th} term = $(4n-2)$</p> $4n - 2 = 2018$ $4n = 2020$ $n = 505$ $T_{506} = 2(506)^2 - 4(506) + 7 = 510055$ $T_{505} = 2(505)^2 - 4(505) + 7 = 508037$ <p>Between the 505th and the 506th terms.</p> <p>OR</p> $T_n - T_{n-1} = 2n^2 - 4n + 7 - [2(n-1)^2 - 4(n-1) + 7]$ $= 2n^2 - 4n + 7 - [2n^2 - 4n + 2 - 4n + 4 + 7]$ $= 4n - 6$ $4n - 6 = 2018$ $4n = 2024$ $n = 506$ <p>Between the 505th and the 506th terms.</p>	<p>✓ $4n - 2 = 2018$</p> <p>✓ $n = 505$</p> <p>✓ answer</p> <p>✓ $T_n - T_{n-1}$ substitution</p> <p>✓ $4n - 6 = 2018$</p> <p>✓ answer</p>	3
			[7]

QUESTION 5

5.1	$A = P(1+i)^n$ $2x = x(1+0,085)^n$ $n = \frac{\log 2}{\log(1+0,085)}$ $= 8,5 \text{ or } 9 \text{ years or } 8 \text{ years and } 6 \text{ months}$	✓✓ correct substitution into correct formula ✓ making n the subject ✓ answer	4
5.2.1	$A = P(1+i)^n$ $= 350\,000 \left(1 + \frac{0,135}{12}\right)^2$ $= R357919,30$	✓ correct substitution into correct formula ✓ answer	2
5.2.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $357919,30 = \frac{x \left[1 - \left(1 + \frac{0,135}{12}\right)^{-238}\right]}{\frac{0,135}{12}}$ $x = \frac{357919,30 \times \frac{0,135}{12}}{\left[1 - \left(1 + \frac{0,135}{12}\right)^{-238}\right]}$ $x = R4328,60$	✓✓ correct substitution into correct formula ✓ x as subject ✓ answer	4
5.2.3	Balance on Loan after the 180th payment $= 357919,30 \left(1 + \frac{0,135}{12}\right)^{178} - \frac{4328,60 \left[\left(1 + \frac{0,135}{12}\right)^{178} - 1\right]}{\frac{0,135}{12}}$ $= R188118,54$ Total Payments = $R4328,60 \times 60 = R259716$ Savings = $R259716 - R188118,54$ $= R71597,46$	✓✓ substitution into formulae ✓ R188118,54 ✓ R1030206,80 ✓ answer	5
			[15]

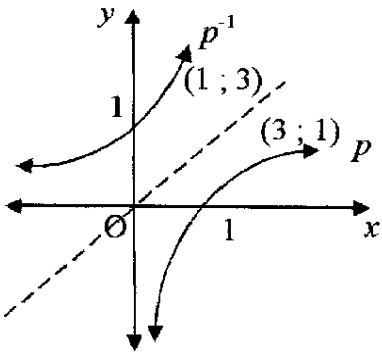
QUESTION 6

6.1	$f(x) = \frac{a}{x+p} + q$. P(-2; 4) is a point on f $p = 1$ $q = 1$ $y = \frac{a}{x+1} + 1$ Now subst. P(-2; 4): $4 = \frac{a}{-2+1} + 1$ $\therefore a = -3$	\checkmark p value \checkmark q value \checkmark subst. p, q and point P \checkmark a value	4	
6.2	$\frac{-3}{x+1} + 1 = 0$ $x+1 = 3$ $x = 2$ $T(2; 0)$	\checkmark $y = 0$ \checkmark $x+1 = 3$ \checkmark answer	3	
6.3	$y = (x+p) + q$ $y = (x+1) + 1$ $y = x + 2$ $\therefore c = 2$ OR $y = x + c$ Point of intersection of asymptotes (-1 ; 1) $-1 = 1 + c$ $c = 2$	\checkmark substitution of p and q values into equation of line of symmetry \checkmark answer \checkmark substitution of p and q values into equation of line of symmetry \checkmark answer	2 2	
			[9]	

QUESTION 7

7.1	$g(x) = -x^2 + 2x + 3$ and $h(x) = ax + q$. CD = 6 units. $-x^2 + 2x + 3 = 0$ $(x+1)(x-3) = 0$ $x = -1$ or $x = 3$ B(3; 0) C(0; 3)	$\checkmark y = 0$ \checkmark factors \checkmark coordinates of B \checkmark coordinates of C	4
7.2	D(0; -3)	$\checkmark\checkmark$ answer	2
7.3	$a = 1$ and $q = -3$	$\checkmark\checkmark$ answers	2
7.4	$-x^2 + 2x + 3 = x - 3$ $x^2 - x - 6 = 0$ $(x+2)(x-3) = 0$ $x = -2$ or $x = 3$ $y = -5$ or $y = 0$ E(-2; -5)	\checkmark equating both equations \checkmark factors \checkmark x values \checkmark y values \checkmark answer	5
7.5	$g: y = -(x-1)^2 + 4$ $-1 < x < 1$ or $x > 3$	\checkmark Axis of symmetry value $\checkmark\checkmark$ end points and inequality \checkmark $x > 3$	4
			[17]

