Basic Education

KwaZulu-Natal Department of Basic Education
C OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2016

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages,
1 information sheet and an answer book of 21 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.

QUESTION 1

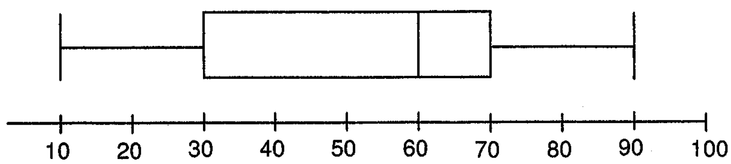
Two schools, M-cee-nai High and Bee Vee high are in competition to see which school performed better in mathematics in the June Examination.

The marks of the leaners at M-cee-nai High school are recorded below. The box whisker diagram below illustrates the results of Bee Vee High School. Both schools have 25 learners. (Marks are given in %).

Marks of M-cee-nai High School learners

9	14	14	19	21
23	33	35	37	37
42	45	55	56	57
59	68	75	75	75
77	78	80	81	92

The box and whisker diagram for the Bee Vee High School Learners



1.1 Write down the five-number summary for M-cee-nai High School. (4)

1.2 Draw the box and whisker diagram that represents M-cee-nai High School marks. Clearly indicate ALL relevant values. (3)

1.3 Comment on the skewness of the data of M-cee-nai High School. (1)

1.4 Calculate the mean mark of M-cee-nai High School. (2)

1.5 Determine which school performed better in the June Examination and give reasons for your conclusion. (3)

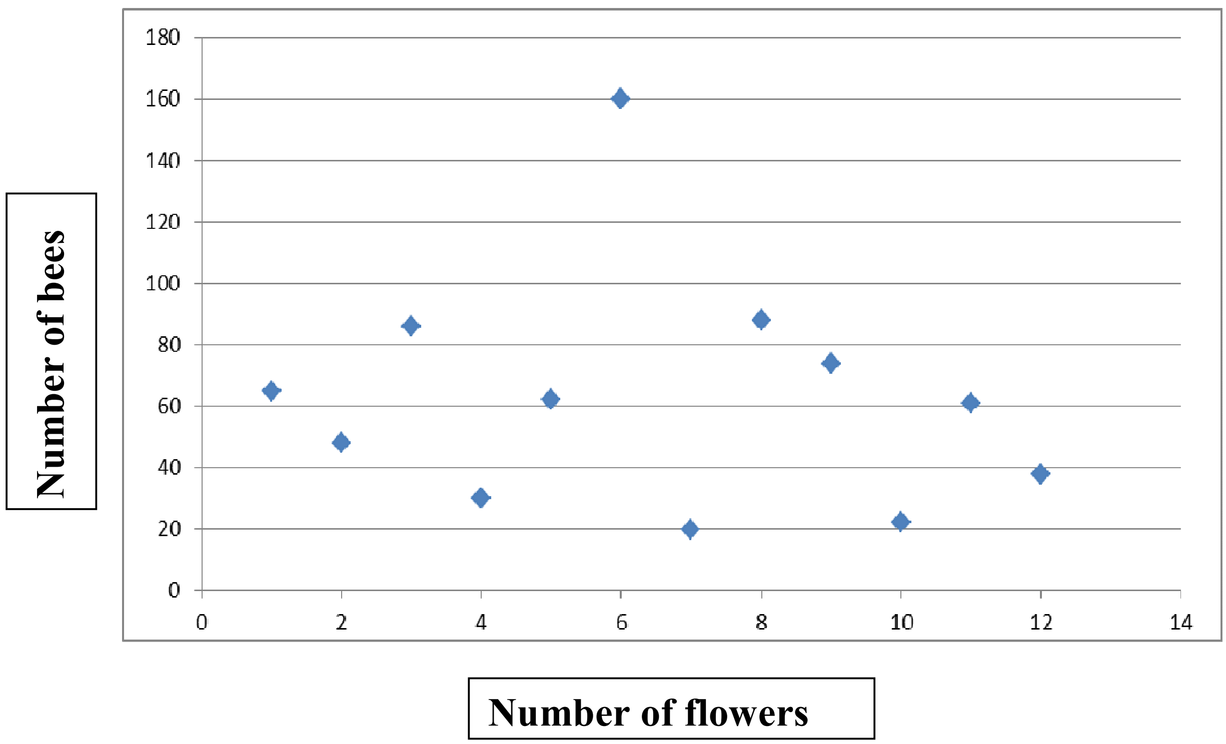
[13]

QUESTION 2

A survey was conducted indicating the number of bees that visited flowers over a period of 12 days. The information is represented in the table and in the scatter plot below.

