Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

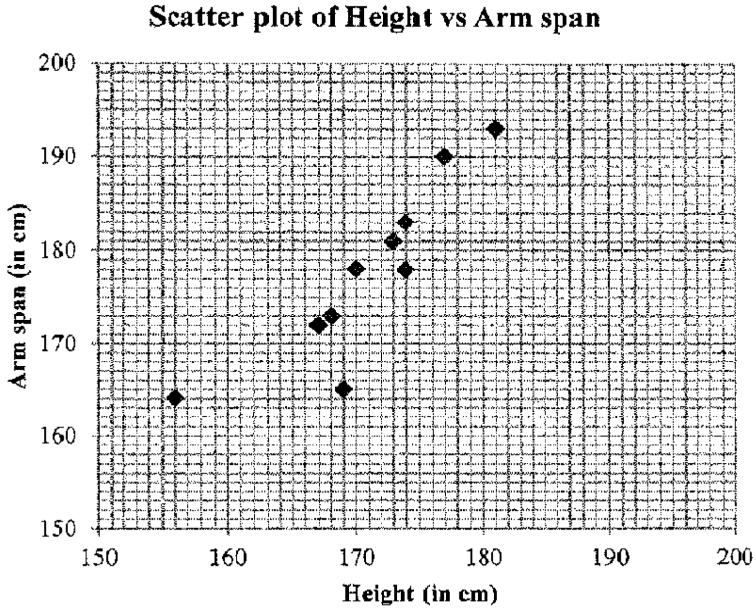
SEPTEMBER 2015

MEMORANDUM

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

QUESTION 1

1.1		A A A	✓ 1 – 4 points correct ✓ 5 – 9 points correct ✓ all points correct (3)
1.2	$a = -36,58$ $(a = -36,57689....)$ $b = 1,25$ $(b = 1,25381....)$ $\hat{y} = -36,58 + 1,25x$	AA A CA	✓✓ a ✓ b ✓ equation Eqn. only 4/4 (4)
1.3	$\hat{y} = -36,58 + 1,25(176)$ $= 183,42$ OR $\hat{y} = 184,09$	CA A AA	✓ substitute 176 ✓ answer (2) ✓✓ answer (2)
1.4	There is strong, positive correlation between height and arm span.	A	✓ strong, positive (1) **[10]**

