Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2016

MEMORANDUM

NATIONAL
SENIOR CERTIFICATE

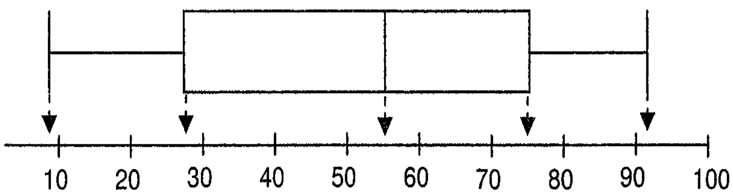
GRADE 12

MARKS : 150

This memorandum consists of 15 pages.

SECTION A

QUESTION 1

1.1	min = 9 ; maximum = 92 ; upper quartile = 75 Lower quartile = 28 and medium = 55 Therefore five number summary is (9; 28; 55; 75; 92)	A✓9 and 92 A✓28 A✓55 A✓75 (4)
1.2		CA✓ correct Q1 & Q3 CA✓ median correctly shown CA✓ both correct whiskers (3)
1.3	Data is skewed to the left /Data is negatively skewed	CA CA✓✓Answer (2)
1.4	mean mark for M–cee-nai High $= \frac{9+14+14+19+21+23+33+35+37+37+42+45+55+56+57+59+68+75+75+75+77+78+80+81+92}{25}$ $\frac{1257}{25} = 50{,}28$ OR $\bar{x} = \frac{1257}{25} = 50{,}28$	A✓sum CA✓answer (2) [Answer only full marks]
1.5	Bee Vee High School. Bee Vee High School performed better because half of the learners got above 60% whilst half of M–cee-nai learners got more than 55%. The median of Bee Vee High was higher than that of M–cee-nai High.	CA✓Bee Vee High CA✓✓Reasoning (3)
		[13]

QUESTION 2

2.1 (6; 160)	A✓ 6 A✓ 160 (2)
2.2 $y = -1{,}64x + 73{,}52$	AA✓✓ gradient AA✓✓y - intercept (4)
2.3 $r = -0{,}2$	AA✓✓ answer (2)
	[8]

QUESTION 3

3.1 AC $= \sqrt{(-5-7)^2 + (1-(2))}^2$ $= \sqrt{(12)^2 + (3)^2}$ $= \sqrt{144+9}$ $= \sqrt{153}$ $= 12{,}37$	A✓ correct Subst CA ✓ answer (2)
3.2 $M_{BC} = \frac{6-(2)}{1-7}$ $= \frac{8}{-6}$ $= \frac{-4}{3}$ $y - y_1 = m(x - x_1)$ $y - 6 = -\frac{4}{3}(x-1)$ $3y - 18 = -4x + 4$ $3y = -4x + 22$	A✓ $\frac{-4}{3}$ CA✓ correct subst. of (1;6) And (7;-2) CA✓ equation in any form (3)

3.3 $\hat{B} = \theta = \alpha - \beta$ .... Ext $\angle$ $\tan\alpha = m_{BC} = -\frac{4}{3}$ $\therefore \alpha = 126,9^\circ$ $\tan_\beta = m_{AB} = \frac{5}{6}$ $\therefore \beta = 39,8^\circ$ $\theta = \alpha - \beta$ $\theta = \alpha - \beta$ $= 126,9^\circ - 39,8^\circ$ $= 87,1^\circ$ $\therefore A\hat{B}C = 87,1^\circ$ OR Distance $AB = \sqrt{(1+5)^2 + (6-1)^2}$ $= \sqrt{61}$ Distance $BC = \sqrt{(1-7)^2 + (6+2)^2}$ $= \sqrt{100}$ $= 10$ Distance $AC = \sqrt{(-5-7)^2 + (1+2)^2}$ $= \sqrt{153}$ $\cos\hat{B} = \frac{a^2 + c^2 - b^2}{2ac}$ $= \frac{10^2 + (\sqrt{61})^2 - (\sqrt{153})^2}{2(10)(\sqrt{61})}$ $= 0,051$ $\hat{B} = 87,1^\circ$	CA✓ $\tan\alpha = -\frac{4}{3}$ CA✓ $\alpha = 126,9^\circ$ A ✓$\tan\beta = \frac{5}{6}$ CA✓ $\beta = 39,8^\circ$ CA ✓ $A\hat{B}C = 87,1^\circ$ (5) A ✓Distance AB A ✓Distance BC A ✓Distance AC CA ✓substitution in cosine rule CA ✓answer (5)
3.4 $P\left(\frac{-5+1}{2} ; \frac{1+6}{2}\right)$ $P\left(-2; \frac{7}{2}\right)$	AA✓✓both co-ordinates (2)