No. of Flowers	4	10	7	12	1	6	2	5	11	9	8	3
No. of bees	30	22	20	38	65	160	48	62	61	74	88	86

SCATTER PLOT



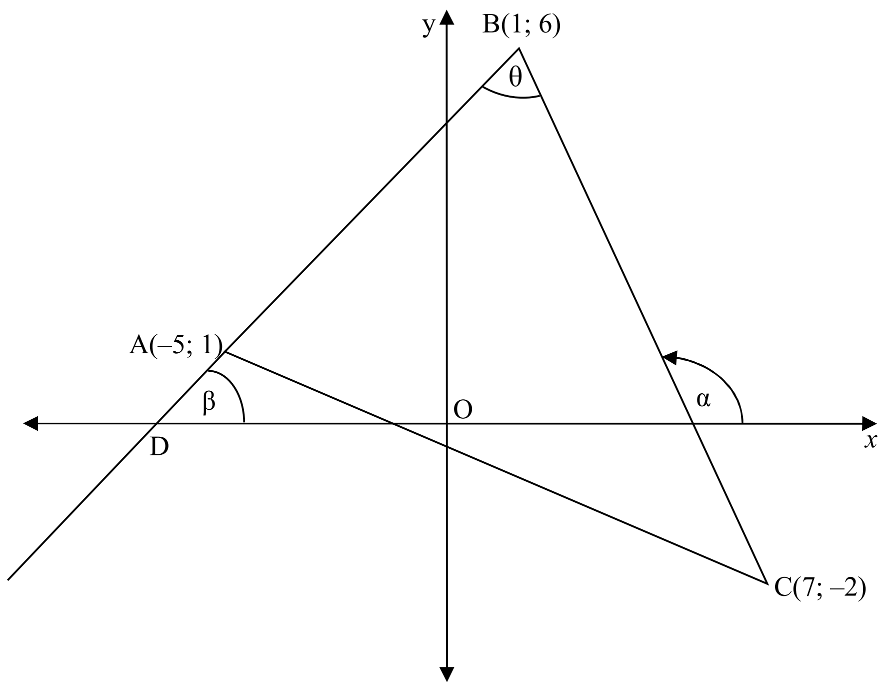
2.1 Write down the coordinates of the outlier. (2)

2.2 Determine the equation for the least squares regression line. (4)

2.3 Calculate the correlation co-efficient **if the outlier is excluded**. (2)
[8]

QUESTION 3

In the diagram below, A (–5; 1) , B(1; 6) and C(7; –2) are vertices of Δ ABC with AB produced to D. BD forms an angle, β , with the negative x – axis and BC forms an angle, α , with the positive x – axis. $A\hat{B}C = \theta$



Determine:

3.1 the length AC (2)

3.2 the equation of line BC (3)

3.3 $A\hat{B}C$ (5)

