Education and Sport Development

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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2016

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $7x(2x-1)=0$ (2)

1.1.2 $2x^2+x=4$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $(x-4)(x+5)\geq 0$ (3)

1.1.4 $3x^{\frac{2}{5}}-5x^{\frac{1}{5}}-2=0$ (4)

1.2 Solve for x and y simultaneously:

$\dfrac{2x}{1+y}=1;\ y\neq -1$ and $(3x-y)(x+y)=0$ (6)

1.3 Given: $f(x)=\dfrac{3}{x-2}$ and $g(x)=3^{x-2}$. Explain why $f(x)=g(x)$ will have only ONE root. Motivate your answer. (3)

[22]

QUESTION 2

2.1 Consider the following quadratic sequence:

$x;\ x+2x;\ x+2x+3x;\ x+2x+3x+4x;\ x+2x+3x+4x+5x; \ldots$

2.1.1 Write down the first 3 terms of the sequence of first differences of the quadratic sequence. (1)

2.1.2 Write down the 100^{th} term of the sequence of first differences of the quadratic sequence. (1)

2.1.3 If $x=2$, determine the general term of the quadratic sequence. (4)

2.2 $54\ ;\ x\ ;\ 6$ are the first three terms of a geometric sequence.

2.2.1 Calculate x. (2)

2.2.2 Is this geometric sequence convergent? Motivate your answer by clearly showing all your calculations. (3)

2.3 Determine the value of k for which:

$$\sum_{r=5}^{60}(3r-4)=\sum_{p=2}^{5}k$$ (5)

2.4 Consider $4; \frac{3}{4}; 4; \frac{1}{4}; 4; \frac{1}{12}; \ldots$ which is a combination of 2 geometric patterns.

2.4.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (1)

2.4.2 Calculate the sum of the first 25 terms of the sequence. Show all calculations. (6)

[23]

QUESTION 3

Given: $f(x) = \frac{-3}{x-2} + 1$

3.1 Calculate the coordinates of the y-intercept of f. (2)

3.2 Calculate the coordinates of the x-intercept of f. (2)

3.3 Sketch the graph of f in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

3.4 Write down the range of f. (2)

3.5 Another function h, is formed by translating f 3 units to the right and 4 units down. Write down the equation of h. (2)

3.6 For which value(s) of x is $h(x) \leq -4$? (3)

3.7 Determine the equations of the asymptotes of $k(x) = \frac{3x-5}{x-1}$. (3)

[17]

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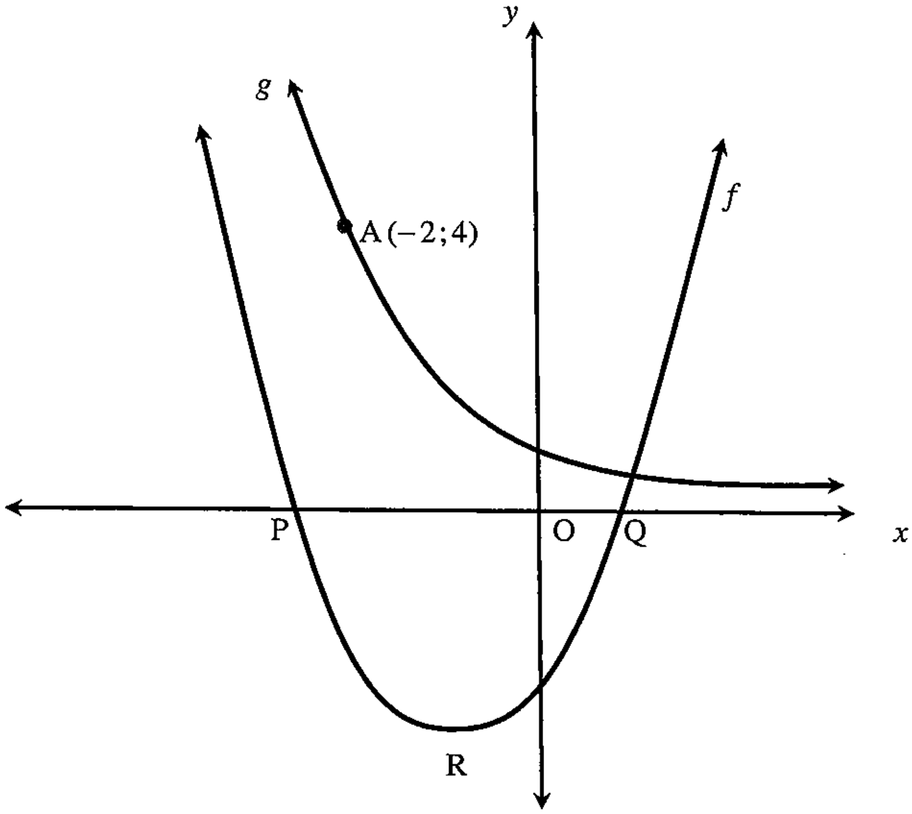
QUESTION 4

