NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION 2015

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

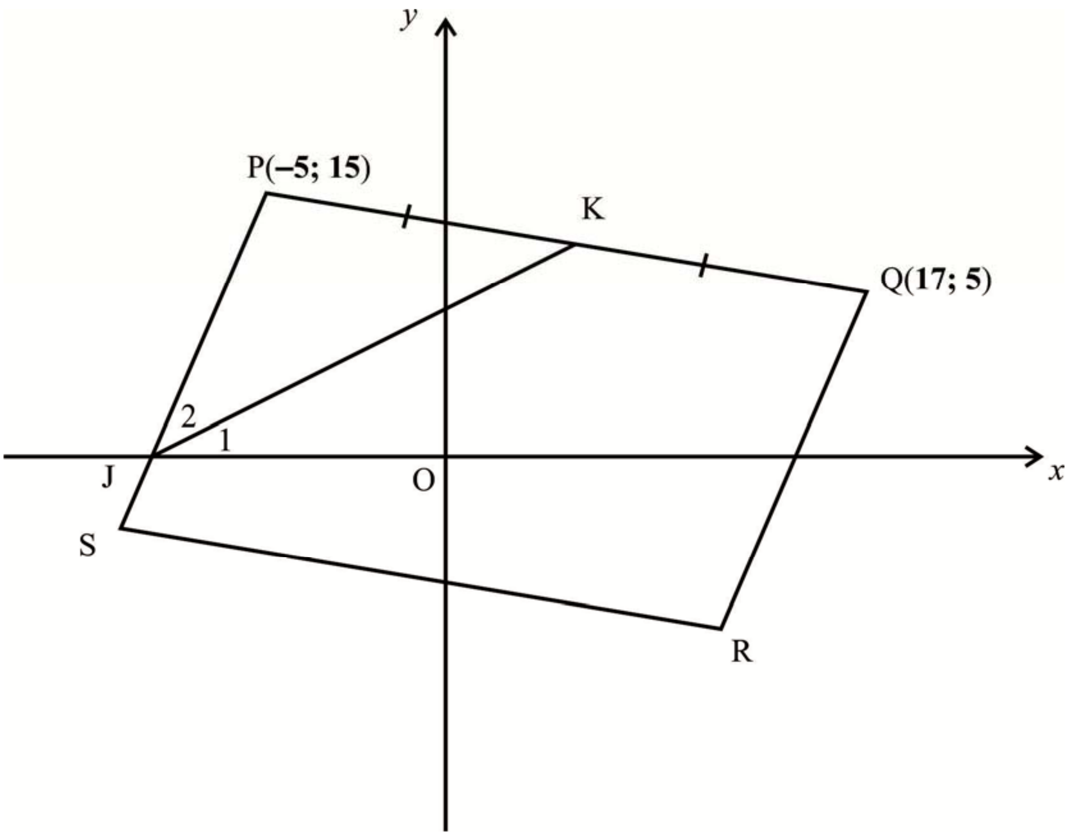
These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a)



(1) $K\left(\frac{-5+17}{2}; \frac{15+5}{2}\right) = K(6; 10)$ (2)

(2) $m = 5$ Therefore, $y - 15 = 5(x+5)$ $\therefore y = 5x + 40$ (3)

(3) For J: $5x + 40 = 0$ $\therefore x = -8 \therefore$ J$(-8; 0)$

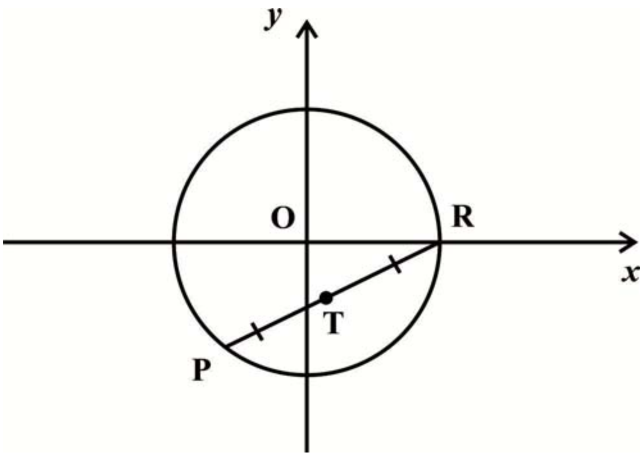
$m = \frac{10-0}{6-(-8)} = \frac{5}{7}$ (3)