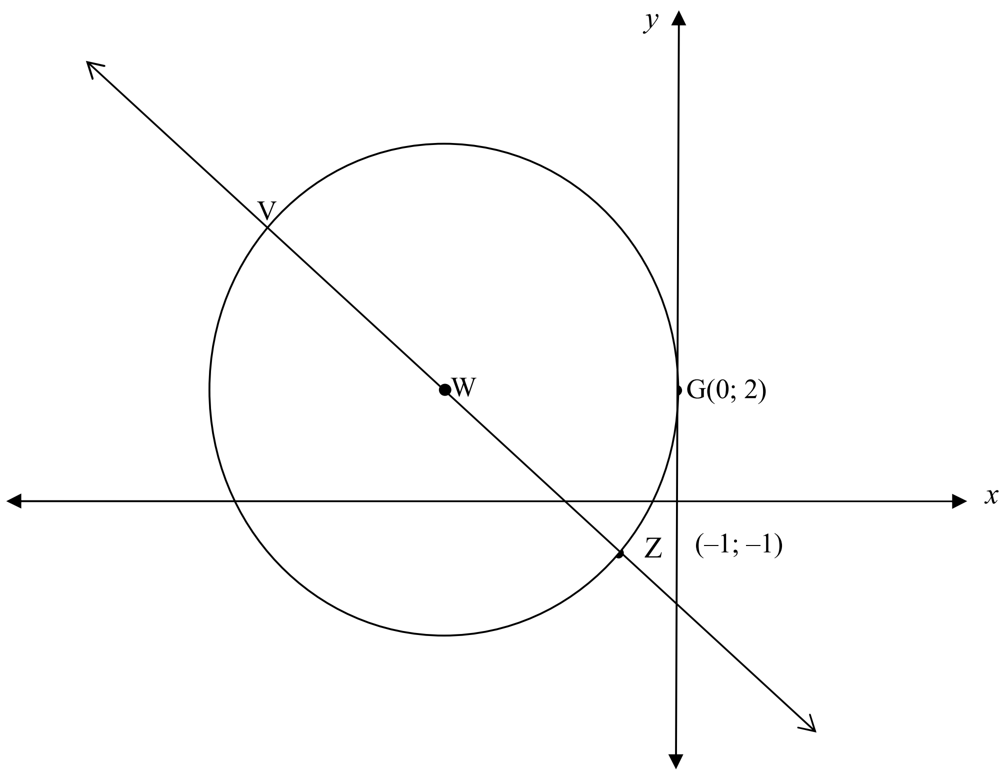
3.4 the midpoint P of AB (2)

3.5 the equation of the line parallel to AC and passing through the point (–1; 3) (3)

3.6 Show that AB is perpendicular to $6x + 5y = 18$ (3)
[18]

QUESTION 4

In the diagram below, centre W of the circle lies on the straight line $3x + 4y + 7 = 0$
The straight line cuts the circle at V and Z(–1; –1). The circle touches the y-axis at G(0; 2)



4.1.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (5)

4.1.2 Determine the length of diameter VZ (1)

4.1.3 Calculate the gradient of GZ. (2)

4.1.4 Write down the coordinates of the midpoint of GZ. (1)

4.1.5 Determine the equation of the line that is the perpendicular bisector of GZ. (3)

4.1.6 Show that the line in QUESTION 4.1.5 and straight line VZ intersect at W. (2)

4.2 The circle defined by $(x + 2)^2 + (y - 1)^2 = 25$ has centre M, and the circle defined by $(x - 1)^2 + (y - 3)^2 = 9$ has centre N.

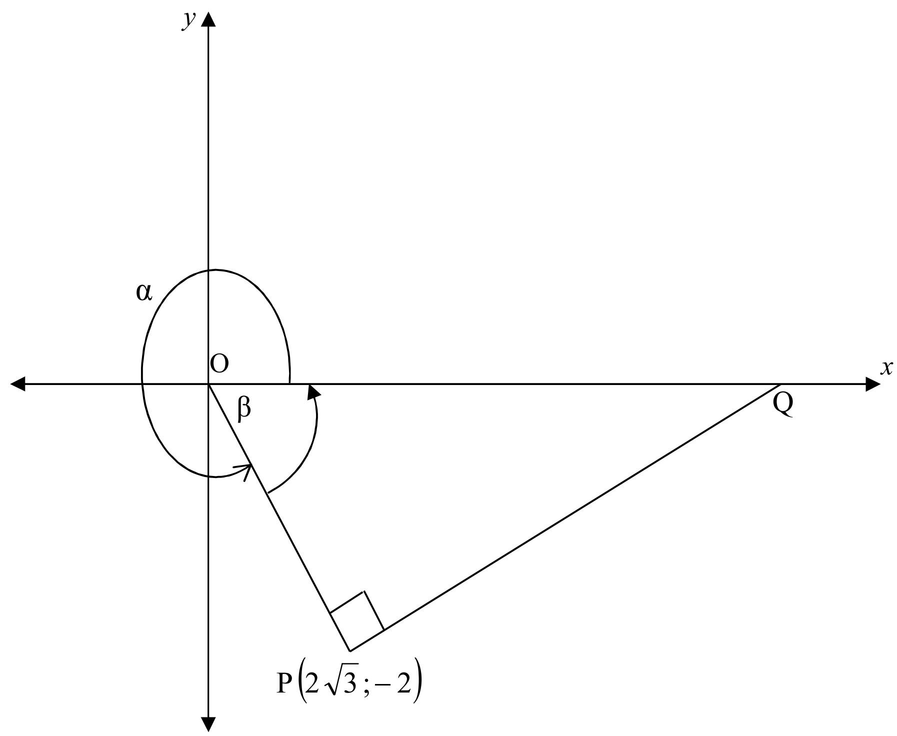
4.2.1 Show that the circles intersect each other at two distinct points. (6)

