Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2016

MEMORANDUM

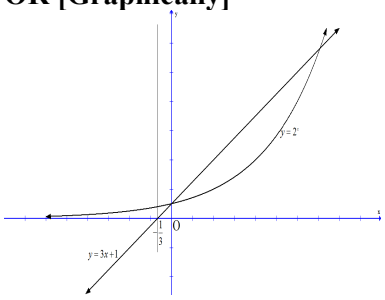
NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 150

This memorandum consists of 12 pages.

QUESTION 1

1.1.1	$2x(3x-5)=0$ $x=0 \quad \text{or} \quad x=\frac{5}{3}$	AA✓✓ Answers	(2)
1.1.2	$x^2-3x=7$ $x^2-3x-7=0$ $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-7)}}{2(1)}$ $x=4{,}54 \quad or \quad x=-1{,}54$	A✓ equation in std. form CA✓ substitution Penalise rounding off CACA✓✓ answers	(4)
1.1.3	$2x-5\sqrt{x}=3$ $2x-3=5\sqrt{x}$ $4x^2-12x+9=25x$ $4x^2-37x+9=0$ $(4x-1)(x-9)=0$ $x=\frac{1}{4} \quad \text{or} \quad x=9$ n/a **OR** $2x-5x^{\frac{1}{2}}-3=0$ $\left(2x^{\frac{1}{2}}+1\right)\left(x^{\frac{1}{2}}-3\right)=0$ $x^{\frac{1}{2}}=\frac{-1}{2} \quad \text{or} \quad x^{\frac{1}{2}}=3$ No solution $\quad x=9$	A✓ isolating $5\sqrt{x}$ M✓ squaring both sides CA✓ standard form CA✓ factors CA✓ both answers CA✓ rejecting A✓ standard form CA✓factors CA✓ $x^{\frac{1}{2}}=\frac{-1}{2}$ CA✓ $x^{\frac{1}{2}}=3$ CA✓answer CA✓rejecting	(6) (6)
1.1.4	$2^x(3x+1)<0$ $2^x>0 \quad \text{for all} \, x\in R$ $\therefore 3x+1<0$ $x<-\frac{1}{3}$ **OR [Graphically]**  $x<-\frac{1}{3}$	A✓ $2^x>0 \quad \text{for all} \, x\in R$ A✓ $3x+1<0$ A✓ $x<-\frac{1}{3}$ A✓ sketch of $y=2^x$ A✓ sketch of $y=3x+1$ A✓ $x<-\frac{1}{3}$	(3) (3)

1.2	$2^{99} \cdot 2^{1} - 2^{99}$ $2^{99}(2-1)$ $= 2^{99}$	M✓ for taking 2^{99} as a common factor A✓ $(2-1)$ A✓ answer	(3)
1.3	$2x - y = 3 \quad \rightarrow (1)$ $x^2 + 5xy + y^2 = 15 \rightarrow (2)$ From (1) : $y = 2x - 3$ Subst. (1) into (2) $x^2 + 5x(2x-3) + (2x-3)^2 = 15$ $x^2 + 10x^2 - 15x + 4x^2 - 12x + 9 - 15 = 0$ $15x^2 - 27x - 6 = 0$ $(5x+1)(3x-6) = 0$ $x = \frac{-1}{5}$ or $x = 2$ $y = \frac{-17}{5}$ or $y = 1$ **OR** $2x - y = 3 \quad \rightarrow (1)$ $x^2 + 5xy + y^2 = 15 \rightarrow (2)$ *From (1) :* $x = \frac{y+3}{2}$ *Subst.* (1) *into (2)* $\left(\frac{y+3}{2}\right)^2 + 5y\left(\frac{y+3}{2}\right) + y^2 = 15$ $\left(\frac{y^2+6y+9}{4}\right) + \frac{5y^2+15y}{2} + y^2 = 15$ $y^2 + 6y + 9 + 10y^2 + 30y + 4y^2 = 60$ $15y^2 + 36y - 51 = 0$ $5y^2 + 12y - 17 = 0$ $(5y+17)(y-1) = 0$ $y = \frac{-17}{5}$ *or* $y = 1$ $x = \frac{-1}{5}$ *or* $x = 2$	A✓ y as subject CA✓ substitution $y = 2x - 3$ CA✓ std. form CA✓factors CA✓x values CA✓y values A✓ x as subject CA✓ substitution $x = \frac{y+3}{2}$ CA✓ std. form CA✓factors CA✓y values CA✓x values	(6) (6)
			[24]

