



NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION 2015

MATHEMATICS: PAPER I

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions.
 4. Number your answers exactly as the questions are numbered.
 5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 6. Round off your answers to one decimal digit where necessary.
 7. All the necessary working details must be clearly shown.
 8. It is in your own interest to write legibly and to present your work neatly.
 9. Please hand in this question paper.
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SECTION A

QUESTION 1

(a) Solve for x :

(1) $10x = 3x^2 - 8$ (3)

(2) $x + \sqrt{x - 2} = 4$ (5)

(3) $x(2x - 1) \geq 15$ (5)

(4) $\log_x 5 = 3$ (2)

(b) Given: $P = \frac{4^{x+3} + 4^x}{8^{x+2} + 8^x}$

(1) Simplify P . (3)

(2) Hence solve for x : $P = 8$. (2)

(c) State whether the following numbers are rational, irrational or non-real.

(1) $\sqrt{3}$ (1)

(2) $\frac{22}{7}$ (1)

(3) The roots of $x^2 + 4 = 0$ (1)

(d) Given that $x - 1$ is a factor of $f(x) = cx^3 - 6x + 2c$, determine c . (3)

[26]

QUESTION 2

(a) Given the sequence: 1,2 ; 12,3 ; 123,4 ; 1234,5 ...

Write down the next two terms, assuming the pattern continues. (1)

(b) Evaluate: $\sum_{k=2}^5 \frac{3^{k-1}}{k}$ (3)

(c) In an arithmetic sequence, the sum of the third and fourth terms equals 167 and $T_{21} = -4$.

(1) Determine the value of the constant difference. (4)

(2) Hence calculate the sum of the first 21 terms. (4)

- (d) A pendulum is set in motion. In the first swing, where both forward and backward motion is taken into account, the tip covers 72 cm. The tip covers 48 cm in the second swing and 32 cm in the third swing. It continues indefinitely in this way with the distances covered forming a geometric sequence. Determine the total distance covered by the tip. (3)
- [15]**

QUESTION 3

- (a) Given $f(x) = 3x^2 + 2$, determine $f'(x)$ from first principles. (5)
- (b) Determine $\frac{dy}{dx}$ given $y = \frac{5x-2}{10x^2}$,
leaving your answer with positive exponents. (5)
- (c) Given: $f(x) = \sqrt{x}(x+2)$, calculate $f'\left(\frac{1}{4}\right)$. (5)
- (d) Given: $f(x) = \frac{2}{3}x^3 - x^2 - 3x$

Determine the equation of the tangent to the curve $y = f(x)$ at the point where $x = 3$. (5)

[20]

QUESTION 4

- (a) Alex bought a laptop for R12 500. It depreciated in value to R5 546,32 after 5 years. Calculate the annual depreciation rate. (3)
- (b) When Sam started his first job, he decided to save R500 at the end of each month in an account earning interest at 6% p.a. compounded monthly.
- (1) Calculate the amount of money he could expect to be in his account at the end of 15 years. (3)
 - (2) At the end of the 15 years, Sam added money to his account so that the total was R150 000. He no longer made monthly payments but the money remained in the account earning interest at 8,5% p.a. compounded quarterly. Calculate the amount in the account after a further 5 years. (3)
 - (3) Determine the number of years after the first 15 years the investment discussed in (b)(2) that Sam's savings reached a million rand. (5)
- [14]**

75 marks

SECTION B

QUESTION 5

- (a) In order to determine whether people aged between 20 and 25 years lived with their parents or not, a survey was undertaken. The table below summarises the results.

	Males	Females	Total
Living with parent(s)	40	160	200
Not living with parent(s)	190	110	300
Total	230	270	500

- (1) Suppose that one of these people was randomly interviewed a second time. Determine the probability that the person:
- (i) is a male who lives with his parent(s). (1)
 - (ii) lives with his or her parent(s). (1)
 - (iii) is female, given that they live with their parent(s). (2)
- (2) Showing all working, determine whether it can be said that living with parent(s) is independent of gender. (3)
- (b) Factorials should be calculated in this question. e.g. $4! = 24$.
 Four boys and three girls sit in a row watching a movie.
 Determine the number of arrangements in which this can be done if:
- (1) they sit in any order. (1)
 - (2) a boy sits at each end of the row. (3)
 - (3) all the boys sit together. (3)
- [14]**

QUESTION 6

(a) Given: 0; 5; 16; 33 are the first four terms of a quadratic sequence.

(1) Show that the n^{th} term of the sequence is given by $T_n = 3n^2 - 4n + 1$. (5)

(2) Determine which term in the sequence equals 5 896. (3)

(b) Given: $f(x) = \frac{2}{x^2} + 1$

Determine $f(x^{-1}) - x^2 f(-1)$.

Simplify your answer fully. (4)

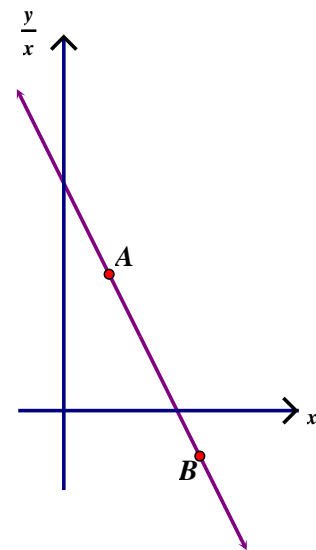
(c) Given: $g(x) = x^2 - 1$ with $x \geq 0$.

(1) Write down the range of $g^{-1}(x)$. (1)

(2) On the same set of axes, draw sketch graphs of $y = g(x)$ and $y = g^{-1}(x)$, clearly labelling intercepts with axes. (4)

(d) The variables x and y are related in such a way that when $\frac{y}{x}$ is plotted against x a straight line is obtained, as shown in the graph.