3.5 $m_{AC} = \frac{-2-1}{7+5}$ $= \frac{-3}{12}$ $= \frac{-1}{4}$ through =(–1 ; 3) equation: $y - 3 = -\frac{1}{4}(x+1)$ $y - 3 = -\frac{1}{4}x - \frac{1}{4}$ $\therefore \quad y = \frac{-1}{4}x + 2\frac{3}{4}$ or $y = -\frac{1}{4}x + \frac{11}{4}$ or $4y + x - 11 = 0$	A ✓ $\frac{-1}{4}$ CA ✓ subst. (-1;3) CA✓equation.in any form (3)
3.6 $m_{AB} = \frac{5}{6}$; $6x + 5y = 18$ $5y = -6x + 18$ $y = \frac{-6}{5}x + \frac{18}{5}$ $\therefore m_1 = \frac{-6}{5}$ $m_{AB}\ m_1 = -1$ $\therefore \quad m_{AB} \perp 6x + 5y = 18$	A✓ $m_1 = -\frac{6}{5}$ A✓ $m_{AB} \cdot m_1$ A ✓= -1 (3)
	[18]

QUESTION 4	
4.1.1 At W, y = 2 $3x + 4(2) + 7 = 0$ $3x = -15$ $x = -5$ W(–5;2) r = 5 $(x + 5)^2 + (y - 2)^2 = 25$	A✓ subst y = 2 CA ✓ $x = -5$ CA✓ co -ordinates of W CA✓ r = 5 CA✓equation of the circle. (5)
4.1.2 VZ = 2r = 2 x 5 = 10 units	CA✓answer (1)
4.1.3 $m_{GZ} = \frac{2+1}{0+1}$ $= 3$	A✓substitution into formula CA✓answer (2)
4.1.4 Midpoint of GZ is $\left(-\frac{1}{2}\ ;\ \frac{1}{2}\right)$	A✓ coordinates (1)
4.1.5 $m_{GZ} = 3$ $m_{\perp} = -\frac{1}{3}$ $y - \frac{1}{2} = -\frac{1}{3}\left(x + \frac{1}{2}\right)$ $y = -\frac{1}{3}x + \frac{1}{3}$	CA✓ gradient of perpendicular bisector CA✓substitution into formula CA ✓ answer (3)

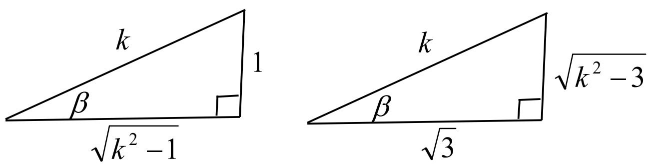
4.1.6	W (–5; 2) into $x + 3y - 1 = 0$ LHS = (2 (–5) + 6(2) – 2 = –10 + 12 – 2 = 0 = RHS W is on the line that bisects GZ perpendicularly and W on GZ. $\therefore$ lines intersect at W. **OR** $-\frac{1}{3}x+\frac{1}{3}=2$ $-x+1=6$ $x=-5$ This is the x - value of the coordinate of W. **OR** Equation of WZ: $y - y_1 = m(x - x_1)$ $y+1=\frac{2+1}{-5+1}(x+1)$ $y+1=-\frac{3}{4}(x+1)$ $y=-\frac{3}{4}x-\frac{7}{4}$ $\therefore -\frac{3}{4}x-\frac{7}{4}=-\frac{1}{3}x+\frac{1}{3}$ $-9x-21=-4x+4$ $-5x=25$ $x=-5$ $\therefore y=-\frac{1}{3}(-5)+\frac{1}{3}=2$ This is the coordinate of W.	A✓ substitution A✓ = 0 (2) A✓equating eq. of perpendicular bisector to the horizontal line $y = 2$ A✓ $x=-5$ (2) A✓equation of WZ A✓ $x=-5$ (2)
4.2.1	circle M: M(–2; 1) ; r_1 = 5 circle N: N(1;3) ; r_2 = 3 $\therefore$ $r_1 + r_2 = 8$ and $r_1 - r_2 = 2$ MN = $\sqrt{(1-(-2))^2+(3-1)^2}$ = $\sqrt{3^2+2^2}$ = $\sqrt{9+4}$ = $\sqrt{13}$ or 3,6 $\therefore$ $r_1 + r_2 > \text{MN} > r_1 - r_2$ $\therefore$ The two circles intersect at two distinct points.	A ✓ $r_1 = 5$ A ✓ $r_2 = 3$ A ✓ $r_1 + r_2 = 8$ A ✓ MN $=\sqrt{13}$ = 3,6 A✓comparing A✓ conclusion (6)

