



NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2016

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours

150 marks

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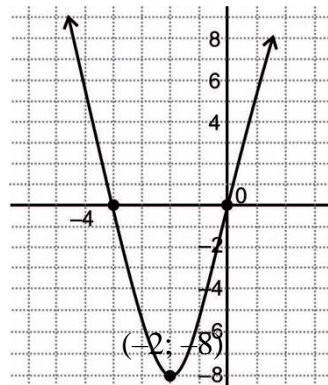
SECTION A

QUESTION 1

(a) (1) $\frac{4x}{2} - \frac{2x+1}{3} = 5$
 $\frac{12x-4x-2}{6} = \frac{30}{6}$ **OR** $12x - 2(2x + 1) = 30$
 $8x = 32$
 $x = 4$ (2)

(2) $(x-5)(x-6) \leq 56$
 $x^2 - 11x + 30 \leq 56$
 $x^2 - 11x - 26 \leq 0$
 $(x-13)(x+2) \leq 0$
 Critical Values: 13 ; -2
 $-2 \leq x \leq 13$ (5)

(b) TP (-2 ; -8) Shape
 Y – Int: (0 ; 0)
 X – Int. Let $y = 0$
 $(x + 2) = \pm 2$ OR $x(x + 4) = 0$
 $\therefore x = 0$ OR $x = -4$
 $\therefore x$ -int: (0;0) and (-4;0)



(c) (1) $x = -1$ (5)
 and

$y = 2$ (2)

(2) $\frac{4}{x+1} + 2 = x$
 $\therefore 4 + 2(x + 1) = x(x + 1)$
 $\therefore 4 + 2x + 2 = x^2 + x$
 $\therefore x^2 - x - 6 = 0$
 $\therefore (x - 3)(x + 2) = 0$
 $\therefore (3; 3) (2; 2)$

NB: (For $x = 3$ OR $x = 2$ award 3 out of 4) (4)

(d) $c = -1$ or $c = -\frac{1}{4}$ (other answers possible) (2)

(e) $3 - k < 0 \therefore k > 3$ (2)

[22]

QUESTION 2

(a) (1) $LHS = 3\left(\frac{1}{3}\right) = 1$
 $RHS = \sqrt{6\left(\frac{1}{3}\right)} - 1 = -1$
 $LHS \neq RHS \therefore x = \frac{1}{3}$ is incorrect (2)

(2) $3x = -\sqrt{6x - 1}$
 $(3x)^2 = 6x - 1$
 $9x^2 - 6x + 1 = 0$
 $x = \frac{1}{3}$
 from (1), no solution

Alternate: Let x be a solution
 Then $3x < 0$ so $x < 0$
 But $x \geq \frac{1}{6}$
 \therefore No solution (4)

(b) $7^{x+a} (1 + 3) = 28 (7^{a^2})$
 $7^{x+a} = \frac{28 (7^{a^2})}{4}$
 $7^{x+a} = 7 (7^{a^2})$
 $7^{x+a} = 7^{1+a^2}$
 $x = a^2 - a + 1$ (3)
[9]

QUESTION 3

(a) $4\,800 - \left(4\,800 \times \frac{13,5}{100}\right)$
 $= R4\,152$ (2)

(b) $415\,200 = x \left[\frac{1 - \left(1 + \frac{7}{1\,200}\right)^{(-5 \times 12)}}{\frac{7}{1\,200}} \right]$ Use of correct formula
 $x \approx R8\,221,46$ (4)
[6]

QUESTION 4

(a) Amount paid for all 110 laptops: $6\,000 \times 110 = 660\,000$

$$\text{Depreciation over 5 years: } A = 660\,000 \left(1 - \frac{15}{100}\right)^5$$

$$\approx 292\,845,51$$

Inflation: $A = P(1+i)^n$

$$A = 660\,000 \left(1 + \frac{6}{100}\right)^5$$

$$A = 883\,228,881$$

$$\text{Amount required in 5 years less "buy-back"} = 883\,228,88 - 292\,845,51$$

$$= \text{R}590\,383,37 \tag{4}$$

(b) Sinking Fund: $F = x \left[\frac{(1+i)^n - 1}{i} \right]$

$$590\,383,37 = x \left[\frac{\left(1 + \frac{12}{1200}\right)^{(5 \times 12)} - 1}{\frac{12}{1200}} \right]$$

Use of correct formula

$$x \approx \text{R}7\,228,92$$

(4)

[8]

QUESTION 5

(a) $T_1 = 5(1) + 2 = 7$
 $T_2 = 5(2) + 2 = 12$
 $T_3 = 5(3) + 2 = 17$
 \therefore Since $T_1 + T_2 + T_3 = 36$