QUESTION 2

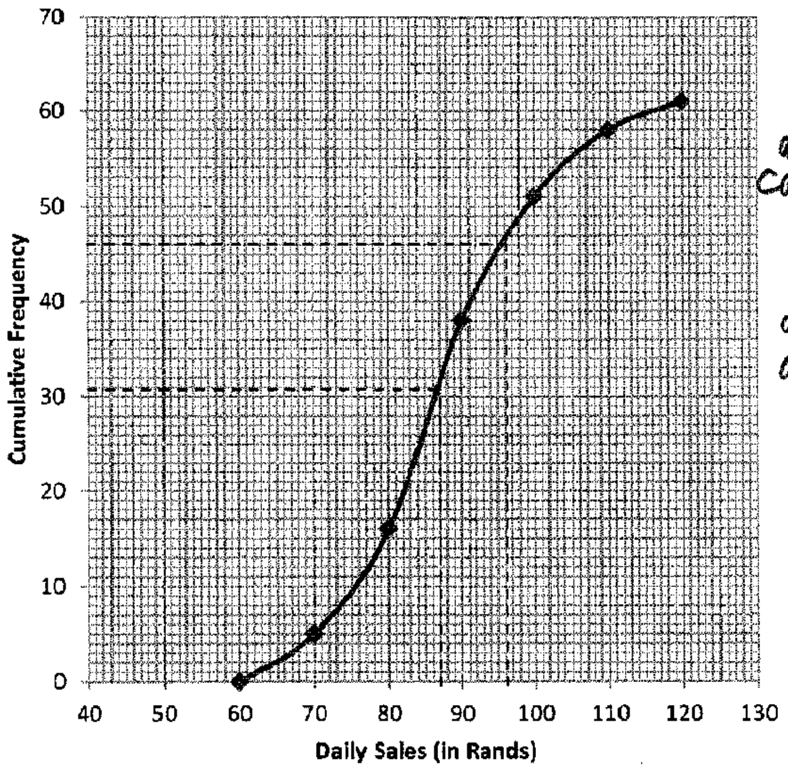
2.1

Daily Sales	Frequency	Cumulative Frequency
$60 \le x < 70$	5	5
$70 \le x < 80$	11	16
$80 \le x < 90$	22	38
$90 \le x < 100$	13	51
$100 \le x < 110$	7	58
$110 \le x < 120$	3	61

a ✓ first two cumulative frequencies correct
a ✓ next two cumulative frequencies correct
a ✓ remainder correct (total = 61)
(3)

2.2

Cumulative frequency graph of Daily Sales



a ✓ grounding at 0
ca ✓ plotting cumulative frequencies at upper limits
a ✓ points correct
a ✓ smooth shape of curve
(4)

2.3 The median for the data is approximately R 87 .
(Accept 85-89)

ca ✓ reading off from graph
a ✓ R87
(2)

2.4 The upper 25% interval is R96 to R120
(Range to accept: 94 to 120)

a ✓ 96 to 120
(1)

[10]

QUESTION 3

3.1	$\frac{x_D - 1}{2} = 2$ $\frac{y_D + 0}{2} = 2$ $x_D = 5$ $x_D = 4$ D(5 ; 4)	a ✓ $x_D = 5$ a ✓ $y_D = 4$ (2)
3.2	$m_{CD} = \frac{4-(-2)}{5-2}$ $= 2$ $\tan\ \alpha = 2$ $\therefore \alpha = 63{,}4°$	ca ✓ substitution into gradient formula ca ✓ $\tan\alpha = 2_{CD}$ a ✓answer (3)
3.3	$m_{AB} = m_{CD} = 2$ AB‖CD, equal gradients $y = 2x + c$ $0 = 2(-1) + c$ $c = 2$ $y = 2x + 2$	ca ✓ $m_{AB} = 2$ ca ✓ subst (–1 ; 0) a ✓ answer (3)
3.4	$m_{AD} = \frac{4-(0)}{5-(-1)}$ $= \frac{2}{3}$ tan (∠ of inclination of AD) $= \frac{2}{3}$ ∠ of inclination of AD = 33,7° $\theta = 63{,}4° - 33{,}7°$ $\therefore = 29{,}7°$	ca ✓ $m = \frac{2}{3}$ ca on D a ✓33,7° ca ✓29,7° acute (3)

3.5	$3AB = DC$ $\therefore 9AB^2 = DC^2$ $9\left[(x+1)^2+(y-0)^2\right]=(5-2)^2+(4+2)^2$ ca $9\left[(x+1)^2+y^2\right]=45$ $AB \parallel DC, \quad \therefore \frac{y-0}{x+1}=2$ $(x+1)^2+y^2=5$........................(1) $y=2x+2$................(2) *Substitute* (2) *in* (1) $(x+1)^2+(2x+2)^2=5$ ca $x^2+2x+1+4x^2+8x+4-5=0$ $5x^2+10x=0$ a $5x(x+2)=0$ $x \neq 0 \quad ; \quad x=-2$ a *Substitute* $x=-2$ *in* (1) $y=-2$ a $\underline{B(-2;-2)}$	✓ substitution Sub. 3.3 ✓ substitute $y = 2x + 2$ ✓ standard form ✓ $x=-2$ ✓ $y=-2$ (5) **[16]**

