NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION 2015

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

a) 1) $10x = 3x^2 - 8$

$3x^2 - 10x - 8 = 0$

$(3x+2)(x-4) = 0$

$x = -\frac{2}{3}$ or $x = 4$ 3)

2) $x + \sqrt{x-2} = 4$

$\sqrt{x-2} = 4 - x$

$x - 2 = 16 - 8x + x^2$

$x^2 - 9x + 18 = 0$

$(x-3)(x-6) = 0$

$x = 3$ or $x = 6$

Check $x = 3$

LHS $= 3 + \sqrt{3-2}$

$= 4 =$ RHS

Check $x = 6$

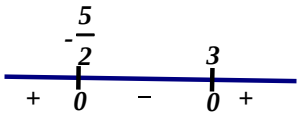
LHS $= 6 + \sqrt{6-2}$

$= 8 \neq$ RHS 5)

3) $x(2x-1) \geq 15$

$2x^2 - x - 15 \geq 0$

$(2x+5)(x-3) \geq 0$



$x \leq -\frac{5}{2}$ or $x \geq 3$ 5)

4) $\log_x 5 = 3$

$x^3 = 5$

$x = \sqrt[3]{5}$

$\approx 1{,}7$ 2)

b) 1) $P = \frac{4^{x+3} + 4^x}{8^{x+2} + 8^x}$

$= \frac{2^{2x+6} + 2^{2x}}{2^{3x+6} + 2^{3x}}$

$= \frac{2^{2x}(2^6+1)}{2^{3x}(2^6+1)}$

$= 2^{-x}$ or $(\frac{1}{2})^x$

ALTERNATIVELY:

$$P = \frac{4^x(4^3 + 1)}{8^x(8^2 + 1)}$$
$$= \frac{4^x(65)}{8^x(65)}$$
$$= (\frac{1}{2})^x$$ 3)

2) $P = 8$
$2^{-x} = 2^3$
$x = -3$ 2)

c) 1) Irrational 1)
2) Rational 1)
3) Non-real 1)

d) $f(x) = cx^3 - 6x + 2c$
$f(1) = c - 6 + 2c = 0$
$3c = 6$
$c = 2$ 3)

[26]

QUESTION 2

a) 12345,6 123456,7 1)

b) $\sum_{k=2}^{5} \frac{3^{k-1}}{k}$
$= \frac{3}{2} + \frac{3^2}{3} + \frac{3^3}{4} + \frac{3^4}{5}$
$= \frac{549}{20}$ 27,45) 3)

c) 1) $T_3 + T_4 = 167$
$a + 2d + a + 3d = 167$
$2a + 5d = 167$ ……①
$T_{21} = -4$
$a + 20d = -4$
$a = -20d - 4$…… ②
① : $2(-20d - 4) + 5d = 167$
$-40d - 8 + 5d = 167$
$-35d = 175$
$d = -5$ 4)

2) $a = -20(-5) - 4$
$= 96$
$S_{21} = \frac{21}{2}[2a + 20d]$
$= \frac{21}{2}[2 \times 96 + 20(-5)]$
$= 966$ 4)

d) 72 ; 48 ; 32
G.S. $a = 72 \quad r = \frac{2}{3}$
$S_\infty = \frac{72}{1 - \frac{2}{3}}$
$= 216$ cm 3)
[15]

QUESTION 3

a) $f(x) = 3x^2 + 2$
$f'(x) = \lim_{h \to 0} \frac{(3(x+h)^2 + 2) - (3x^2 + 2)}{h}$
$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 2 - 3x^2 - 2}{h}$
$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$
$= \lim_{h \to 0}(6x + 3h)$
$= 6x$ 5)

b) $y = \frac{5x - 2}{10x^2}$
$= \frac{x^{-1}}{2} - \frac{x^{-2}}{5}$
$\frac{dy}{dx} = \frac{-x^{-2}}{2} + \frac{2x^{-3}}{5}$
$= -\frac{1}{2x^2} + \frac{2}{5x^3}$ 5)

c) $f(x) = \sqrt{x}\left(x+2\right)$

$= x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$

$f'(x) = \frac{3x^{\frac{1}{2}}}{2} + 2\times\frac{x^{-\frac{1}{2}}}{2}$

$f'\left(\frac{1}{4}\right) = \frac{3}{2}\left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{4}\right)^{-\frac{1}{2}}$