QUESTION 2

2.1	$2a = 4$ $a = 2$ $3a + b = 0$ $b = -6$ $a + b + c = 4$ $c = 8$ $T_n = 2n^2 - 6n + 8$ **OR** $2a = 4$ $a = 2$ $3a + b = 0$ $b = -6$ $T_0 = c = 8$ $T_n = 2n^2 - 6n + 8$ **OR** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= 4 + (n-1)(0) + \frac{(n-1)(n-2)}{2}(4)$ $= 5 + 0 + 2n^2 - 6n + 4$ $= 2n^2 - 6n + 8$	A✓ a value CA✓b value CA✓c value CA✓general term A✓ a value CA✓b value CA✓c value CA✓general term A✓formula A✓substituting first and second difference values CA✓simplifying CA✓general term	(4) (4) (4)
2.2	$T_n = 4n - 4 = 28088$ $4n = 28092$ $n = 7023$ Between the 7023rd and 7024th terms. **OR** $T_n - T_{n-1} = 28088$ $2n^2 - 6n + 8 - [2(n-1)^2 - 6(n-1) + 8] = 28088$ $2n^2 - 6n + 8 - 2n^2 + 4n - 2 + 6n - 6 - 8 - 28088 = 0$ $4n - 28096 = 0$ $4n = 28096$ $n = 7024$ Between the 7023rd and 7024th terms.	A✓ $4n - 4$ CA✓ $4n - 4 = 28088$ CA(n ∈N)✓ $n = 7023$ CA✓(n ∈N)conclusion A✓ $T_n - T_{n-1} = 28088$ CA✓ $4n - 28096 = 0$ CA✓(n ∈N) $n = 7024$ CA✓(n ∈N)conclusion	(4) (4)
			[8]

QUESTION 3

3.1.1	6 ; 2 ; 10 ; 2 ; 14 ; 2 ; ... 18 ; 2	AA✓✓ answers	(2)
3.1.2	$a = 2$ and is a constant sequence $\therefore S_{50} = 50 \times 2$ $= 100$	A✓100	(1)
3.1.3	$a = 6; d = 4; n = 50$ $S_{50} = \frac{50}{2}[2(6) + 4(50-1)]$ $= 5200$ $S_{100} = 5200 + 100$ $= 5300$	A✓ for a, d and n values CA✓ for substitution into formula CA✓ 5200 CA✓answer	(4)
3.2	$S_n = a + ar + ar^2 + ... + ar^{n-1} \rightarrow (1)$ $rS_n = ar + ar^2 + ... + ar^n \rightarrow (2)$ $(2) - (1):$ $rS_n - S_n = ar^n - a$ $S_n(r-1) = a(r^n - 1)$ $\therefore S_n = \frac{a(r^n - 1)}{r-1}$	A✓ for equation (1) A✓ for equation (2) A✓ subtraction on LHS and RHS A✓ factorising if $S_n = \frac{a(1-r^n)}{1-r}$ penalize 1 mark	(4)
			[11]

QUESTION 4

4.1	$a = 1 \qquad r = \frac{2}{3}$ $S_\infty = \frac{a}{1-r}$ $= \frac{1}{1-\frac{2}{3}}$ $= 3$	A✓*a and r* value CA✓substitution of *r* into formula CA✓answer	(3)
4.2	$S_n = 3n^2 - n$ $T_4 = S_4 - S_3$ $= [3(4)^2 - (4)] - [3(3)^2 - (3)]$ $= 44 - 24$ $= 20$ **OR**	M✓ $T_4 = S_4 - S_3$ AA✓✓substitution A✓ answer	(4)