(4) (i) $\tan \hat{J}_1 = \frac{5}{7}$ $\therefore$ $\hat{J}_1 = 35{,}5°$ (2)

(ii) $\tan\left(\hat{J}_1 + \hat{J}_2\right) = 5$ $\therefore$ $\hat{J}_1 + \hat{J}_2 = 78{,}7°$ (2)

(iii) $\therefore$ $\hat{J}_2 = 43{,}2°$ (1)

(b)



(1) $O\hat{T}R = 90°$; line from centre bisects chord (2)

(2) $m_{OT} = \dfrac{-6-0}{4-0} = -\dfrac{3}{2}$

$\therefore\ m_{PR} = \dfrac{2}{3}$

$\therefore\ y + 6 = \dfrac{2}{3}(x - 4)$

$\therefore\ 3y = 2x - 26$ (3)

(3) For R: $0 = 2x - 26\ \therefore\ x = 13\ \therefore\ R(13; 0)$
Therefore, radius = 13 (3)

[21]

QUESTION 2

(a) $\cos(P+Q) = \cos 133° = -0,681\ ...$

$\cos P + \cos Q = 0,569\ ...$

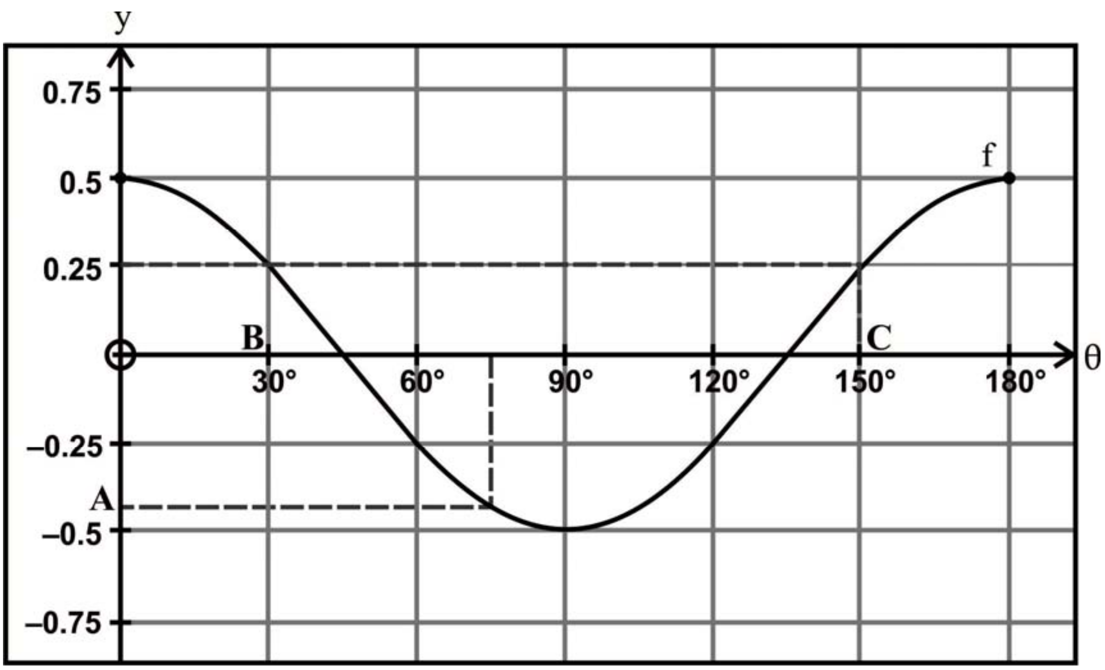
$\therefore \cos(P+Q) \neq \cos P + \cos Q$ (2)