The line passes through the points A(1, 3) and B(3, -1).



(1) Calculate the gradient of the line passing through A and B. (2)

(2) Hence express y in terms of x . (4)

[23]

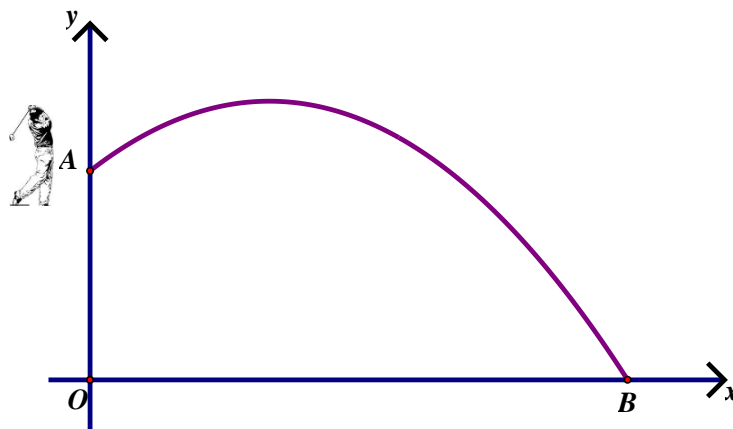
QUESTION 7

- (a) Ernie is practising his golf from the top of a cliff at Pinnacle Point. The path of his ball can be modelled by

$$f(x) = -\frac{x^2}{1000} + \frac{x}{5} + 30$$

where x is the horizontal distance covered by the ball and $f(x)$ is the number of metres that the ball is above the ground.

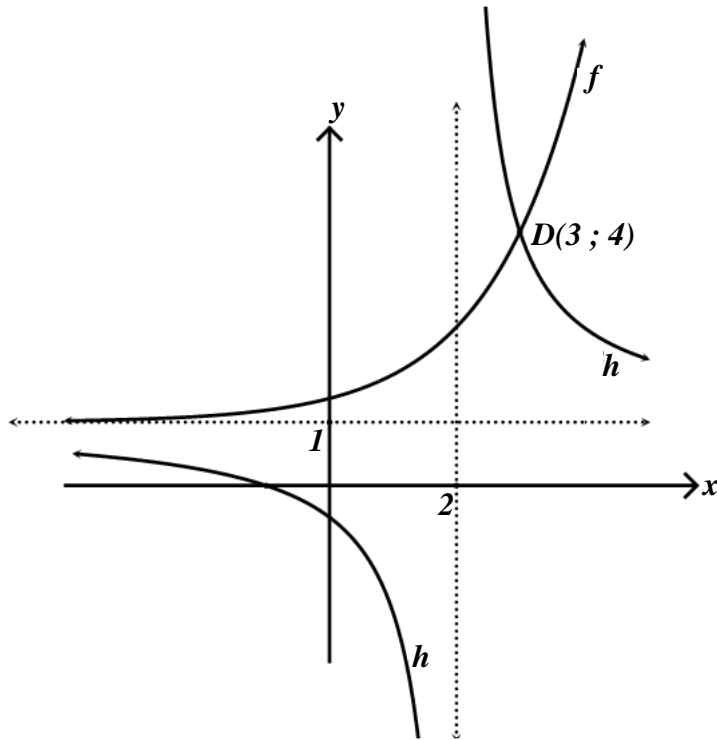
The figure shows the path of the ball from point A when Ernie hits it, to when it hits the ground at point B.



Determine:

- (1) the height of the cliff. (1)
- (2) the horizontal distance covered by the ball when it hits the ground. (3)
- (3) the maximum height the ball reaches above the ground. (4)
- (4) the domain in which this model can be applied. (1)

- (b) Refer to the figure showing the graphs of a hyperbola $y = h(x)$ with asymptotes $x = 2$ and $y = 1$, with a point $D(3 ; 4)$, which is also on an exponential graph $y = f(x) = 3 \cdot 2^{x-p} + q$.



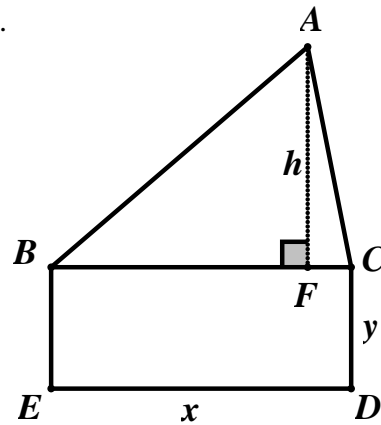
- (1) Determine the equation of the hyperbola. (4)
- (2) Determine the values of p and q in the equation of $y = f(x)$. (4)
- (3) Write down the values of x for which $f(x) \leq h(x)$. (2)

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QUESTION 8

Refer to the figure with $\triangle ABC$ placed on rectangle BCDE.

The perimeter of the rectangle is 64 units.
 The sum of the base BC and altitude AF of the triangle is 20 units.



- (a) Show that the area of ABEDC is $-\frac{3x^2}{2} + 42x$. (6)
 - (b) Hence determine the dimensions of x , y and h when the area is maximised. (4)
- [10]**

QUESTION 9

- (a) The gradient of the tangent to the curve defined by

$$y = kx^3 + lx^2 \text{ at the point } (1 ; 5) \text{ is } 12.$$

Calculate the value of k and l . (5)

- (b) Given a cubic graph $y = ax^3 + bx^2 + cx + d$.
 Showing all working, determine an expression for the x -coordinate at a point of inflection. (4)
- [9]**

75 marks

Total: 150 marks