4.2.2 Determine the equation of the common chord. (3)

[23]

QUESTION 5

5.1 In the diagram below, $P\left(2\sqrt{3}\,;-2\right)$ is a point in the cartesian plane, with reflex angle $Q\hat{O}P = \alpha$. Q is the point on the x – axis so that $O\hat{P}Q = 90^{\circ}$



Calculate without measuring:

5.1.1 β. (3)

5.1.2 the length of OP. (2)

5.1.3 the co-ordinates of Q. (3)

5.2 If $\cos\alpha + \sqrt{3}\sin\alpha = k\sin(\alpha + \beta)$.

Calculate the values of k and β. (5)

[13]

QUESTION 6

6.1 On the same system of axes, sketch the graphs of $f(x) = 3\cos x$ and $g(x) = \tan\frac{1}{2}x$ for $-180° \leq x \leq 360°$. Clearly show the intercepts with the axes and all turning points. (5)

Use the graphs in 6.1 to answer the following questions.

6.2 Determine the period of g. (1)

6.3 Determine the co-ordinates of the turning points of f on the given interval. (2)

6.4 For which values of x will both functions increase as x increases for $-180° \leq x \leq 360°$? (2)

6.5 If the y–axis is moved 45º to the left, then write down the new equation of f in the form $y = \ldots..$ (1)

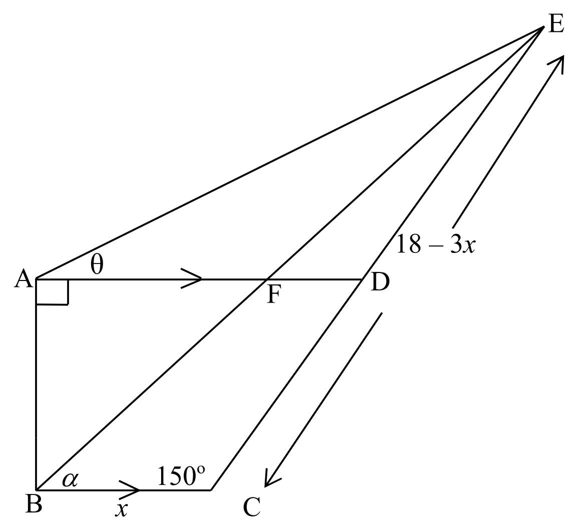
[11]

QUESTION 7

7.1 Determine the general solution of:

$\cos 54^\circ \, . \, \cos x + \sin 54^\circ \, . \, \sin x = \sin 2x$ (5)

7.2 ABCD is a trapezium with AD || BC, $B\hat{A}D = 90^\circ$ and $B\hat{C}D = 150^\circ$.
CD is produced to E. F is point on AD such that BFE is a straight line, and $C\hat{B}E = \alpha$.
The angle of elevation of E from A is θ, BC = x and CE = 18 – 3x.