(b) $\dfrac{\cos(180+\theta) - \cos(90-\theta)}{\cos(-\theta) - \sin(-\theta)}$

$= \dfrac{-\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= -1$ (5)

(c) (1) $a = \dfrac{1}{2}$ $b = 2$ (2)



(2) On graph (Accept A as shown) (1)

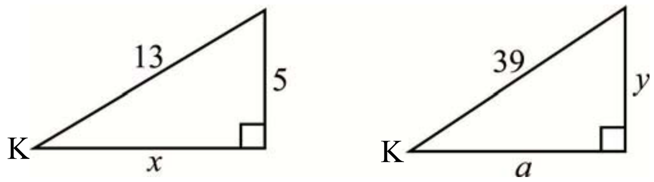
(3) On graph (Indicated by B and C) (2)

(d) $\cos 72°.\sin 198°$

$= \sin 18° \times -\sin 18°$

$= -t^2$ (3)

(e) (1) $\hat{K}$ is acute.



$x^2 + 5^2 = 13^2 \therefore \quad x^2 = 144 \quad \therefore x = 12$

$\frac{a}{39} = \frac{12}{13} \quad \therefore \quad a = 36$ (4)

(2) $\hat{K}$ is obtuse

$\frac{a}{39} = \frac{-12}{13} \quad \therefore \quad a = -36$ (2)

[21]

QUESTION 3

(a) IQR = 14 000 – 7 000 = R7 000

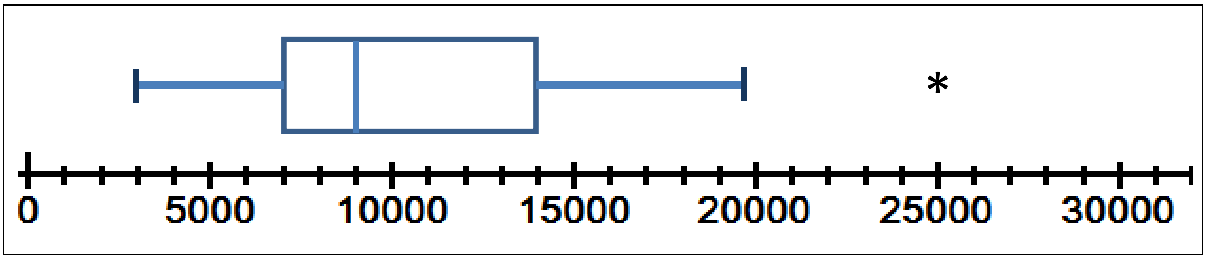
1,5 × IQR = R10 500

$\therefore\ Q_3 + 1,5 \times \text{IQR} = \text{R}24\ 500$

$Q_1 - 1,5 \times \text{IQR} = -\text{R}3\ 500$

Therefore, R25 000 is an outlier. (5)

(b) for maximum



(4)

(c) Skewed to the right. Or positively skewed. (1)

[10]