The graphs of $f(x) = 2(x+1)^2 - 8$ and $g(x) = \left(\frac{1}{2}\right)^x$ are represented in the sketch below. P and Q are the x-intercepts of f and R is the turning point of f. The point $A(-2; 4)$ is a point on the graph of g.



4.1 Write down the equation of the axis of symmetry of f. (1)

4.2 Write down the coordinates of the turning point of f. (1)

4.3 Determine the length of PQ. (4)

4.4 Write down the equation of k, if k is the reflection of f in the y-axis. Give your answer in the form $y = ax^2 + bx + c$. (3)

4.5 Write down the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (1)

4.6 Sketch the graph of g^{-1} in your ANSWER BOOK, clearly showing the intercept with the axis as well as ONE other point on the graph of g^{-1}. (3)

4.7 For which value(s) of x will:

4.7.1 $g^{-1}(x) \geq -2$ (2)

4.7.2 $x.f(x) < 0$ (4)

[19]

QUESTION 5

Thabo bought a house for R980 000. He paid a deposit of 10% of the selling price of the house. He obtained a loan from the bank at an interest rate of 11% per annum, compounded monthly, to pay the balance of the selling price. He agreed to pay monthly instalments of R10 000 on the loan.

5.1 How much money did Thabo borrow from the bank? (2)

5.2 How many months will it take to repay the loan? (6)

5.3 Calculate the balance of his loan immediately after his 90th instalment. (3)

5.4 Thabo experienced financial difficulties after the 90th instalment and did not pay the 91st to the 95th instalment. At the end of the 96th month he increased his monthly instalment so as to pay off the loan in the same time interval as planned initially. Calculate the value of his new monthly instalment. (5)

[16]

QUESTION 6

6.1 Determine $f'(x)$ from first principles if $f(x) = \frac{3}{x}$. (5)

6.2 Determine $\frac{dy}{dx}$ if:

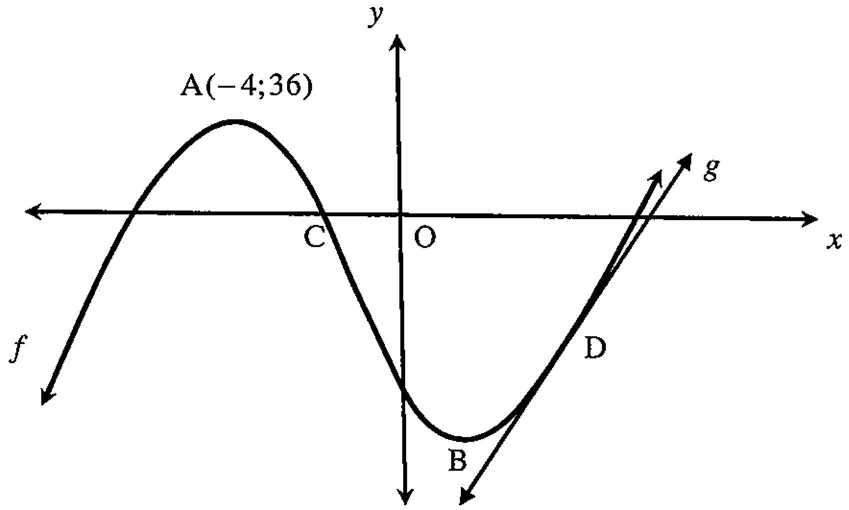
6.2.1 $y = \pi^3 x - \sqrt[3]{x}$ (3)

6.2.2 $y = \frac{7x^5 - 3x}{4x}$ (2)

[10]

QUESTION 7

The function defined by $f(x) = x^3 + px^2 + qx - 12$ is sketched below. A$(-4; 36)$ and B are turning points of f. g is a tangent to the graph of f at D.