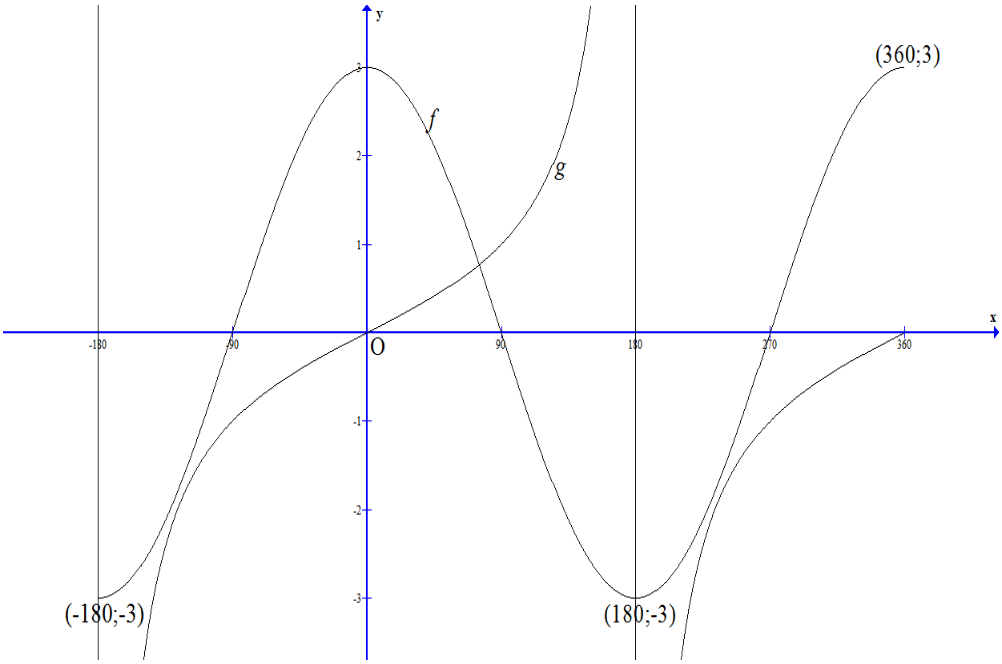
4.2.2 circle M = circle N $(x + 2)^2 + (y-1)^2 - 25 = (x-1)^2 + (y-3)^2 - 9$ $x^2 + 4x + 4 + y^2 - 2y + 1 - 25 = x^2 - 2x + 1 + y^2 - 6y + 9 - 9$ $6x + 4y = 21$ $\therefore$ The equation of the common chord is : $6x + 4y = 21$	M✓ equating A✓ simplifying CA✓ equation of the chord (3) **[23]**

QUESTION 5	
5.1.1 $\tan\alpha = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$ $\therefore \alpha = 360^\circ - 30^\circ$ $= 330^\circ$ $\therefore \beta = 30^\circ$ or $\tan(-\beta) = -\tan\beta$ $= -(\frac{-2}{2\sqrt{3}})$ $\tan\beta = \frac{2}{2\sqrt{3}}$ $\beta = 30^\circ$	A✓correct ratio CA✓ $\alpha = 330^\circ$ CA✓ $\beta = 30^\circ$ A✓correct ratio CACA✓✓ $\beta = 30^\circ$ (3)
5.1.2 $OP^2 = (2\sqrt{3})^2 + (-2)^2 = 12 + 4$ $= 16$ $\therefore OP = 4$	A✓ using distance for formula CA✓answer (2)
5.1.3 $\frac{OP}{OQ} = \cos\beta$ $\therefore OQ = \frac{OP}{\cos\beta} = \frac{4}{\cos 30^\circ}$ $= \frac{4}{\frac{\sqrt{3}}{2}}$ $= \frac{8}{\sqrt{3}}$	CA✓ $\cos 30^\circ = \frac{\sqrt{3}}{2}$ CA✓ $\frac{8}{\sqrt{3}}$