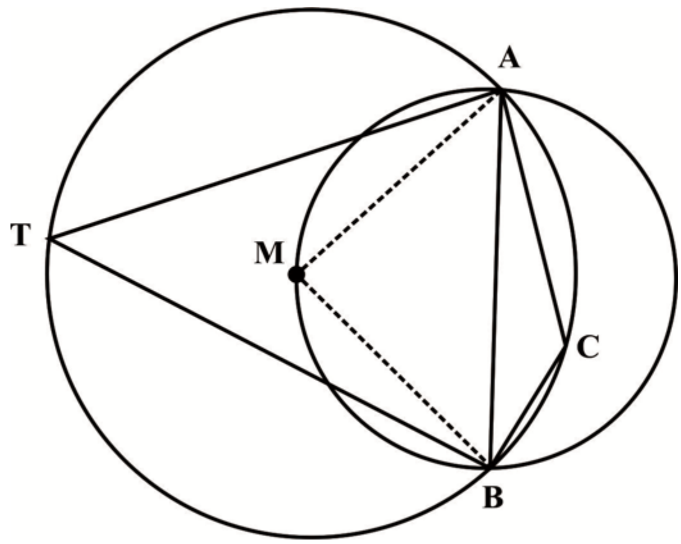
QUESTION 4

(a)

STATEMENT	REASON
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	Ext $\angle$ of cyclic quad
$\hat{D}_2 + \hat{E} = 180°$	Opp $\angle$'s of cyclic quad
$\hat{B}_1 = \hat{D}_1$	Tan/chord thm
$\hat{B}_1 + \hat{B}_2 = \hat{D}_1 + \hat{D}_2$	Not correct
$\hat{D}_2 = \hat{A}_2$	Not correct

(5)

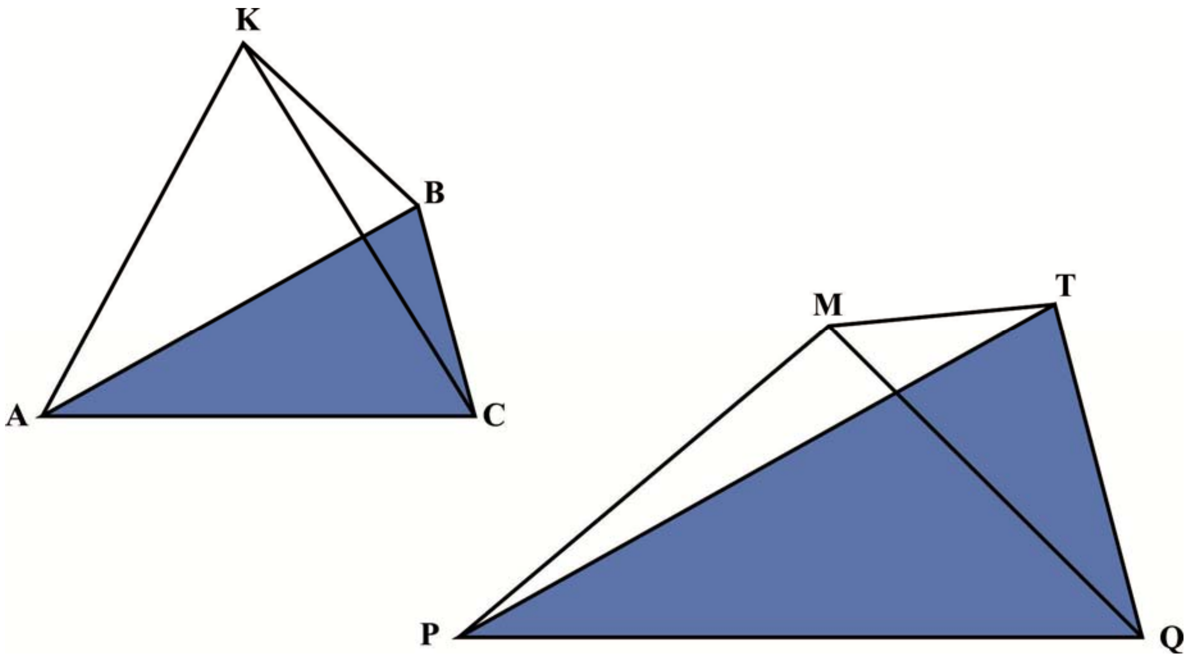
(b) (1) In the diagram below, circle centre M intersects a second smaller circle at A and B.
A, B, C and T are points on circle centre M.
AB is the diameter of the smaller circle.