Then $y = 3$

OR Alternate:

$7 + 12 + 17 + \dots + (5y + 2)$
 \therefore sequence is arithmetic
 with $a = 7$ and $d = 5$

$$\therefore \frac{y}{2} [7 + 5y + 2] = 36$$

$$\therefore 9y + 5y^2 = 72$$

$$\therefore 5y^2 + 9y - 72 = 0$$

$$\therefore (5y + 24)(y - 3) = 0$$

$$\therefore y = 3$$

(3)

(b) (1) $3p - (2p + 14) = (p + 7) - 3p$
 $3p - 2p - 14 = p + 7 - 3p$
 $3p = 21$
 $p = 7$

(2)

(2) $a = 28$ and $d = -7$
 $S_{38} = \frac{38}{2} [2(28) + (38 - 1)(-7)]$
 $S_{38} = -3\,857$

(3)

(c) $T_n = an^2 + bn + c$
 $a + b + c = 7 \dots \text{eq ①}$
 $4a + 2b + c = 13 \dots \text{eq ②}$
 $9a + 3b + c = 21 \dots \text{eq ③}$
 $\text{②} - \text{①}: 3a + b = 6$
 $\text{③} - \text{②}: 5a + b = 8$
 Sub. $b = 6 - 3a$
 Into: $5a + b = 8$
 $5a + 6 - 3a = 8 \quad \therefore 2a = 2$
 $a = 1, b = 3, c = 3$
 $\therefore T_n = n^2 + 3n + 3$

Alternate 1

$$T_n = \frac{(n+1)^3 - 1}{n}$$

$$= \frac{n^3 + 3n^2 + 3n + 1 - 1}{n}$$

$$= n^2 + 3n + 3$$

Alternate 2

$$7 \quad 13 \quad 31$$

$$7 \quad \sqrt{13} \quad \sqrt{21} \quad \sqrt{31}$$

$$6 \quad \sqrt{8} \quad \sqrt{10}$$

$$\quad \sqrt{2} \quad \sqrt{2}$$

\therefore Quadratic sequence

$$T_n = T_1 + (n-1) \cdot f + \frac{(n-1)(n-2)}{2} \cdot s$$

$f =$ first term of the first difference $= 6$

$s =$ second difference $= 2$

$$T_n = n^2 + 3n + 3 \tag{4}$$

(d) $r = \frac{2}{3}$

The sequence of sums is:

$$9, 15; 19; 21\frac{2}{3}; 23\frac{4}{9}; \frac{665}{27}; \frac{2059}{81}$$

$T_6 \approx 24,6$ and $T_7 = 25,4$

$\therefore n = 7$ is the smallest.

OR

$$S_n = 27 \left(1 - \frac{2}{3}\right)^n$$

$$S_n > 25 \text{ leads to } \left(\frac{2}{3}\right)^n < \frac{2}{27}$$

Try $n = 6$, it does not work,

But $n = 7$ works.

Alternate

$$a = 9 \quad \text{and} \quad r = \frac{2}{3}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$25 = \frac{9 \left[\left(\frac{2}{3}\right)^n - 1 \right]}{\frac{2}{3} - 1}$$

$$\left(\frac{2}{3}\right)^n = \frac{2}{27}$$

$$n = \log_{\frac{2}{3}} \frac{2}{27}$$

$$n = 6,41 \dots$$

\therefore Smallest value: $n = 7$

(6)

$$\begin{aligned}
 \text{(e)} \quad V_1 &= 729 \text{ cm}^3 \\
 V_{n+1} &= \frac{1}{3} A_{n+1} \cdot h_{n+1} \\
 &= \frac{1}{3} \left(\frac{1}{3} \right) A_n \left(\frac{1}{3} h_n \right) \\
 &= \frac{1}{9} V_n \\
 \therefore \text{Sequence is geometric} \\
 \text{with } r &= \frac{1}{9} \\
 \therefore S_\infty &= \frac{729}{1 - \frac{1}{9}} = 820,1 \text{ cm}^3
 \end{aligned}$$

Alternate

$$\text{Volume of pyramid ①} = \frac{1}{3} \times (9 \times 9) \times 27 = 729 \text{ cm}^3$$

$$\text{Volume of pyramid ②} = \frac{1}{3} \times \left(\frac{81}{3} \right) \times \frac{27}{3} = 81 \text{ cm}^3$$

$$\text{Volume of pyramid ③} = \frac{1}{3} \times \left(\frac{27}{3} \right) \times \frac{9}{3} = 9 \text{ cm}^3$$