	$S_1 = T_1 = a = 2$ $S_2 = T_1 + T_2 = 3(2)^2 - 2 = 10$ $T_2 = 10 - 2 = 8$ $S_3 = T_1 + T_2 + T_3 = 3(3)^2 - 3 = 24$ $T_3 = 24 - 8 - 2 = 14$ $d = 6$ $T_4 = a + 3d = 2 + 3(6) = 20$	A✓ a value A✓second term value CA✓d value CA✓answer	(4)
			[7]

QUESTION 5

5.1	$A = P(1-i)^n$ $A = 255000(1-0{,}125)^7$ $= \text{R}\quad 100\,137{,}45$	A✓ formula A✓correct substitution CA✓answer	(3)
5.2.1	$A = P(1+i)^n$ $= 10\,000\left(1+\frac{0{,}095}{12}\right)^5$ $= \text{R}10\,402{,}15$	A✓correct substitution into correct formula CA✓answer	(2)
5.2.2	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $10\,402{,}15 = \frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-n}\right]}{\frac{0{,}095}{12}}$ $\frac{10\,402{,}15 \times \frac{0{,}095}{12}}{450} = 1-\left(1+\frac{0{,}095}{12}\right)^{-n}$ $\left(1+\frac{0{,}095}{12}\right)^{-n} = 0{,}816999213$ $-n = \frac{\log 0{,}816999213}{\log\left(1+\frac{0{,}095}{12}\right)}$ $n = 25{,}6315128$ $\therefore n = 26$ payments	CA✓ correct substitution of P into formula A✓ $x = 450$ CA✓ n as subject M✓using logs CA(n Є N)✓ answer	(5)

5.2.3	Balance on Loan after the 25th payment $= 10\ 402{,}15\left(1+\frac{0{,}095}{12}\right)^{25} - \frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{25}-1\right]}{\frac{0{,}095}{12}}$ $= R12668{,}90 - R12386{,}54$ $= R282{,}36$ **OR** $P = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-0{,}6315128}\right]}{\frac{0{,}095}{12}}$ $= R282{,}36$	M✓ $(A - F_v)$ formula A✓ $n = 25$ CA✓ $10\ 402{,}15\left(1+\frac{0{,}095}{12}\right)^{25}$ CA✓ $\frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{25}-1\right]}{\frac{0{,}095}{12}}$ CA✓ answer **OR** A✓formula A✓subst. 450 CA✓subst $\frac{0{,}095}{12}$ CA✓subst $0{,}6315128$ CA✓answer	(5) (5)
			[15]

QUESTION 6 (For learners who did not receive the errata and used P(-4;4) , should be marked accordingly)

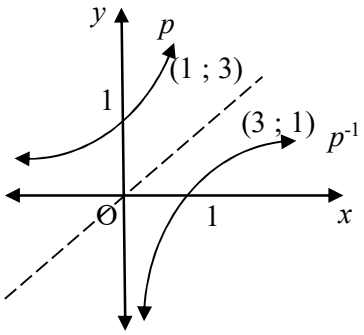
6.1	$p = -3$ $q = -2$	A✓ p value A✓ q value	(2)
6.2	$h(x) = \frac{a}{x+p} + q$. P(4;–4) is a point on h $-4 = \frac{a}{4-3} - 2$ $\therefore a = -2$	CA✓ subst. p, q and point P CA(negative)✓a value	(2)
6.3	$h(x) = \frac{-2}{x-3} - 2$ $h(0) = \frac{-2}{0-3} - 2$ $= -1\frac{1}{3}$ $\left(0;-1\frac{1}{3}\right)$	A✓substituting $x = 0$ CA(negative)✓answer	(2)
6.4	$x = 1$	CACA✓✓ answer	(2)

6.5	$y = -(x + p) + q$ $y = -(x - 3) - 2$ $y = -x + 1$ $\therefore c = 1$ **OR** $y = -x + c$ *Point of intersection of asymptotes* $(3\ ;\ -2)$ $-2 = -3 + c$ $c = 1$	CA✓substitution of p and q values into equation of line of symmetry CA✓answer CA✓substitution of $(3\ ;\ -2)$ into equation of line of symmetry CA✓answer	(2) (2)
			[10]