$A\hat{M}B = 90°$; angle in semi circle
$\hat{T} = 45°$; angle at centre
$\hat{C} = 135°$; opp. angles of cyclic quad (6)

(2) $\hat{M} + \hat{C} \neq 180°$; opp $\angle$'s do not add up to $180°$ (1)

(d)



(1) $\frac{AB}{PT}=\frac{4}{6}=\frac{2}{3}$ $\quad \frac{AC}{PQ}=\frac{5}{7{,}5}=\frac{2}{3}$ $\quad \frac{BC}{TQ}=\frac{2}{3}$

$\therefore \frac{AB}{PT}=\frac{AC}{PQ}=\frac{BC}{TQ}=\frac{2}{3}$

$\therefore$ $\Delta ABC \;|||\; \Delta PQT$; sides in prop. (4)

(2) $$\frac{\text{Volume of Pyramid PQTM}}{\text{Volume of Pyramid ABCK}}=\frac{\frac{1}{3}\times\left(\frac{1}{2}\times AB\times AC\times\sin\hat{A}\right)\times(\perp\text{ height from K})}{\frac{1}{3}\times\left(\frac{1}{2}\times PQ\times PT\times\sin\hat{P}\right)\times(\perp\text{ height from M})}$$

Since $\Delta ABC\;|||\;PQR$, we have $\hat{A}=\hat{P}$ Therefore, $\sin\hat{A}=\sin\hat{P}$,

Therefore, $\frac{\text{Volume of Pyramid PQTM}}{\text{Volume of Pyramid ABCK}}$

$=\frac{PQ\times PT}{AB\times AC}$

$=\left(\frac{3}{2}\right)^2$

$=\frac{9}{4}$ (4)

[20]

75 marks

SECTION B

QUESTION 5

(a) (1) Very strong and positive. (2)

(2) (i) $y = 0{,}62x - 1\ 238{,}42$ $\quad y = mx + c$ (5)

(ii) 12,74 million (2)

(3) Not reliable because prediction far from interval of values in data given.
Too few points to model on. (1)

[10]

QUESTION 6

(a) (1) $\frac{\sin B}{10} = \frac{\sin 120°}{30}$

$\therefore \sin B = 0,2886...$

$\therefore \hat{B} = 16,8°$

$\therefore A = 180 - 120 - 16,8$

$\therefore A = 43,2°$

$\frac{OB}{\sin 43,2°} = \frac{30}{\sin(120°)}$

$\therefore$ OB = 23,7 cm

Therefore B moves $35,4 - 23,7 = 11,7$ cm (6)

(2) $50 - 30 = 20$ cm. (2)

(b) $\frac{1-\sin(2\theta)}{1-\sin(2\theta)+\cos(2\theta)}$

$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta}{1 - 2\sin\theta.\cos\theta + 2\cos^2\theta - 1}$

$= \frac{(\sin\theta - \cos\theta)(\sin\theta - \cos\theta)}{-2\cos\theta(\sin\theta - \cos\theta)}$

$= -\frac{1}{2} \times \frac{\sin\theta - \cos\theta}{\cos\theta}$

$= -\frac{1}{2}(\tan\theta - 1)$

$= -\frac{1}{2}p + \frac{1}{2}$ (6)

(c) (1) $\cos\beta - \sqrt{3}\sin\beta = 2\sqrt{3}\cos(\beta - 30°)$

$\therefore \quad \cos\beta - \sqrt{3}\sin\beta = 2\sqrt{3}[\cos\beta.\cos 30° + \sin\beta.\sin 30°]$

$\therefore \quad \cos\beta - \sqrt{3}\sin\beta = 2\sqrt{3}\left[\cos\beta.\frac{\sqrt{3}}{2} + \sin\beta.\frac{1}{2}\right]$

$\therefore \quad \cos\beta - \sqrt{3}\sin\beta = 3\cos\beta + \sqrt{3}.\sin\beta$