The sequence is geometric

$$a = 729 ; \text{ common ratio is } \frac{1}{9}$$

$$\begin{aligned}
 S_\infty &= \frac{a}{1 - r} \\
 &= 820 \frac{1}{8} \text{ cm}^3
 \end{aligned}$$

(5)
[23]

QUESTION 6

$$(a) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Working:

$$f(x) = 3x^2 + 2x$$

$$f(x+h) = 3(x+h)^2 + 2(x+h)$$

$$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - (3x^2 + 2x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} (6x + 3h + 2)$$

$$f'(x) = 6x + 2$$

(5)

$$(b) \quad y = -x^{-1} + x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = +x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$$

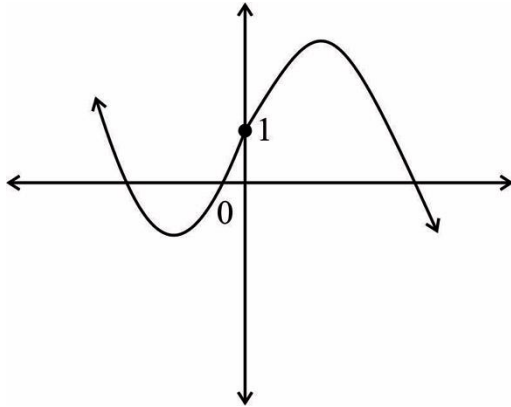
(4)

[9]

77 marks

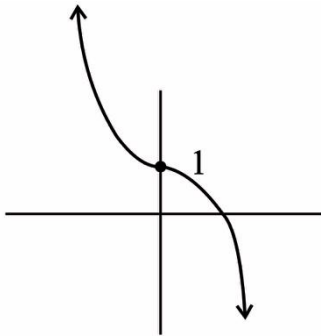
SECTION B

QUESTION 7



Shape
y-int and Pt of Inflection (0;1)
Concave down for $x > 0$

OR



Shape
y-int and Pt of Inflection (0;1)
Concave down for $x > 0$

[3]

QUESTION 8

(a) Axis of symm: $x = \frac{-3+1}{2} = -1$

$\therefore f'(x) > 0$ and $g(x) < 0$

OR $f'(x) < 0$ and $g(x) > 0$

$\therefore x < -1$ **OR** $x > 0$ (4)

(b) $g(x) = d^x + q$ sub. (0;0)

$0 = d^0 + q$

$q = -1$

Sub. (1;2)

$2 = d^1 - 1$

$d = 3$

$\therefore g(x) = 3^x - 1$

(4)

(c) Inverse of g :

$$\begin{aligned} x &= 3^y - 1 \\ 3^y &= x + 1 \\ y &= \log_3(x + 1) \end{aligned} \tag{3}$$

(d) Domain: $x > -1$

Alternative

$$\begin{aligned} \text{Domain of } g^{-1} &= \text{Range of } g \\ &= (-1; \infty) \end{aligned} \tag{2}$$

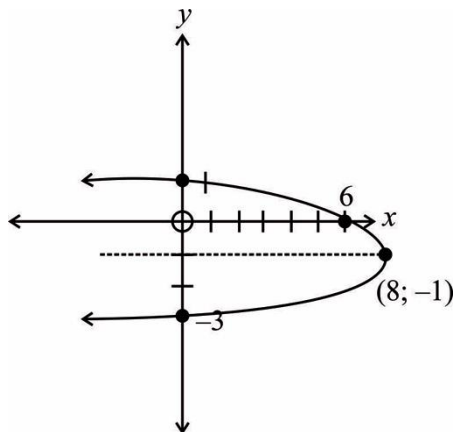
$$\begin{aligned} f(x) &= a(x + 3)(x - 1) \\ 6 &= a(3)(-1) \\ a &= -2 \\ \therefore f(x) &= -2(x + 3)(x - 1) \\ &= -2x^2 - 4x + 6 \\ \therefore a &= -2, b = -4, c = 6 \end{aligned}$$

Alternate: Given y-int (0;6)

$$\begin{aligned} y &= ax^2 + bx + 6 \quad \text{sub } (-3;0) \\ 0 &= a(-3)^2 + b(-3) + 6 \\ b &= \frac{9a+6}{3} \quad \text{eq. 1} \\ \text{sub } (1;0) \quad 0 &= a(1)^2 + b(1) + 6 \\ a + b + 6 &= 0 \quad \text{eq. 2} \\ \text{Sub. Eq. 1 in Eq. 2} \\ -24 &= 12a \quad \therefore a = -2 \\ b &= -4 \end{aligned}$$

(4)

(f)



$$\begin{aligned} \text{Turning point of } f: \quad f(-1) &= -2(-1 + 3)(-1 - 1) \\ &= 8 \\ \therefore (8; -1) &\text{ is the T.P of } g \end{aligned}$$

