

## **Education and Sport Development**

Department of Education and Sport Development

Departement van Onderwys en Sportontwikkeling

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**NORTH WEST PROVINCE**

### **NATIONAL SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P2**

**SEPTEMBER 2016**

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 9 pages, 1 information sheet and  
a 20-pages answer book.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you used to determine the answers.
4. Answer only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

**QUESTION 1**

A Mathematics teacher wants to create a model by which she can predict a learner's final marks. She decided to use her 2015 results to create the model.

Preparatory exam ( $x$ )	55	35	67	85	91	48	78	72	15	75	69	37
Final exam ( $y$ )	57	50	74	80	92	50	80	81	23	80	75	42

- 1.1 Determine the equation of the least squares regression line in the form  $y = a + bx$ . (3)
- 1.2 Use the equation of the regression line to predict the final exam mark for a learner who attained 46% in the preparatory examination. (2)
- 1.3 Could you use this equation to estimate the preparatory exam mark for a learner who attained 73% in the final exam? Give a reason for your answer. (2)
- 1.4 Show that the point  $(\bar{x}; \bar{y})$  lies on the regression line. (4)
- 1.5 Determine the correlation coefficient of the data. (1)
- 1.6 Describe the correlation between preparatory and final exam results. (2)
- [14]**

**QUESTION 2**

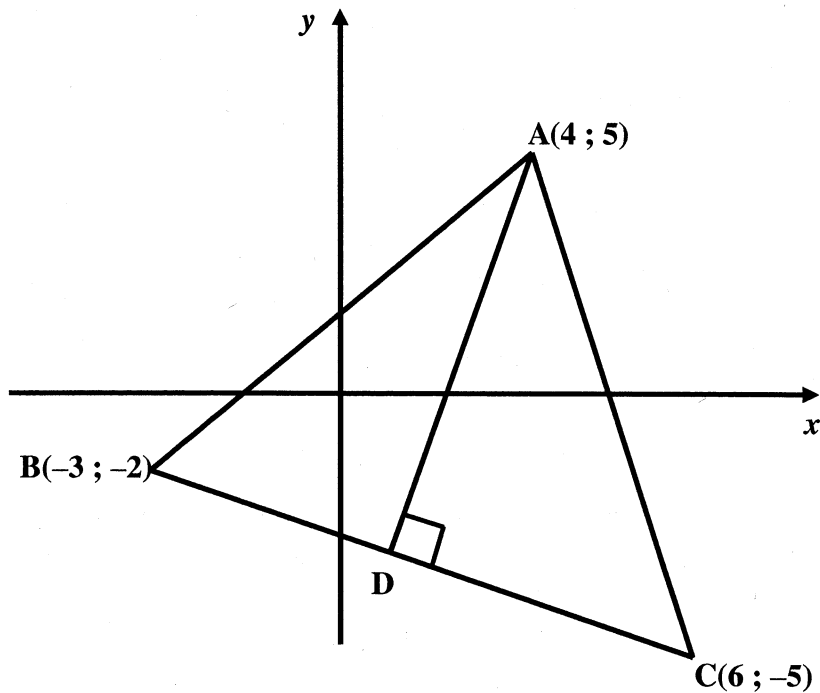
The table below shows the results from a survey of cellphone expenditure for 100 learners from a secondary school in Rustenburg.

Expenditure (in rand)	Frequency	Cumulative frequency
$50 \leq x < 100$	24	
$100 \leq x < 150$	52	
$150 \leq x < 200$	14	
$200 \leq x < 250$	$a$	
$250 \leq x < 300$	4	

- 2.1 Determine the value of  $a$ . (1)
- 2.2 Complete the cumulative frequency table in the ANSWER BOOK. (2)
- 2.3 Draw an ogive (cumulative frequency graph) for the data. (3)
- 2.4 What is the modal class for the data? (1)
- [7]**

**QUESTION 3**

In the diagram below,  $A(4 ; 5)$ ,  $B(-3 ; -2)$  and  $C(6 ; -5)$  are the vertices of  $\triangle ABC$ .  
 $AD$  is drawn perpendicular to  $BC$ .