QUESTION 4

4.1	$r^2 = (2-4)^2 + (3-5)^2$ $= 8$ $(x-2)^2 + (y-3)^2 = 8$	✓ subst into distance formula ✓ 8 ✓ $(x-2)^2$ ✓ $(y-3)^2$ (4)
4.2	$m_{NP} = \frac{5-3}{4-2} = 1$ $m_{PT} = -1$ NP ⊥ PT, product of gradients = – 1 $y = -x + c$ $5 = -4 + c$ $c = 9$ $y = -x + 9$ $0 = -x + 9$ $x = 9$ ∴ T(9 ; 0)	✓ $m_{NP} = 1$ ✓ $m_{PT} = -1$ ✓ subst (5; 4) ✓ $c = 9$ ✓ $y = -x + 9$ ✓coordinates of T (6)
4.3	$PT = \sqrt{(9-4)^2 + (5-0)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$	✓ substitution into distance formula ✓ $\sqrt{50}$ (2)
4.4	$\text{Area} = \pi \times PT^2$ $= \pi \times 50$ $= 157$	✓ substitution into area formula ✓ 157 (2)
4.5	$\tan \cancel{N\hat{P}T}\ N\hat{T}P = \frac{\sqrt{8}}{\sqrt{50}}$ $N\hat{T}P\ \cancel{N\hat{P}T} = 21{,}8°$	✓ $\tan N\hat{P}T = \frac{\sqrt{8}}{\sqrt{50}}$ 4.1 4.3 ✓21,8° (2)
4.6	NP = NM radii PT = TM radii ∴ MNPT is a kite two pairs of adjacent sides equal in length	✓ S/R ✓ S/R ✓ reason (3)

 Please turn over

4.7	$N\hat{T}P = N\hat{T}M = 21{,}8°$ diagonal of kite	ca	✓ S/R on 4.5
	$N\hat{P}T = N\hat{M}T = 90°$ tangent perpendicular to radius	a	✓ S/R
	$N\hat{M}P = 360° - 90° - 90° - 43{,}6°$ angles in quadilateral	ca	✓ S/R
	$= 136{,}4°$	a	✓ answer
			(4)
	OR		
	$NP \perp PT$ (rad $\perp$ tan)	ca	✓ S/R
	$\therefore T\hat{N}P = 68{,}2°$ $(\angle \text{ sum } \Delta)$	a	✓ S/R
	$T\hat{N}P = T\hat{N}M$ (prop of kite)	ca	✓ S/R
	$\therefore M\hat{N}P = 2(68{,}2°)$		
	$\therefore M\hat{N}P = 136{,}4°$	a	✓ answer
			(4)
			[23]