$Q = \left(\frac{8\sqrt{3}}{3}\ ;\ 0\right)$ or $\frac{OP}{OQ} = \cos 30^\circ$ $OQ = \frac{4}{\cos 30^\circ} = \frac{4}{\frac{\sqrt{3}}{2}}$ $OQ = \frac{8\sqrt{3}}{3}$ $Q = \left(\frac{8\sqrt{3}}{3}\ ;\ 0\right)$	CA✓ co-ordinates CA✓ $\cos 30^\circ = \frac{\sqrt{3}}{2}$ CA✓ $\frac{8\sqrt{3}}{3}$ CA✓ co-ordinates (3)

5.2 $2\left[\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right]$ $= 2\left[\sin 30^\circ \cos\alpha + \cos 30^\circ \sin\alpha\right]$ $= 2\sin\left(30^\circ + \alpha\right)$ $k = 2\ ;\ \beta = 30^\circ$	M✓ for introducing 2 A✓ introducing 30° special angle A✓ sum compound formula A✓ calculating k A✓ calculating β
OR $\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha = \frac{k}{2}\left(\sin\alpha + \beta\right)$ $= \sin 30^\circ \cos\alpha + \cos 30^\circ \sin\alpha = \frac{k}{2}\sin\left(\alpha + \beta\right)$ $= \sin\left(30^\circ + \alpha\right) = \frac{k}{2}\sin\left(\alpha + \beta\right)$ $\frac{k}{2} = 1,\ \beta = 30^\circ$ $\therefore k = 2,\ \beta = 30^\circ$ OR	A✓ for introducing 2 A✓ introducing 30° special angle A✓ sum compound formula A✓ calculating k A✓ calculating β (5)

$\cos\alpha + \sqrt{3}\sin\alpha = k\sin(\alpha+\beta)$	
$\cos\alpha + \sqrt{3}\sin\alpha = k(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$	
$\cos\alpha + \sqrt{3}\sin\alpha = k\sin\alpha\cos\beta + k\cos\alpha\sin\beta$	A✓ expansion
$\cos\alpha + \sqrt{3}\sin\alpha = (k\sin\beta)\cos\alpha + (k\cos\beta)\sin\alpha$	
$\therefore 1 = k\sin\beta$ and $\sqrt{3} = k\cos\beta$	A✓ comparing coefficients
$\therefore \sin\beta = \frac{1}{k}$ $\cos\beta = \frac{\sqrt{3}}{k}$	A✓ values of trig ratios
	
$\sqrt{k^2-1} = \sqrt{3}$ $k^2 = 4$ $k = 2$	
$\sin\beta = \frac{1}{2}$	A✓ calculating k
$\beta = 30^\circ$	A✓ calculating β
	(5)
	[13]

QUESTION 6 6.1 	A✓asymptotes A✓shape of f A✓shape of g A✓correct x- intercept of f (–90°; 90°; 270°) A✓correct x – intercept of g (0°; 360°) (5)
6.2 360°	A✓360° (1)
6.3 (0; 3) and (180° ; – 3) (–180° ; –3) and (360°; 3)	CA✓for any two (2)
6.4 $-180^\circ < x < 0^\circ$ or $180^\circ < x < 360^\circ$ OR $-180^\circ < x < 0^\circ \cup 180^\circ < x < 360^\circ$	CA✓$-180^\circ < x < 0^\circ$ CA✓ $180 < x < 360^\circ$ [penalize one mark for incorrect notation] (2)
6.5 $y = 3\cos(x - 45^\circ)$	A✓$3\cos(x - 45^\circ)$ (1) **[11]**