QUESTION 8

8.1	$p(x) = \log_3 x$ $x = \log_3 y$ $p^{-1}: y = 3^x$	$\checkmark\checkmark$ answer	2
8.2		$\checkmark\checkmark$ graph of p $\checkmark\checkmark$ graph of p^{-1}	4
8.3	$\log_3 x = 2$ $x = 9$ $0 < x \leq 9$	$\checkmark\checkmark$ for end points and inequality	2
8.4	x intercept of p is (1; 0) x intercept of h is (-1; 0)	$\checkmark\checkmark$ answer	2
			[10]

QUESTION 9

9.1	$f(x) = x^2 + 3x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h + 3)}{h}$ $= 2x + 3$	<ul style="list-style-type: none"> ✓ $f(x+h) = (x+h)^2 + 3(x+h)$ ✓ substitution into correct formula ✓ simplifying ✓ factors ✓ answer 	5	
9.2.1	$\frac{dy}{dx} \text{ if } y = 3x^2 \cdot \sqrt[3]{8x^4}$ $y = 6x^{\frac{10}{3}}$ $\frac{dy}{dx} = 20x^{\frac{7}{3}} \text{ or } 20\sqrt[3]{x^7}$	<ul style="list-style-type: none"> ✓ 6 ✓ $x^{\frac{10}{3}}$ ✓ answer 	3	
9.2.2	$f'(x) \text{ if } f(x) = \frac{x^3 - 5x^2 + 4x}{x-4}$ $f(x) = \frac{x(x^2 - 5x + 4)}{(x-4)}$ $= \frac{x(x-1)(x-4)}{(x-4)}$ $= x^2 - x$ $f'(x) = 2x - 1$	<ul style="list-style-type: none"> ✓ factorising ✓ factors ✓ simplification of f ✓ answer 	4	
			[12]	

QUESTION 10

10.1	$y = \frac{4}{x}$ & the gradient of the tangent to the curve is -1 . $y = 4x^{-1}$ $\frac{dy}{dx} = -4x^{-2} = \frac{-4}{x^2}$ $\frac{-4}{x^2} = -1$ $x^2 = 4$ $\therefore x = -2$ or $x = 2$ $y = -2$ or $y = 2$ $(-2; -2)$ $(2; 2)$	✓ exponential form ✓ derivative ✓ derivative = -1 ✓ x - values ✓ y - values	5
10.2.1	$y = (x-1)(x-4)^2$ $= (x-1)(x^2 - 4x + 4)$ $= x^3 - 9x^2 + 24x - 16$	✓ $(x-1)(x-4)^2$ ✓ squaring binomial	2
10.2.2	$y = x^3 - 9x^2 + 24x - 16$ $\frac{dy}{dx} = 3x^2 - 18x + 24 = 0$ $x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ $x = 2$ or $x = 4$ $y = 4$ or $y = 0$ $B(2; 4)$	✓ derivative and equal to 0 ✓ factors ✓ y - values ✓ coordinates of B	4
10.2.3	$k < -16$	✓ inequality ✓ -16	2
10.2.4	$f''(x) = 6x - 18 > 0$ $x > 3$	✓ $6x - 18 > 0$ ✓ answer	2
			[15]

QUESTION 11

11.1	$V = l \times b \times h$ $= 5x(9 - 2x)(x)$ $= 45x^2 - 10x^3$	✓ formula ✓ substitution	2
11.2	$V' = 90x - 30x^2$ $90x - 30x^2 = 0$ $30x(3 - x) = 0$ $x = 0$ or $x = 3$ Therefore the box will have a maximum at $x = 3$	✓ derivative ✓ derivative equal to 0 ✓ factors ✓ x values ✓ choosing $x = 3$	5
			[7]

QUESTION 12

12.1	$(x + 0,2) \times 0,5 = 0,2$ $x + 0,2 = 0,4$ $x = 0,2$ $0,2 + 0,2 + 0,3 + y = 1$ $y = 0,3$	✓ $P(A) \times P(B) = P(A \text{ and } B)$ ✓ x value... P(A only) ✓ $y = 1 - 0,7$ ✓ y value	4
12.2	$P(X \text{ and } Y \text{ together}) = \frac{2!5!}{6!}$ $= \frac{1}{3}$ $P(X \text{ and } Y \text{ not together}) = 1 - \frac{1}{3}$ $= \frac{2}{3}$	✓ $\frac{2!5!}{6!}$ ✓ $\frac{1}{3}$ ✓ answer	3
12.3.1	4 digit numbers(with repetitions) $= 9 \times 10 \times 10 \times 10$ $= 9000$	✓ product ✓ answer	2
12.3.2	4 digit numbers(without repetitions) $= 9 \times 9 \times 8 \times 7$ $= 4536$	✓ 9×9 ✓ 8×7 ✓ answer	3
12.3.3	4 digit numbers(with repetitions and last digit = 0) $= 9 \times 10 \times 10 \times 1$ $= 900$	✓ product ✓ answer	2
			[14]

TOTAL MARKS: 150