Please turn over

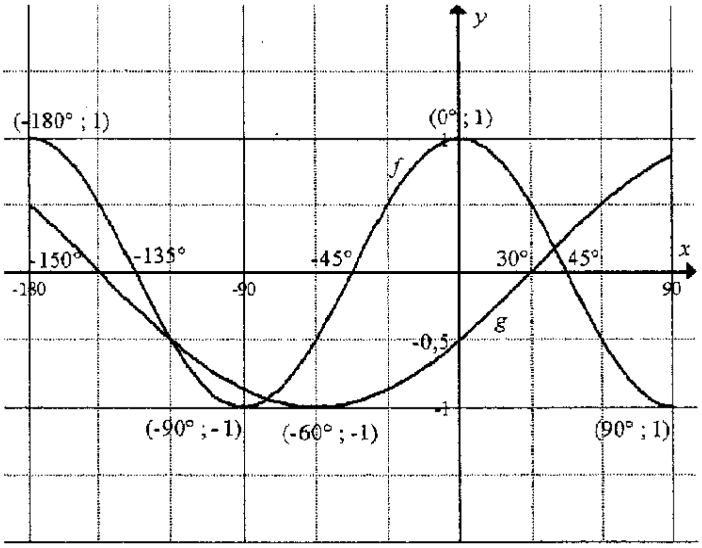
QUESTION 5

5.1	$\dfrac{\tan(180° + A) \cdot \cos(180° - A) \cdot \sin(360° - A)}{\cos(90° - A)}$ $= \dfrac{(\tan A)(-\cos A)(-\sin A)}{\sin A}$ $= \dfrac{\sin A}{\cos A} \cdot \cos A$ $= \sin A$	✓ tan A ✓ − cos A ✓ − sin A ✓ sin A ✓ $\dfrac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$\cos 52° = \cos 2(26°)$ $= 2\cos^2 26° - 1$ $= 2(r)^2 - 1$ $= 2r^2 - 1$	✓ writing 52° in terms of 26° ✓ expansion ✓ answer (3)
5.2.2	$\tan 71° = \dfrac{\sin 71°}{\cos 71°}$ $= \dfrac{\sin(45° + 26°)}{\cos(45° + 26°)}$ $= \dfrac{\sin 45° \cos 26° + \cos 45° \sin 26°}{\cos 45° \cos 26° - \sin 45° \sin 26°}$ $= \dfrac{\left(\frac{\sqrt{2}}{2}\right) r + \left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{1 - r^2}\right)}{\left(\frac{\sqrt{2}}{2}\right) r - \left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{1 - r^2}\right)}$ $= \dfrac{\left(\frac{\sqrt{2}}{2}\right)\left(r + \sqrt{1 - r^2}\right)}{\left(\frac{\sqrt{2}}{2}\right)\left(r - \sqrt{1 - r^2}\right)}$ $= \dfrac{r + \sqrt{1 - r^2}}{r - \sqrt{1 - r^2}}$	✓ identity ✓ writing in terms of 26° ✓ expansions ✓ $\sqrt{1 - r^2}$ ✓ substitution ✓ answer (6)

Please turn over

5.3	$LHS = \dfrac{\sin 2x}{\cos 2x + \sin^2 x}$	
	$= \dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x + \sin^2 x}$	a ✓ identity for sin2x a ✓ identity for cos 2x
	$= \dfrac{2\sin x\cos x}{\cos^2 x}$	a ✓ simplification
	$= \dfrac{2\sin x}{\cos x}$	a ✓ identity
	$= 2\tan x$	(4)
	$= RHS$	
	OR	
	$LHS = \dfrac{\sin 2x}{\cos 2x + \sin^2 x}$	
	$= \dfrac{2\sin x\cos x}{2\cos^2 x - 1 + \sin^2 x}$	a ✓ identity for sin2x a ✓ identity for cos 2x
	$= \dfrac{2\sin x\cos x}{2\cos^2 x - (1 - \sin^2 x)}$	
	$= \dfrac{2\sin x\cos x}{2\cos^2 x - \cos^2 x}$	
	$= \dfrac{2\sin x\cos x}{\cos^2 x}$	a ✓ simplification
	$= \dfrac{2\sin x}{\cos x}$	a ✓ identity (4)
	$= 2\tan x$	
	$= RHS$	
	OR	
	$LHS = \dfrac{\sin 2x}{\cos 2x + \sin^2 x}$	
	$= \dfrac{2\sin x\cos x}{1 - 2\sin^2 x + \sin^2 x}$	a ✓ identity for sin2x a ✓ identity for cos 2x
	$= \dfrac{2\sin x\cos x}{1 - \sin^2 x}$	
	$= \dfrac{2\sin x\cos x}{\cos^2 x}$	a ✓ simplification
	$= \dfrac{2\sin x}{\cos x}$	
	$= 2\tan x$	a ✓ identity
	$= RHS$	(4)
		[19]