QUESTION 7	
7.1 $\cos 54^\circ \cdot \cos x + \sin 54^\circ \cdot \sin x = \sin 2x$	
$\cos(54^\circ - x) = \sin 2x$	A✓$\cos(54^\circ - x)$
$= \cos(90^\circ - 2x)$	A✓$\cos(90^\circ - 2x)$
$\therefore$ $54^\circ - x = 90^\circ - 2x + n \cdot 360^\circ$ *or* $54^\circ - x = 360 - (90^\circ - 2x) + n \cdot 360^\circ$	
$x = 36^\circ + n \cdot 360^\circ$ *or* $-3x = 216^\circ + n \cdot 360^\circ$	
$x = 36^\circ + n \cdot 360^\circ$ *or* $-x = -72^\circ + n \cdot 120^\circ$	CA✓$x = 36^\circ + n \cdot 360^\circ$
$\therefore$ $x = 36^\circ + n \cdot 360^\circ$ *or* $x = -72^\circ + n \cdot 120^\circ$, $n \in Z$	CA✓$x = -72^\circ + n \cdot 120^\circ$ A✓$n \in Z$ (5)
7.2.1 $E\hat{F}D = F\hat{B}C = \alpha$ … corresponding ∠'s … AD // BC	A✓ S/R
$E\hat{F}D = \theta + A\hat{E}F$ ... ext ∠ of Δ AEF	A✓ S/R
$\therefore A\hat{E}F = \alpha - \theta$	A✓ $A\hat{E}F = \alpha - \theta$
In Δ ABE : $\dfrac{\sin A\hat{E}B}{AB} = \dfrac{\sin E\hat{A}B}{BE}$	A✓ sine rule application
$\therefore BE = \dfrac{AB \sin(90^\circ + \theta)}{\sin(\alpha - \theta)}$	A✓ substitution
$= \dfrac{AB \cos\theta}{\sin(\alpha - \theta)}$	(5)
7.2.2 Area of Δ BCE $= \dfrac{1}{2} \cdot x(18 - 3x)\sin 150^\circ$	A✓ area rule
$A(x) = \dfrac{1}{2}x \cdot (18 - 3x) \cdot \dfrac{1}{2}$	A✓ $\sin 150^\circ = \dfrac{1}{2}$
$= \dfrac{1}{4} \cdot x \cdot 18 - \dfrac{1}{4}x \cdot 3x$	
$= \dfrac{18}{4}x - \dfrac{3}{4}x^2$	
$= \dfrac{9}{2}x - \dfrac{3}{4}x^2$	A✓ simplifying (3)

7.2.3 At maximum area: $A'(x) = 0$ $A'(x) = \frac{9}{2} - 2.\frac{3}{4}x$ $0 = \frac{9}{2} - \frac{3}{2}x$ $3x = 9$ $x = 3$	M✓ $A'(x) = 0$ A✓ derivative CA✓ $x = 3$ (3)
7.2.4 BC = 3 CE = 18 – 3 (3) = 18 – 9 = 9 $BE^2 = BC^2 + CE^2 - 2\ BC\ .\ CE \cos 150^o$ $= (3)^2 + (9)^2 - 2 \times 3 \times 9\ (-\cos 30^o)$ $= 9 + 81 + 54 \cos 30^o$ $= 90 + 54\ .\left(\frac{\sqrt{3}}{2}\right)$ $= 136{,}765$ $\therefore BE = 11{,}695$ $= 11{,}69$	A✓ for both BC = 3 and CE = 9 CA✓ applying cosine rule and substitution CA✓ answer (3) **[19]**

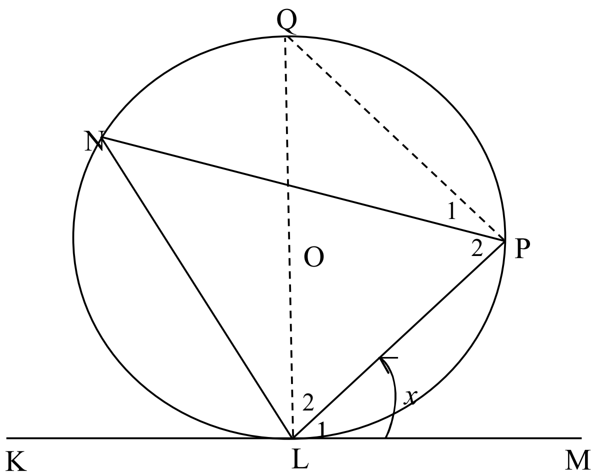
QUESTION 8

8.1	PT = TQ = 12cm …(line from center perpendicular to chord PQ) $\therefore$ PQ = 12 cm + 12 cm = 24cm	 A✓ R A✓ answer (2)
8.2	$OT^2 = OQ^2 - QT^2$ ….. pythagoras $= 13^2 - 12^2$ $= 169 - 144$ $= 25$ $\therefore OT = 5$ $\therefore TR = OR - OT$ = 13cm – 5cm = 8cm In Δ PTR, $PR^2 = TR^2 + PT^2$ $= 8^2 + 12^2$ $= 64 + 144$ $= 208\ cm^2$ $\therefore PR = \sqrt{208}$ cm or $4\sqrt{13}$ cm or 14,42 cm	 A✓ OT = 5 CA✓ TR = 8cm CA✓ $PR^2 = 208$ CA✓ PR = $4\sqrt{13}$ or 14,42 (4) **[6]**