$= \frac{3}{2}\times\frac{1}{2} + 2$

$= \frac{11}{4}$ 2,75)

5)

d) $f(x) = \frac{2}{3}x^3 - x^2 - 3x$

$f(3) = \frac{2}{3}\times 3^3 - 3^2 - 3\times 3$

$= 0$

$f'(x) = \frac{2}{3}\times 3x^2 - 2x - 3$

$= 2x^2 - 2x - 3$

$f'(3) = 2\times 3^2 - 2\times 3 - 3$

$= 9$

Eqn. of tangent : $y - 0 = 9(x-3)$

$y = 9x - 27$

5)

[20]

QUESTION 4

a) $12\ 5001 - i)^5 = 5\ 546{,}32$

$$(1-i)^5 = 0{,}4437056$$
$$1-i = 0{,}8500001102$$
$$i = 0{,}1499998898$$
$$I \approx 15\%$$

3)

b) 1) $$\frac{500\left[\left(1+\frac{0{,}06}{12}\right)^{180}-1\right]}{\frac{0{,}06}{12}}$$

$= 145\ 409{,}3562$
$\approx$ R145 409,36

3)

2) $$150\ 000\left(1+\frac{0{,}085}{4}\right)^{20}$$

$= 228\ 419{,}223$
$\approx$ R228 419,22

3)

3) $$150\ 000\left(1+\frac{0{,}085}{4}\right)^{4n} = 10^6$$

$$\left(1+\frac{0{,}085}{4}\right)^{4n} = \frac{20}{3}$$
$$4n = \log_{1{,}02125}6{,}66666)$$
$$= 90{,}22147031$$
$$n = 22{,}55536758$$
$$= 22{,}6$$

5)
[14]

75 marks

SECTION B

QUESTION 5

(a) 1) i) $\frac{40}{500}$

$= \frac{2}{25}$ 0,08) 1)

ii) $\frac{200}{500}$

$= \frac{2}{5}$ 0,4) 1)

iii) $\frac{160}{200}$

$= \frac{4}{5}$ 0,8) 2)

2) PMale)

$= \frac{230}{500}$ 0,46)

PMale) × PLiving with Parents)

$= \frac{23}{50} \times \frac{2}{5}$

$= \frac{23}{125}$ 0,184)

$\neq$ PMale and Living with Parents) = 0,08

$\therefore$ Living with Parents is NOT independent of gender.

ALTERNATIVELY:

PFemale)

$= \frac{270}{500}$ (0,54)

PFemale and Living with Parents)

$= \frac{160}{500}$ (0,32)

PFemale) × PLiving with Parents)

$= \frac{27}{50} \times \frac{2}{5}$

$\frac{27}{125}$ (0,216) 3)

$\neq$ PFemale and Living with Parents) = 0,32

(b) 1) $7!$ 1)
$= 5\,040$

2) $4 \times 3 \times 5!$ 3)
$= 1\,440$

3) $4! \times 3! \times 4$ 3)
$= 576$ **[14]**

QUESTION 6

a) 1)

	0	1	2	3	4
	1	0	5	16	33
1st diff.		1	5	11	17
2nd diff.			6	6	6

Any valid method:
a b c

$$T_n = 0 + (n-1)\times 5 + \frac{(n-1)(n-2)}{2}\times 6$$
$$= 5n - 5 + 3(n^2 - 3n + 2)$$
$$= 5n - 5 + 3n^2 - 9n + 6$$
$$= 3n^2 - 4n + 1$$
$$\therefore\ a = 3,\ b = -4 \text{ and } c = 1$$

ALTERNATIVELY :
$$2a = d = 6$$
$$\therefore\ a = 3$$
$$T_0 = 1 = c$$
$$T_1 = 3 + b + 1 = 0$$
$$\therefore\ b = -4$$ 5)