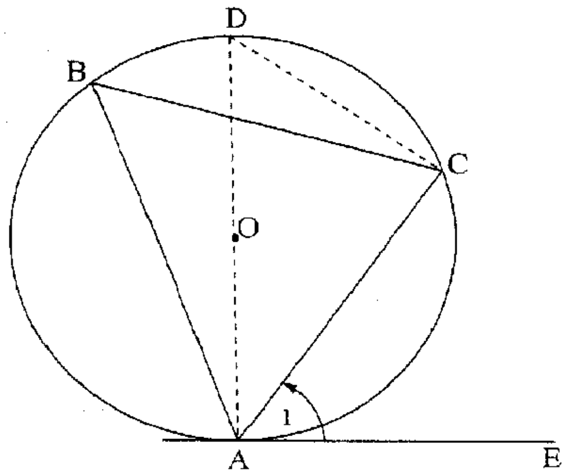
QUESTION 6

6.1	$\cos 2x = \sin(x-30°)$ $= \cos[90° - (x - 30°)]$ $= \cos(120° - x)$ key angle $= 120° - x$ $2x = 120° - x + n.360°; n \in Z$ $3x = 120° + n.360°; n \in Z$ $x = 40° + n.120°; n \in Z$ or $2x = 360° - (120° - x) + n.360°; n \in Z$ $2x = 240° + x + n.360°; n \in Z$ $x = 240° + n.360°; n \in Z$	✓ using co-ratio ✓ $120° - x$ ✓ $2x = 120° - x + n.360°$ ✓ $x = 40° + n.120°$ ✓ $-2x = 360° - (120° - x) + n.360°$ ✓ $x = 240° + n.360°$ ✓ $n \in Z$ (7)
6.2	 	f ✓ x-intercepts ✓ turning points ✓ shape end pts g ✓ intercepts ✓ turning points ✓ shape (6)
6.3	$-120° < x < -80°$ or $40° < x \leq 90°$	✓ critical values: $-120°$; $-80°$ ✓ critical values: $40°$; $90°$ ✓ correct notation (3) **[16]**

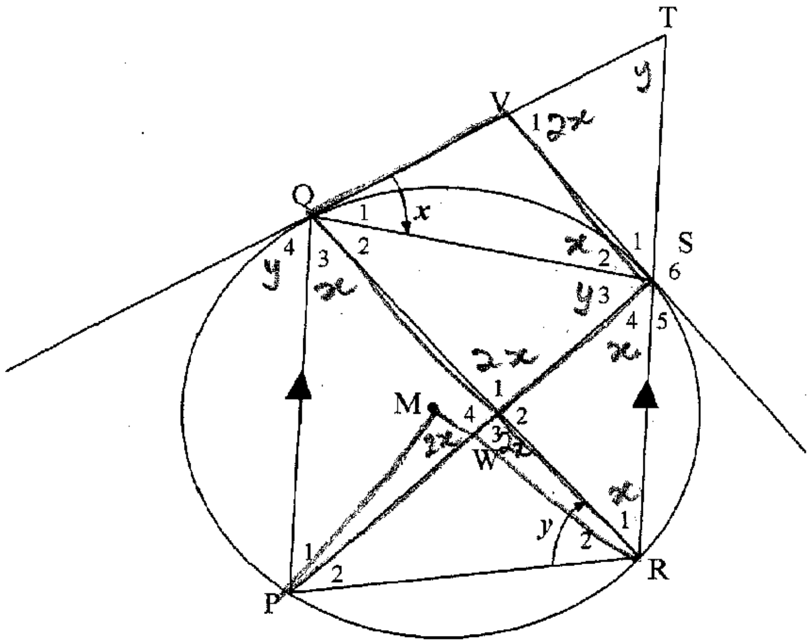
QUESTION 7

7.1	$\tan x = \frac{AB}{BD}$ $= \frac{h}{BD}$ $BD = \frac{h}{\tan x}$	a a	✓ using tan ratio ✓ $BD = \frac{h}{\tan x}$ (2)
7.2	BC = BD $CD^2 = BC^2 + BD^2 - 2BC.BD.\cos y$ $= \left(\frac{h}{\tan x}\right)^2 + \left(\frac{h}{\tan x}\right)^2 - 2\left(\frac{h}{\tan x}\right)\left(\frac{h}{\tan x}\right).\cos y$ $= \frac{h^2}{\tan^2 x} + \frac{h^2}{\tan^2 x} - \frac{2h^2}{\tan^2 x}.\cos y$ $= \frac{2h^2}{\tan^2 x} - \frac{2h^2}{\tan^2 x}.\cos y$ $= \frac{2h^2}{\tan^2 x}(1 - \cos y)$ $= \frac{2h^2(1-\cos y)}{\tan^2 x}$	a Ca a a	✓ using cosine formula ✓ substitution 7.1. ✓simplification ✓common factor (4) **[6]**