$\therefore \quad -2\sqrt{3}\sin\beta = 2\cos\beta$

$\therefore \quad \tan\beta = -\frac{1}{\sqrt{3}}$ (4)

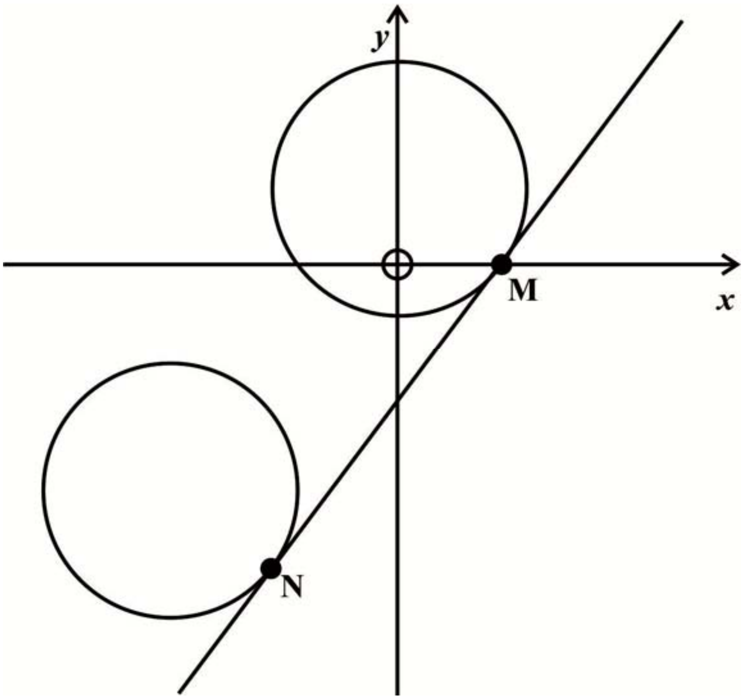
(2) $\tan\beta = -\frac{1}{\sqrt{3}}$

$\therefore \quad \beta = -30° + k.180°; \quad k \in Z$ (3)

[20]

QUESTION 7

(a)



(1) $x^2 + y^2 - 6y = 16$

$x^2 + (y-3)^2 - 9 = 16$

$x^2 + (y-3)^2 = 25$

Therefore, $B(0;3)$ is centre of one circle.

For M: $x^2 + (0-3)^2 = 25$

$\therefore \quad x^2 = 16$

$\therefore \quad x = 4$

$\therefore \quad M(4;0)$

$m_{rad} = \dfrac{3-0}{0-4} = -\dfrac{3}{4}$

$\therefore \quad m_{tan} = \dfrac{4}{3}$

$\therefore \quad y - 0 = \dfrac{4}{3}(x-4)$

$\therefore \quad y = \dfrac{4}{3}x - \dfrac{16}{3}$

OR

M_{tan} = slope of line joining centres

$\therefore \ M_{tan} = \dfrac{3+9}{0+9} = \dfrac{4}{3}$

(7)

(2) A(–9 –9)

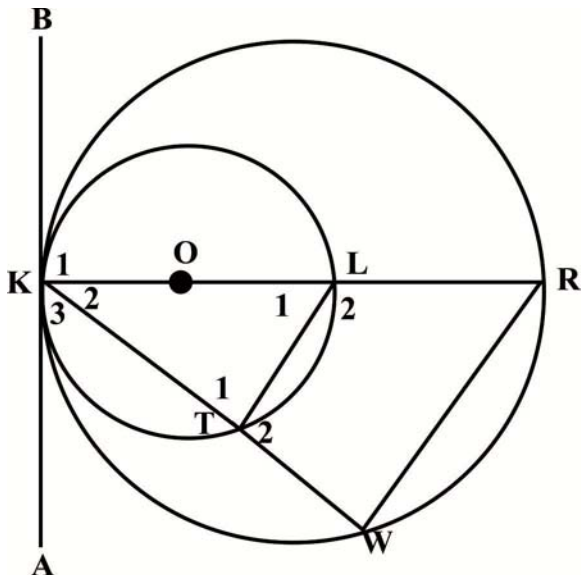
B(0; 3)