7.2.1 Show that: $\text{BE} = \dfrac{\text{AB} \cos \theta}{\sin(\alpha - \theta)}$ (5)

7.2.2 Show that the area of Δ BCE = $\dfrac{9}{2}x - \dfrac{3x^2}{4}$ (3)

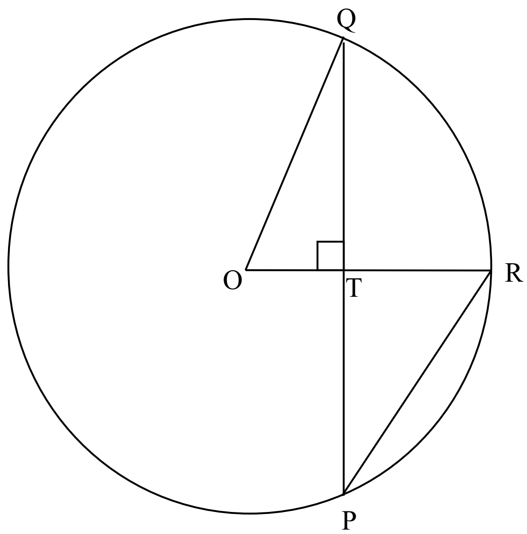
7.2.3 Determine, without the use of a calculator, the value of x for which the area of ΔBCE will be maximum. (3)

7.2.4 Calculate the length of BE if $x = 3$. (3)

[19]

QUESTION 8

In the diagram below, PQ is the chord of circle O. OR is perpendicular to PQ and OR intersects PQ at T. If the radius of the circle is 13 cm and PT = 12 cm.



Calculate the length of:

8.1 PQ (2)

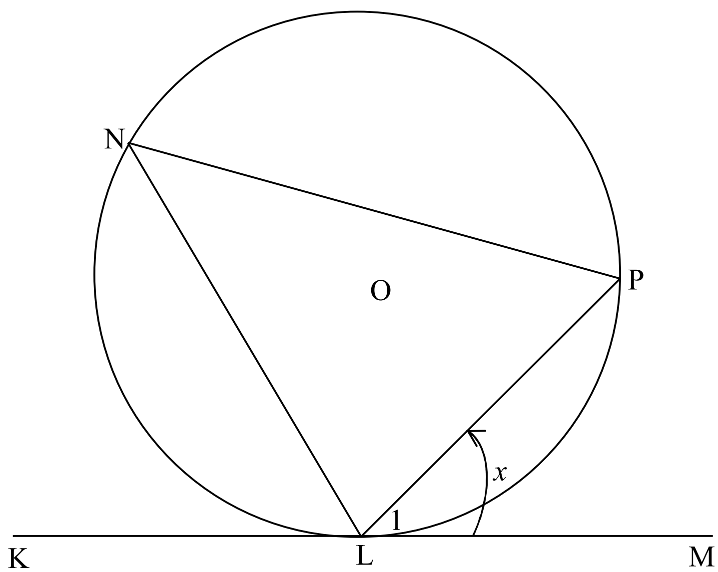
8.2 PR (4)
[6]

QUESTION 9

9.1 Complete the statement so that it is true:

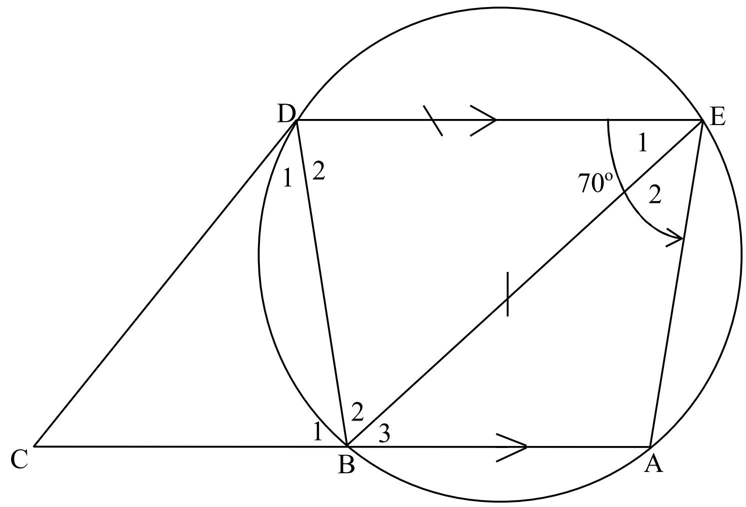
The exterior angle of a cyclic quadrilateral is equal to … (1)

9.2 In the diagram below the circle with centre O passes through L, N and P.
KLM is a tangent to the circle at L. NP, NL and LP are joined.