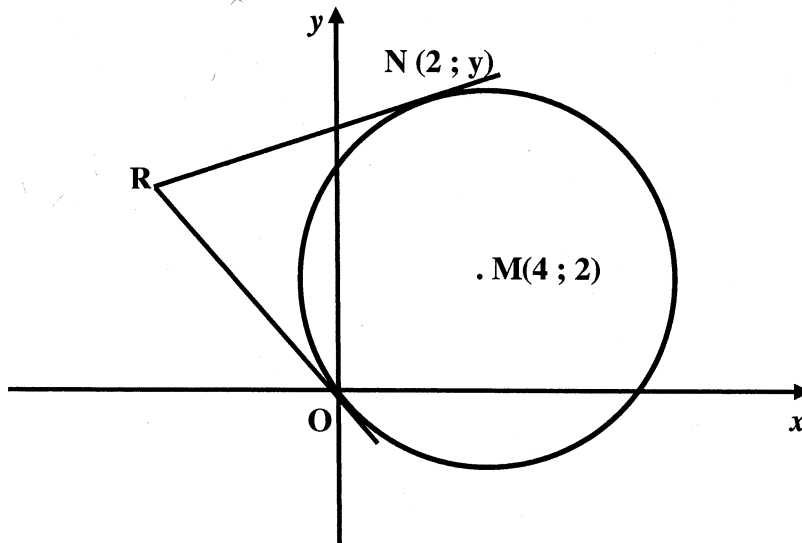


- 3.1 Calculate the length of  $BC$ . (2)
- 3.2 Determine the equation of  $BC$ . (3)
- 3.3 Determine the equation of  $AD$ . (3)
- 3.4 Determine the coordinates of  $D$ . (3)
- 3.5 Calculate the size of  $\hat{BAD}$ . (5)
- 3.6 Calculate the coordinates of a point  $E$  if the area of  $\triangle EBC = \text{area of } \triangle ABC$  and  $E$  is a point on the positive  $x$  axis. (4)

**[20]**

**QUESTION 4**

In the diagram below,  $O(0; 0)$  and  $N(2; y)$  are two points on the circumference of a circle with centre  $M(4; 2)$ . The tangents at  $O$  and  $N$  meet at  $R$ .



- 4.1 Determine the equation of the circle. (3)
- 4.2 Calculate the value of  $y$ . (4)
- 4.3 Determine the equation of  $OR$ . (3)
- 4.4 Calculate the coordinates of  $R$ . (6)
- 4.5 Determine, with a reason, the type of quadrilateral represented by  $MNRO$ . (2)

**[18]****QUESTION 5**

- 5.1 Determine the value of  $\frac{\cos(180^\circ + x) \cdot \tan(360^\circ - x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ . (6)
- 5.2 5.2.1 Prove the identity:  $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$ . (3)
- 5.2.2 Hence calculate, without using a calculator, the value of  $\cos 15^\circ - \cos 75^\circ$ . (4)
- 5.3 Find the value of  $\tan \theta$ , if the distance between  $A(\cos \theta; \sin \theta)$  and  $B(6; 7)$  is  $\sqrt{86}$ . (4)

**[17]**

**QUESTION 6**

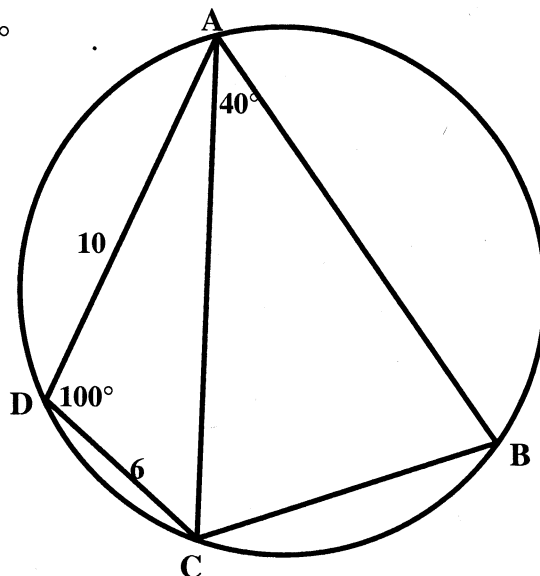
Consider :  $f(x) = \cos(x - 45^\circ)$  and  $g(x) = \tan \frac{1}{2}x$  for  $x \in [-180^\circ ; 180^\circ]$