2) $3n^2 - 4n + 1 = 5896$
$3n^2 - 4n - 5895 = 0$
$$n = \frac{4 \pm \sqrt{16 + 12 \times 5895}}{6}$$
$$= 45 \text{ or } -\frac{131}{3}$$
$\therefore\ 45^{\text{th}}$ term 3)

b) $f(x) = \frac{2}{x^2} + 1$

$f(x^{-1}) = \frac{2}{\left(x^{-1}\right)^2} + 1$

$= 2x^2 + 1$

$f(-1) = \frac{2}{(-1)^2} + 1$

$= 3$

. $f(x^{-1}) - x^2 f(-1)$

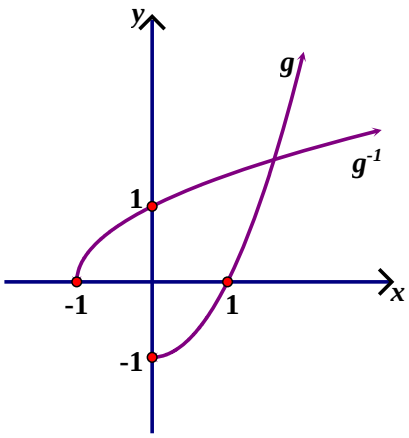
$= 2x^2 + 1 - x^2 \times 3$

$= -x^2 + 1$ 4)

c) $g(x) = x^2 - 1$ with $x \geq 0$

1) $y \geq 0$ 1)

2)



4)

d) 1) $m_{AB} = \frac{3+1}{1-3}$

$= -2$ 2)

2) $\frac{y}{x} - 3 = -2(x - 1)$

$\frac{y}{x} = -2x + 2 + 3$

$y = -2x^2 + 5x$ 4)

[23]

QUESTION 7

a) 1) $f(0) = 30$
$\therefore$ 30 m

1)

2) $-\frac{x^2}{1\,000} + \frac{x}{5} + 30 = 0$

$x^2 - 200x - 30\,000 = 0$

$(x - 300)(x + 100) = 0$

$x = 300 \quad \text{or} \quad x = -100$

$\therefore$ 300 m

3)

3) $x_{TP} = -\frac{1}{5} \div -\frac{2}{1\,000}$

$= 100$

$f(100) = \frac{-100^2}{1000} + \frac{100}{5} + 30$

$= 40$

i.e. Max height is 40 m.

4)

4) $x \in [0\,;\,300]$

1)

b) 1) $h(x) = \frac{k}{x - 2} + 1$

$3\,;\,4) : 4 = \frac{k}{3 - 2} + 1$

$k = 3$

$h(x) = \frac{3}{x - 2} + 1$

4)

2) $y = f(x) = 3.2^{x-p} + q$

$q = 1$

$3\,;\,4) : 4 = 3.2^{3-p} + 1$

$3 = 3.2^{3-p}$

$2^{3-p} = 1$

$3 - p = 0$

$p = 3$

4)

3) $2 < x \leq 3$

2)

[19]

QUESTION 8

(a) $2x + 2y = 64$

$y = 32 - x$

$x + h = 20$

$h = 20 - x$

$$\text{Area ABEDC} = \frac{1}{2}xh + xy$$

$$= \frac{x}{2}(20 - x) + x(32 - x)$$

$$= 10x - \frac{x^2}{2} + 32x - x^2$$

$$= -\frac{3x^2}{2} + 42x$$

6)

b) $$\frac{dArea}{dx} = -3x + 42 = 0$$

$-3x = -42$

$x = 14$

ALTERNATIVELY:

$$x = \frac{-42}{2(-\frac{3}{2})}$$

$= 14$

$h = 6$

$y = 18$

4)

[10]

QUESTION 9

(a) $y = kx^3 + lx^2$

At 1 ; 5) : $5 = k + l$ ….. ①

$\frac{dy}{dx} = 3kx^2 + 2lx$

At 1 ; 5) : $m = 3k + 2l = 12$ ……②

$3(5 - l) + 2l = 12$

$15 - 3l + 2l = 12$

$l = 3$

$k = 2$ 5)

b) $y = ax^3 + bx^2 + cx + d$

$\frac{dy}{dx} = 3ax^2 + 2bx + c$

$\frac{d^2y}{dx^2} = 6ax + 2b$

At a point of inflection, $\frac{d^2y}{dx^2} = 0$

$6ax = -2b$

$x = \frac{-2b}{6a}$

$= \frac{-b}{3a}$ 4)

[9]

75 marks

Total: 150 marks