QUESTION 8

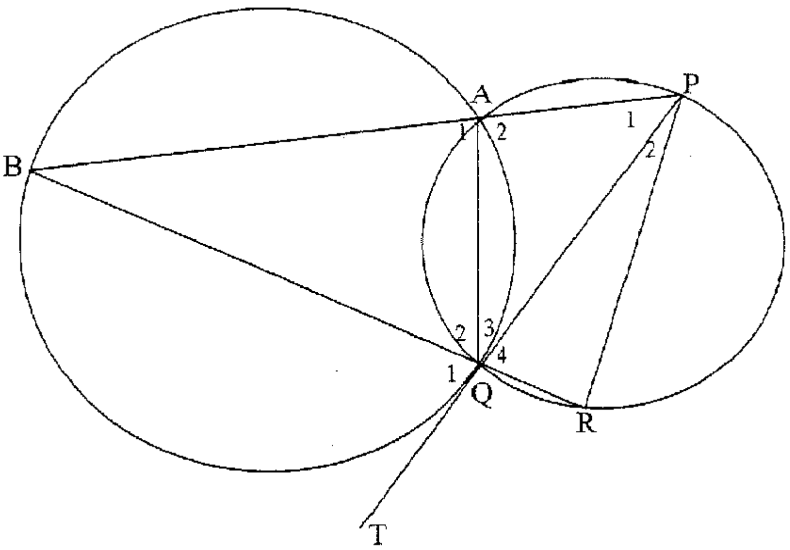
8.1			
	Construction: Draw diameter AD. Join D to C		✓ construction
	Proof:		
	$E\hat{A}C + D\hat{A}C = 90°$	tan ⊥ radius	✓ S / R
	$D\hat{C}A = 90°$	∠ in semi circle	✓ S / R
	$A\hat{D}C + D\hat{A}C = 90°$	sum of ∠s in Δ	✓ S / R
	$\therefore E\hat{A}C = A\hat{D}C$		
	But $A\hat{B}C = A\hat{D}C$	∠s in the same segment	✓ S ✓ R
	$\therefore E\hat{A}C = A\hat{B}C$		(6)

Please turn over



8.2.1	Two tangents drawn from the same external point are equal in length. a	✓ answer (1)
8.2.2 (a)	$\hat{S}_2 = x$ angles opposite equal sides a	✓ S ✓ R (2)
8.2.2 (b)	$\hat{R}_1 = x$ tan-chord theorem a	✓ S ✓ R (2)
8.2.2 (c)	$\hat{V}_1 = \cancel{x}\,2x$ ext ∠ of Δ a	✓ S ✓ R (2)
8.2.3	$\hat{R}_1 = \hat{Q}_3 = x$ $\hat{S}_4 = \hat{P}_1$ alt ∠s; PQ ∥ RS a $\hat{Q}_3 = \hat{S}_4 = x$ $\hat{R}_1 = \hat{P}_1$ ∠s in the same segment chord PR a $\therefore \hat{R}_1 = \hat{S}_4$	✓ S ✓ R ✓ S ✓ R (4)
8.2.4	$\hat{W}_1 = 2x$ ext ∠ of Δ a $\hat{W}_1 = \hat{V}_1$ ∴ QVSW is a cyclic quad converse: ext ∠ of cyclic quad a	✓ S ✓ R ✓ S ✓ R (4)
8.2.5 (a)	$\hat{Q}_4 = y$ tan-chord theorem a	✓ S ✓ R (2)
8.2.5 (b)	$\hat{T} = \hat{Q}_4 = y$ corresp ∠s; PQ ∥ RS a OR ext ∠ of Δ	✓ S ✓ R (2)
8.2.6	Join M to R and M to P $\hat{Q}_3 = x$ proven in 8.2.3 $P\hat{M}R = 2x$ ∠ at centre = 2 × ∠ at circumference $P\hat{M}R = \hat{W}_3 = 2x$ ∴ PMWR is a cyclic quad converse: angles in same segment	a ✓ S / R a ✓ S a ✓ R (3) **[28]**