- 6.1 Use the grid provided to draw sketch graphs of  $f$  and  $g$  on the same set of axes for  $x \in [-180^\circ ; 180^\circ]$ . Show clearly all the intercepts on the axes, the coordinates of the turning points and the asymptotes. (6)
- 6.2 Use your graphs to answer the following questions for  $x \in [-180^\circ ; 180^\circ]$
- 6.2.1 Write down the solutions of  $\cos(x - 45^\circ) = 0$  (2)
- 6.2.2 Write down the equations of asymptote(s) of  $g$ . (2)
- 6.2.3 Write down the range of  $f$ . (1)
- 6.2.4 How many solutions exist for the equation  $\cos(x - 45^\circ) = \tan \frac{1}{2}x$ ? (1)
- 6.2.5 For what value(s) of  $x$  is  $f(x).g(x) > 0$  (3)

**[15]****QUESTION 7**

In the diagram below, ABCD is a cyclic quadrilateral with DC = 6 units, AD = 10 units,

$$\hat{ADC} = 100^\circ \text{ and } \hat{CAB} = 40^\circ$$



Calculate the following, correct to ONE decimal place:

- 7.1 The length of BC (6)
- 7.2 The area of  $\triangle ABC$  (3)

**[9]**

**Give reasons for ALL statements in QUESTIONS 8, 9 and 10**

**QUESTION 8**

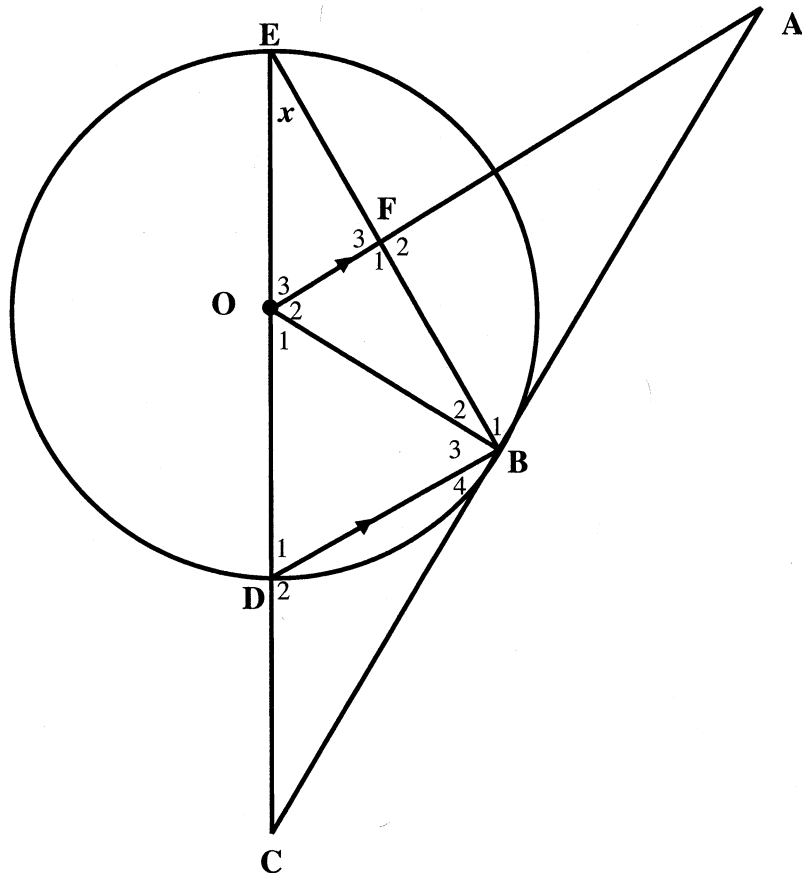
8.1 Complete the following statement:

8.1.1 The angle subtended at the circle by a diameter is ..... (1)

8.1.2 The angle between a tangent to a circle and a chord drawn from the point of contact is equal to ..... (1)

8.2 In the diagram below, ED is a diameter of the circle with centre O. ED is produced to C and CA is a tangent to the circle at B. AO intersects BE at F.  $BD \parallel AO$ .