QUESTION 7

7.1	$D(0\ ;-10)$	A(must be in coordinate form)✓ answer	(1)
7.2	$x^2 - 3x - 10 = 0$ $(x + 2)(x - 5) = 0$ $x = -2 \quad or \quad x = 5$ $A(-2;0)$ $B(5;0)$	A✓ $x^2 - 3x - 10 = 0$ CA✓factors CACA(negative and positive)✓✓ each x – value $A(-2;0)$ $B(5;0)$	(4)
7.3	$a = 2 \quad and \quad q = -10$	CA(positive)✓a – value A✓ q – value	(2)
7.4	$x = -\frac{b}{2a} = -\frac{(-3)}{2(1)} = \frac{3}{2} \quad or \quad x = \frac{-2+5}{2} = \frac{3}{2}$ $y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10 = -\frac{49}{4} / -12{,}25 / -12\frac{1}{4}$ C(1,5 ; −12,25)	A✓ $x = -\frac{b}{2a} = -\frac{(-3)}{2(1)} = \frac{3}{2}$ or CA✓ $x = \frac{-2+5}{2} = \frac{3}{2}$ CA✓substitution CA✓minimum value	(3)
7.5	$(-1{,}5\ ;\ -9{,}25)$	CA✓ x – value CA✓ y – value	(2)
7.6	$x \geq \frac{3}{2}$ **OR** $g'(x).h'(x) \geq 0$ $(2x - 3).2 \geq 0$ $x \geq \frac{3}{2}$	CACA✓✓answer CA✓product of derivatives CA(positive)✓answer penalize 1 mark for incorrect notation	(2) (2)
			[14]

QUESTION 8

8.1	$y = \log_3 x$	AA✓✓ answer	(2)
8.2		A✓Shape of p and p^{-1} A✓y – intercept of p A✓ x – intercept of p^{-1} A✓ point on each graph	(4)
8.3	$\log_3 x = 3$ $x = 27$ $0 < x \leq 27$	M✓setting up equation CA✓$x = 27$ CACA✓✓for end points and inequality ANSWER ONLY full marks	(4)
			[10]

QUESTION 9 [Penalise once for notational error in questions 9, 10 and 11]

9.1	$f(x) = x^3$ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{(x+h)^3 - x^3}{h}$ $= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$ $= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$ $= \lim_{h\to 0}(3x^2+3xh+h^2)$ $= 3x^2$	A✓ formula A✓ substitution into correct formula CA✓ simplifying CA✓factors CA✓ answer	(5)
9.2.1	$f(x) = x^{\frac{1}{2}} - 4x^{-2}$ $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 8x^{-3}$	AA✓✓ exponential form CACA✓✓ answer	(4)

9.2.2	$\left(\frac{1}{2}x^2-3\right)^2 = y$ $y = \frac{1}{4}x^4 - 3x^2 + 9$ $\frac{dy}{dx} = x^3 - 6x$ **OR** $\frac{dy}{dx} = 2\left(\frac{1}{2}x^2-3\right).x$ $= x^3 - 6x$	A✓squaring both sides A✓simplifying CACA✓✓answers **OR** AA✓✓derivative using chain rule CACA✓✓answers	(4) (4)
			[13]

QUESTION 10

10.1.1	$f(-3) = (-3)^2 - 8 = 1$	A✓ $y = 1$ (y – value)	(1)
10.1.2	$f'(x) = 2x$ $m = f'(-3) = 2(-3) = -6$	A✓ value of gradient of tangent	(1)
10.1.3	Equation of tangent : - $y = mx + c$ $1 = -6(-3) + c$ $c = -17$ $y = -6x - 17$ **OR** $y - 1 = -6(x+3)$ $y = -6x - 18 + 1$ $y = -6x - 17$	CA✓ substitution CA✓answer CA✓ substitution CA✓answer	(2) (2)
10.2.1	$f(x) = x^3 - 3x - 2$ $f'(x) = 3x^2 - 3 = 0$ $3(x+1)(x-1) = 0$ $x = -1$ *or* $x = 1$ $y = 0$ *or* $y = -4$ $A(-1\ ;\ 0)$ $B(1\ ;\ -4)$	A✓derivative and equating to 0 CA✓factors CA✓x - values CA✓ y - values	(4)