$AB^2 = (-9-0)^2 + (-9-3)^2 = 81 + 144 = 225$

$\therefore$ AB = 15

$\therefore$ MN = 15; MNAB is a rectangle (5)

(b)



(1)

$\hat{K}_1 = 90°$; radius $\perp$ tangent

$\hat{W} = \hat{K}_1$; tan/chord theorem

$\therefore$ $\hat{W} = 90°$

$\therefore$ KR is a diameter; converse of angle in semi-circle (5)

(2) $K\hat{T}L = 90°$; angle in semi circle

$K\hat{T}L = \hat{W} = 90°$; proven

$\therefore$ TL//WR ; corres angles equal

$\therefore$ $\frac{KL}{LR} = \frac{KT}{TW}$; prop int thm (6)

(c) $\hat{T}_2 = \hat{P} + \hat{S}$; ext $\angle$ of Δ

$\hat{S} = \hat{R}$; angles in same segment

$\hat{P} = \hat{R}$; alt $\angle$'s PS // QR

$\therefore \hat{P} = \hat{S}$

$\therefore \hat{T}_2 = 2\hat{S}$ (5)

[28]

QUESTION 8

(a)

(1)

Statement	$\hat{C} = \hat{A} + \hat{B}$
Statement	$\hat{A} = \hat{B}$
Deduction	$\therefore \hat{C} = 2\hat{B}$

(2)

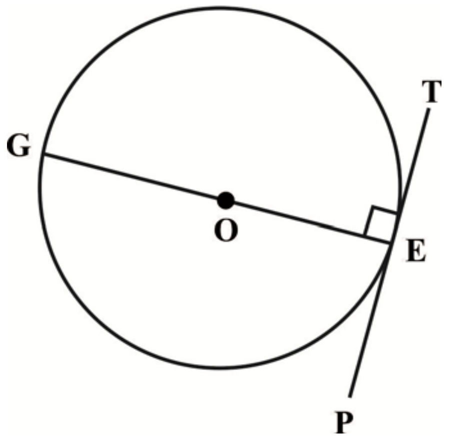
Statement	$\hat{C} = \hat{A} + \hat{B}$
Statement	$\hat{C} = \hat{D} + \hat{B}$
Deduction	$\therefore \hat{A} = \hat{D}$

(3)

Statement	$\hat{C} = \hat{A} + \hat{B}$
Statement	$\hat{A} = \hat{P}$
Deduction	$\therefore \hat{C} = \hat{P} + \hat{B}$

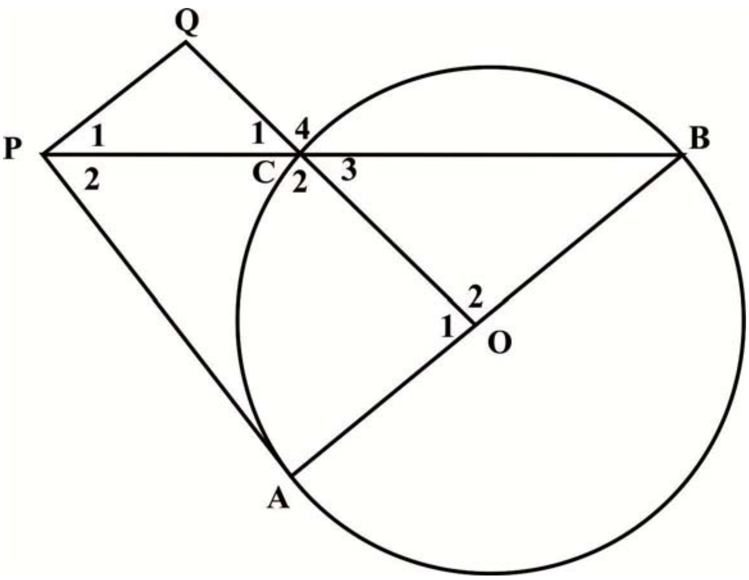
(3)