7.1 Show that $p = 5$ and $q = -8$. (6)

7.2 If C$(-1; 0)$ is an x-intercept of f, calculate the other x-intercepts of f. (4)

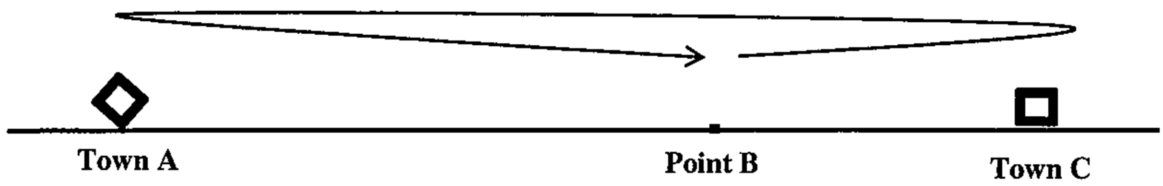
7.3 Determine the equation of g, the tangent to f at D$(1 ; -14)$. (4)

7.4 For which values of k will $f(x) = g(x) + k$ have TWO positive roots? (2)

[16]

QUESTION 8

A marathon athlete trains between two towns A and C.



The athlete starts at point B which lies between towns A and C. To complete one cycle, he runs from point B to town C, passes point B on his way to town A and then back to point B. The road between the towns is in a straight line. The displacement S, in kilometres, from point B after t hours, is given by:

$$S(t) = -t^3 + 12t^2 - 32t$$

8.1 How many hours will it take the athlete to complete a full cycle and return to point B? (3)

8.2 Calculate the distance between point B and town C. (5)

8.3 Calculate the maximum speed that the athlete has reached while training. (4)

[12]

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QUESTION 9

At a certain technical school all Grade 9 learners have to choose subjects by the end of the year. All Grade 9 learners have to choose between Mathematics and Technical Mathematics. Thereafter they have to choose between Electrical Technology, Mechanical Technology and Civil Technology. The probability that a learner from this school will choose Mathematics is $\frac{4}{7}$. If a learner chooses Mathematics as a subject, the probability that he will choose Electrical Technology as a subject is $\frac{5}{10}$ and the probability that he will choose Mechanical Technology is $\frac{3}{10}$. If a learner chooses Technical Mathematics, the probability that he will choose Electrical Technology as a subject is $\frac{4}{10}$ and the probability that he will choose Mechanical Technology is $\frac{5}{10}$.

9.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes. (4)

9.2 If a learner is selected randomly, what is the probability that the learner …

9.2.1 will choose Technical Mathematics and Mechanical Technology? (2)

9.2.2 will choose Electrical Technology as a subject? (3)
[9]

QUESTION 10

10.1 The digits 0 to 9 are used to form codes.

10.1.1 Determine the number of different 6-digit codes that can be formed if repetition of digits is allowed. (1)

10.1.2 Determine the number of 6-digit codes that can be formed that starts with a 9 and ends with a 2 if repetition of digits is not allowed. (2)

10.2 The digits 0 to 9 are used to form 10-digit codes. Determine the number of 10-digit codes that can be formed if the 2 and the 3 may not appear next to each other and if repetition of digits is not allowed. (3)
[6]

TOTAL: 150

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INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $A=P(1-ni)$ $A=P(1-i)^n$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $S_n=\frac{a(r^n-1)}{r-1}$; $r\neq 1$ $S_\infty=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x[(1+i)^n-1]}{i}$ $P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $a^2=b^2+c^2-2bc.\cos A$

$area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\1-2\sin^2\alpha\\2\cos^2\alpha-1\end{cases}$$ $\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum x}{n}$ $\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$

$\hat{y}=a+bx$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

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