10.2.2	$PQ = g(x) - f(x)$ $= (2x-2) - (x^3 - 3x - 2)$ $= -x^3 + 5x$ $PQ' = -3x^2 + 5 = 0$ $x^2 = \frac{5}{3}$ $x = \pm\sqrt{\frac{5}{3}} = 1{,}29$ *For* $x = 1{,}29$ $y = -(1{,}29)^3 + 5(1{,}29) = 4{,}3$ *Maximum Length of* $PQ = 4{,}3 units$	A✓expression for PQ A✓ $PQ' = 0$ CA✓ x – values CA✓y value for x = 1,29	(4)
10.2.3	$k = -4$ *or* $k = 0$	$k = 0$ A✓ k = -4 CA✓answers	(2)
10.2.4	$f''(x) = 6x > 0$ $x > 0$ **OR** $x = \frac{x_1 + x_2}{2} = \frac{(-1)+(1)}{2} = 0$ $x > 0$ **OR** $x = -\frac{b}{3a} = -\frac{0}{3(1)} = 0$ $x > 0$	A✓ $6x$ A✓ $6x > 0$ A✓answer A✓midpoint formula CA✓ $x = 0$ CA✓answer A✓formula A✓ $x = 0$ A✓answer	(3) (3) (3)
			[17]

QUESTION 11 (Ignore units in the question)

11.1	$V = s'(t) = 2t + 15 \ \ m/s$	A✓answer	(1)
11.2	$V = s'(25) = 2(25) + 15 = 65 m/s$	CA✓answer	(1)
11.3	$V = s'(t) = 2t + 15 \ \ m/s$ $A = s''(t) = 2 \ m/s^2$	CACA✓✓2	(2)
11.4	$A = s''(5) = 2 \ m/s^2$	CA✓answer (answer only 1 mark)	(1)
11.5	$s(t) = t^2 + 15t = 250$ $t^2 + 15t - 250 = 0$ $(t+25)(t-10) = 0$ $t = 10 \ s$ $V = s'(10) = 2(10) + 15 = 35 m/s$	A✓equating s to 250 CA✓factors CA✓t = 10 CA✓answer	(4)
			[9]

QUESTION 12

12.1.1	150	A✓answer	(1)
12.1.2(a)	$\text{P(Male)} = \frac{70}{150} = \frac{7}{15}$	A✓answer	(1)
12.1.2(b)	$\text{P(Eating Chocolate)} = \frac{80}{150} = \frac{8}{15}$	A✓answer	(1)
12.1.3	$P(\text{Male eating chocolate}) = \frac{45}{150} = 0,3$ $P(\text{Male}) \times P(\text{Eating chocolate})$ $= \frac{70}{150} \times \frac{80}{150} = \frac{56}{225} = 0,249$ $\text{P(Male and Eating chocolate)} \neq$ $\text{P(Male)} \times \text{P(Eating chocolate)}$ Events are not independent	A✓P(male eating chocolate) CA✓ P(Male) x P(eating chocolate) value CA✓conclusion	(3)
12.2.1	6 x 7 x 7 x 7 = 2058	A✓6 x 7 x 7 x 7 A✓2058	(2)
12.2.2	6 x 6 x 5 x 4 = 720	A✓6 x 6 x 5 x 4 A✓720	(2)
12.2.3	Four digit codes divisible by 5:- 6 x 7 x 7 x 1 = 252 Probability of a four digit code divisible by 5 $= \frac{252}{2058} = \frac{6}{49} = 0,1224 = 12,24\ \%$	A✓6 x 7 x7 x 1 A✓252 CA✓answer	(3)
			[13]

TOTAL MARKS: 150