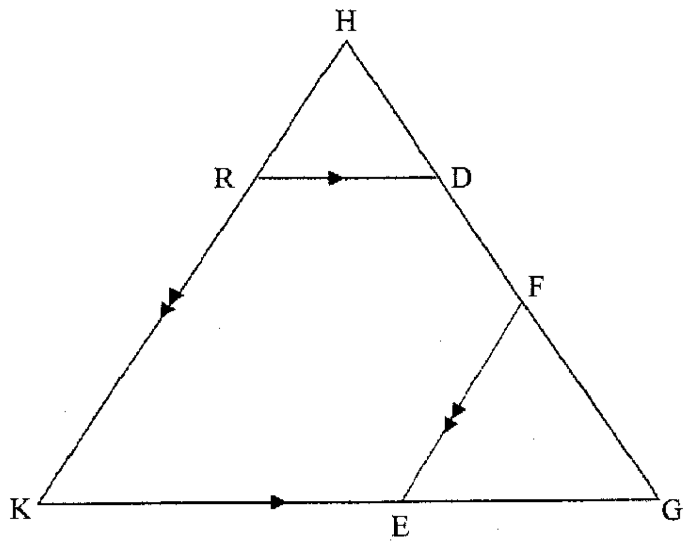
QUESTION 9



9.1	$\hat{Q}_4 = \hat{Q}_1$	vert opp angles	✓ S ✓ R
	$\hat{Q}_1 = \hat{A}_1$	tan-chord theorem	✓ S ✓ R
	$\hat{A}_1 = \hat{R}$	ext angle of cyclic quad	✓ S ✓ R
	$\Rightarrow \hat{Q}_4 = \hat{R}$		
	$\therefore PQ = PR$	sides opp equal angles	✓ R
			(7)
9.2	In ΔPBQ and ΔPQA		
	(i) $\hat{P}_1$ is common		✓ S
	(ii) $\hat{B} = \hat{Q}_3$	tan-chord theorem	✓ S ✓R
	(iii) $P\hat{Q}B = \hat{A}_2$	remaining angles in triangle	✓ S / R
	$\therefore \Delta PBQ \parallel\!\mid \Delta PQA$	equiangular	
			(4)
9.3	$\frac{PA}{PQ} = \frac{PQ}{PB}$	from 9.2	✓ deduction
	But $PQ = PR$		✓ PQ = PR
	$\therefore \frac{PA}{PR} = \frac{PR}{PB}$		
	$\therefore$ PA, PR and PB form a geometric sequence	the ratio is constant	✓ conclusion in full
			(3)
			[14]

 Please turn over

QUESTION 10



10.1	In Δ HKG: $\frac{DG}{HD} = \frac{RK}{RH}$ (prop theorem; RD ∥ KG) $\frac{DG}{2} = \frac{9}{3}$ $\therefore DG = 6$	a a a	✓ S / R ✓ substitution ✓ answer (3)
10.2	Let FD $= y$ $\therefore FG = 6 - y$ $\frac{GF}{FH} = \frac{GE}{EK}$ (prop theorem; FE ∥ HK) DG−FD $\frac{6-y}{y+2} = \frac{1}{2}$ $2(6-y) = y + 2$ $12 - 2y = y + 2$ $-3y = -10$ $\therefore y = \frac{10}{3} = FD$	Ca a Ca a a	✓ statement DG−FD ✓ S / R ✓ substitution ✓ simplification ✓ answer (5) **[8]**

TOTAL: 150