Using the above diagram, prove the theorem that states that $P\hat{L}M = \hat{N}$. (5)

9.3 In the diagram below, BAED is a cyclic quadrilateral with BA || DE. BE = DE and $\hat{AED} = 70^\circ$. The tangent to the circle at D meets AB produced at C.



Calculate, with reasons the sizes of the following angles.

9.3.1 $\hat{A}$ (2)

9.3.2 $\hat{B}_1$ (2)

9.3.3 $\hat{D}_2$ (2)

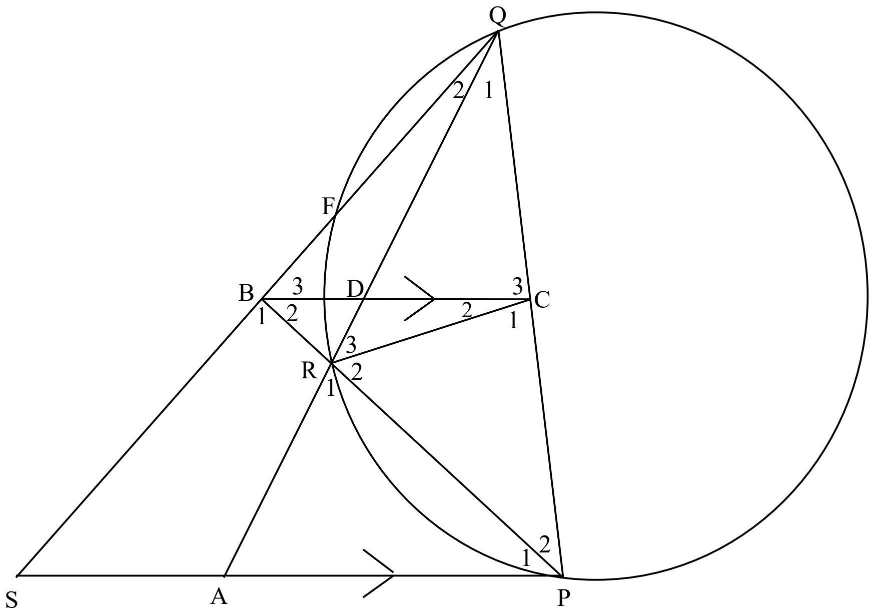
9.3.4 $\hat{B}_2$ (2)

9.3.5 $\hat{D}_1$ (3)

[17]

QUESTION 10

In the diagram below, SP is a tangent to the circle at P and PQ is a chord. Chord QF produced meets SP at S and chord RP bisects $Q\hat{P}S$. PR produced meets QS at B. BC || SP and cuts the chord QR at D. QR produced meets SP at A. Let $\hat{B}_2 = x$.



10.1 Name, with reasons, 3 angles equal to x. (4)

10.2 Prove that PC = BC (2)

10.3 Prove that RCQB is a cyclic quadrilateral. (2)

10.4 Prove that Δ PBS ||| Δ QCR. (5)

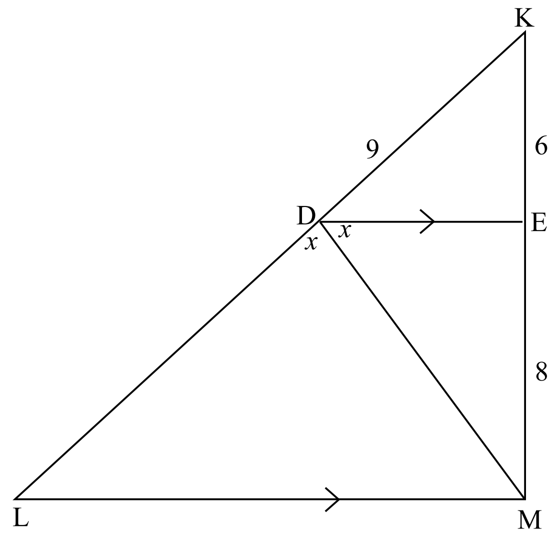
10.5 Show that PB.CR = QB.CP (4)

[17]

QUESTION 11

In the diagram alongside, Δ KLM,
DE || LM, $L\hat{D}M = M\hat{D}E = x$.
KD = 9, EM = 8 and EK = 6

Calculate, with reasons LM.



(5)

TOTAL: [150]

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) - P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$