Shape
y-int: -3 and 1
x-int: 6
TP (8;-1)

(5)

(g) $k > -6$ **OR** $k \in (-6; \infty)$

(2)

[24]

QUESTION 9

(a) $f(1) = a(1)^3 + b(1)^2 \therefore a + b$
 $f(2) = a(2)^3 + b(2)^2 \therefore f(2) = 8a + 4b$

$$\therefore 5,5 = \frac{8a + 4b - (a + b)}{2 - 1}$$

$$7a + 3b = 5,5 \dots\dots (1)$$

$$f'(x) = 3ax^2 + 2bx$$

$$-18 = 3a(6)^2 + 2b(6)$$

$$-18 = 108a + 12b \dots\dots (2)$$

$$4(1)-(2): \begin{cases} 28a + 12b = 22 \\ 108a + 12b = -18 \end{cases}$$

$$\therefore -80a = 40$$

$$\therefore a = -\frac{1}{2}$$

$$\therefore b = 3$$

Note: No marks for answer only.

(8)

(b) $f(x)$ is increasing when $f'(x) \geq 0$

$$-\frac{3}{2}x^2 + 6x \geq 0$$

$$-3x^2 + 12x \geq 0$$

$$-3x(x - 4) \geq 0$$

$$0 \leq x \leq 4$$

Alternate

$$f'(x) = -\frac{3}{2}x^2 + 6x$$

$$= \frac{-3x}{2}(x - 4)$$

$$\therefore x_c = 4$$

$$f \text{ is increasing on } 0 \leq x \leq x_c = 4$$

(4)

(c) f is concave down when

$$f''(x) < 0$$

$$-6x + 12 < 0$$

$$x > 2$$

Alternate

$$\text{Point of inflection is: } x = \frac{0+4}{2} = 2$$

From graph, f is concave down when $x > 2$

(3)

[15]

QUESTION 10

$$h + r = 9$$

$$\therefore h = 9 - r$$

$$V = \pi r^2 h$$

$$V = \pi r^2 (9 - r)$$

$$V = 9\pi r^2 - \pi r^3$$

$$V = 9\pi r^2 - \pi r^3$$

$$V' = 18\pi r - 3\pi r^2$$

$$0 = 18\pi r - 3\pi r^2$$

$$3\pi r(6 - r) = 0$$

$$r \neq 0 \quad \therefore r = 6 \text{ units}$$

[7]

QUESTION 11

(a) (1) $\frac{46}{80} \times \frac{45}{79} = 0,3$ (3)

(2) $\left(\frac{9}{80} \times \frac{25}{79}\right) + \left(\frac{25}{80} \times \frac{9}{79}\right)$
 $= \frac{45}{1264} + \frac{45}{1264}$
 $= \frac{45}{632} \approx 0,07$ (4)

(b) $\frac{8!}{2!2!}$
 $= 10080$ (3)

(c) $P(\text{Khanya will win}) = P(RB) + P(RRRB) + P(RRRRRB) + \dots$

$$P(\text{Khanya will win}) = \left(\frac{6}{7} \times \frac{1}{7}\right) + \left[\left(\frac{6}{7}\right)^3 \times \frac{1}{7}\right] + \left[\left(\frac{6}{7}\right)^5 \times \frac{1}{7}\right] + \dots$$

This is an infinite geometric series since $-1 < r < 1$

For $\frac{1}{6}$ and $\frac{1}{7}$

$$a = \frac{6}{7^2} \quad \text{and} \quad r = \left(\frac{6}{7}\right)^2$$

Tree diagram

$$P(\text{Khanya will win}) = \frac{a}{1 - r}$$

$$P(\text{Khanya will win}) = \frac{\frac{6}{7^2}}{1 - \left(\frac{6}{7}\right)^2} = \frac{6}{13} \approx 0,46$$
 (7)

(7)

[17]

QUESTION 12

$$\left[\frac{1}{8}(4\pi x^2) + \left(x^2 - \frac{1}{4}\pi x^2\right) \times 3 \right] \times 8 + 30x^2 = 28$$

$$\therefore 24x^2 - 2\pi x^2 + 30x^2 = 28$$

$$x^2(54 - 2\pi) = 28$$

$$\therefore x^2 = \frac{28}{54 - 2\pi}$$

$$\therefore x = 0,766$$

$$\therefore x \approx 0,8$$

Marks allocated as follows:

$$54x^2$$

$$30x^2$$

$$4\pi x^2$$

$$6(24x^2 - 6\pi x^2)$$

$$\text{sum of the three parts} = 28$$

$$\therefore x \approx 0,8$$

[7]

73 marks

Total: 150 marks