QUESTION 9

9.1 Interior opposite angle	A✓ S (1)

9.2



Construction : Draw diameter LOQ and join QP or Join OL and OP	A✓construction

STATEMENT	REASON	
Let $P\hat{L}M = \hat{L}_1 = x$		
$\hat{P}_1 + \hat{P}_2 = 90^o$	angle subtended by the diameter	A✓S/R
$\hat{L}_2 = 90^o - x$	LM ⊥ OL, tan – radius	A✓S/R
$\therefore \hat{Q} = x$	Sum of the angles of a triangle	A✓S
$\hat{N} = x$	Subtended by the same chord LP	A✓S/R
$P\hat{L}M = \hat{N}$		(5)

9.3.1 $\hat{A} = 180^o - A\hat{E}D$ … co interior ∠'s, AB//ED $= 180^o - 70^o$ $= 110^o$	A✓ S/R A✓ 110° (2)
9.3.2 $\hat{B}_1 = 70^\circ$ ... ext ∠ cyclic quad ABDE	A✓ R A✓ 70° (2)
9.3.3 $\hat{D}_2 = \hat{B}_1 = 70^\circ$ ....(alt ∠s ; DE//CA)	CA✓ 70° A✓ S/R (2)
9.3.4 $\hat{B}_2 = \hat{D}_2 = 70^\circ$ ... (∠s opp = sides)	CA✓ 70° A✓ S/R (2)

9.3.5 $\hat{E}_1 = 180^\circ - (\hat{B}_2 + \hat{D}_2)$...($\angle$ sum of Δ) $= 180^\circ - 140^\circ$ $= 40^\circ$ $\therefore \hat{D}_1 = \hat{E}_1 = 40^\circ$ ... tan chord theorem	CA✓ $\hat{E}_1 = 40^\circ$ CA✓ $\hat{D}_1 = 40^\circ$ A✓ R (3) **[17]**

QUESTION 10

10.1 $\hat{P}_1 = \hat{B}_2 = x$ ... alt $\angle$s; SP//BC $\hat{P}_2 = \hat{P}_1 = x$ ... given $Q_1 = P_1 = x$ ... tan chord theorem	A✓ S A✓ R A✓ S/R A✓ S/R (4)
10.2 PC = BC … $\hat{P}_2 = \hat{B}_2 = x$ proved above (Δ PCB)	A✓ $\hat{P}_2 = \hat{B}_2 = C = x$ A✓reason (2)
10.3 $\hat{Q}_1 = \hat{B}_2 = x$ ... proved $\therefore$ RCQB is a cyclic quad … converse $\angle$'s in the same segment	A✓ S A✓ R (2)
10.4 $\hat{S} = \hat{B}_3$ ... corresp $\angle$'s SP $\parallel$ BC $= \hat{R}_3$ ...$\angle$'s in the same segment, cyclic quad RCQB In Δ PBS and Δ QCR $\hat{P}_1 = \hat{Q}_1 = x$ ... proved $\hat{S} = \hat{R}_3$ … proved Remaining $\angle s$ equal $\therefore \Delta$ PBS ||| Δ QCR	A✓ S/R A✓ S/R A✓ S/R A✓ S/R A✓ R (5)
10.5 In Δ PBQ and Δ PCR $\hat{P}_2$ is common $P\hat{Q}B = \hat{R}_2$ ... ext $\angle$ of cyclic quad RCQB Δ PBQ ||| Δ PCR … (3rd $\angle$ Δ) $\therefore \frac{PB}{CP} = \frac{QB}{CR}$ (||| Δ s) $\therefore$ PB . CR = QB . CP	A✓S A✓ S/R A✓ S/R A✓ S (4) **[17]**

QUESTION 11

In Δ KLM $\frac{LD}{9} = \frac{8}{6}$ … (LM//DE; proportionality theorem) $\therefore$ LD = 12 $D\hat{M}L = M\hat{D}E = x$ … alt $\angle s$, LM ∥ DE $\therefore$ LM = LD = 12 … (sides opp = $\angle s$)	A✓S/R A✓ LD = 12 A✓ S A✓answer A✓ R (5)
	[5]

TOTAL: [150]