(b)



No/Not necessarily. We do not know whether GO = OE.
(If "OG is a diameter" is stated, give a mark.) (2)

(c) (1)



$\hat{B} = \hat{P}_1$; alt<'s PQ // BA

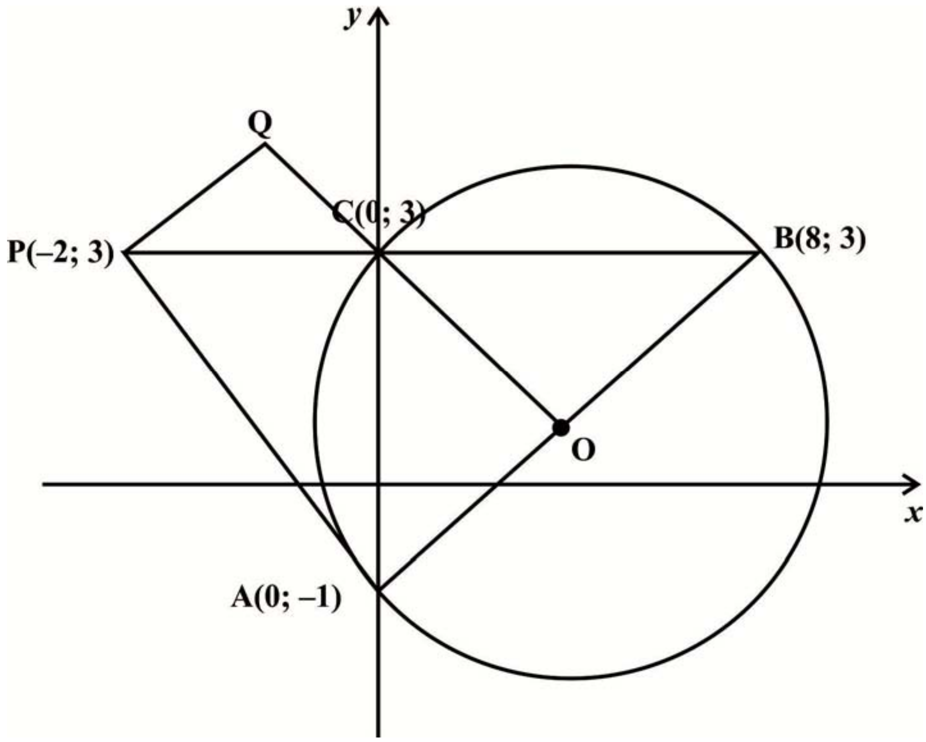
$\therefore\ \hat{B} = \hat{C}_3$; OB = OC

$\hat{C}_3 = \hat{C}_1$; vert. opp<'s

$\therefore\ \hat{P}_1 = \hat{C}_1$

$\therefore$ PQ = QC; (6)

(2) The Cartesian plane is introduced in the diagram above so that $A(-2; -1)$, $B(6; 3)$, $C(-2; 3)$ and $P(-4; 3)$.



(i) $O\left(\frac{0+8}{2}; \frac{-1+3}{2}\right) = O(4; 1)$

$m_{OC} = \frac{3-1}{0-4} = -\frac{1}{2}$

$\therefore \quad y = -\frac{1}{2}x + 3$

(4)

(ii) mid-point of $PC = (-1; 3)$.

$\therefore x_Q = -1$; $PQ = PC$ and QC is horizontal

$\therefore y_Q = -\frac{1}{2}(-1) + 3 = 3\frac{1}{2}$

$\therefore Q\left(-1; \frac{7}{2}\right)$ (4)

[19]

75 marks

Total: 150 marks