Let  $\hat{E} = x$



8.2.1 Write down, with reasons, THREE other angles each equal to  $x$ . (6)

8.2.2 Determine, with reasons,  $\hat{CBE}$  in terms of  $x$ . (3)

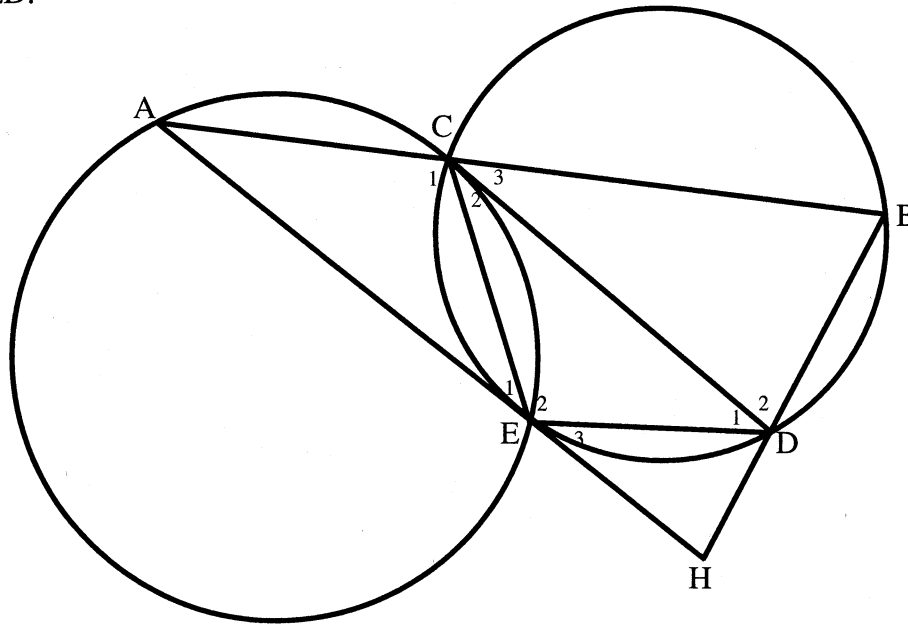
8.2.3 Prove that F is the midpoint of BE. (4)

8.2.4 Calculate the length of the diameter if it is further given that  $EB = 8$  cm and  $OF = 3$  cm. (4)

**[19]**

**QUESTION 9**

In the diagram below, CE is a common chord of the circles ACE and CBDE. CD is a tangent to circle ACE at C and AEH is a tangent to circle CBDE at E. ACB and BDH are straight lines. AC = ED.



- 9.1 Prove that  $AB \parallel ED$ . (5)
- 9.2 What type of quadrilateral is ACDE? Give reasons. (2)
- 9.3 Prove that  $\frac{AC}{CB} = \frac{HE}{EA}$ . (4)

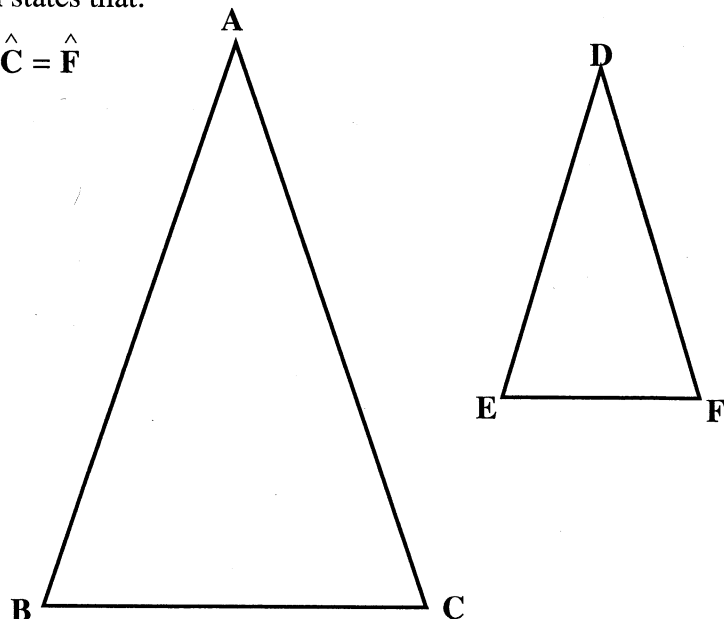
[11]

**QUESTION 10**

- 10.1 In the diagram alongside,  $\triangle ABC$  and  $\triangle DEF$  are given.  
Prove the theorem which states that:

If  $\hat{A} = \hat{D}$ ,  $\hat{B} = \hat{E}$  and  $\hat{C} = \hat{F}$

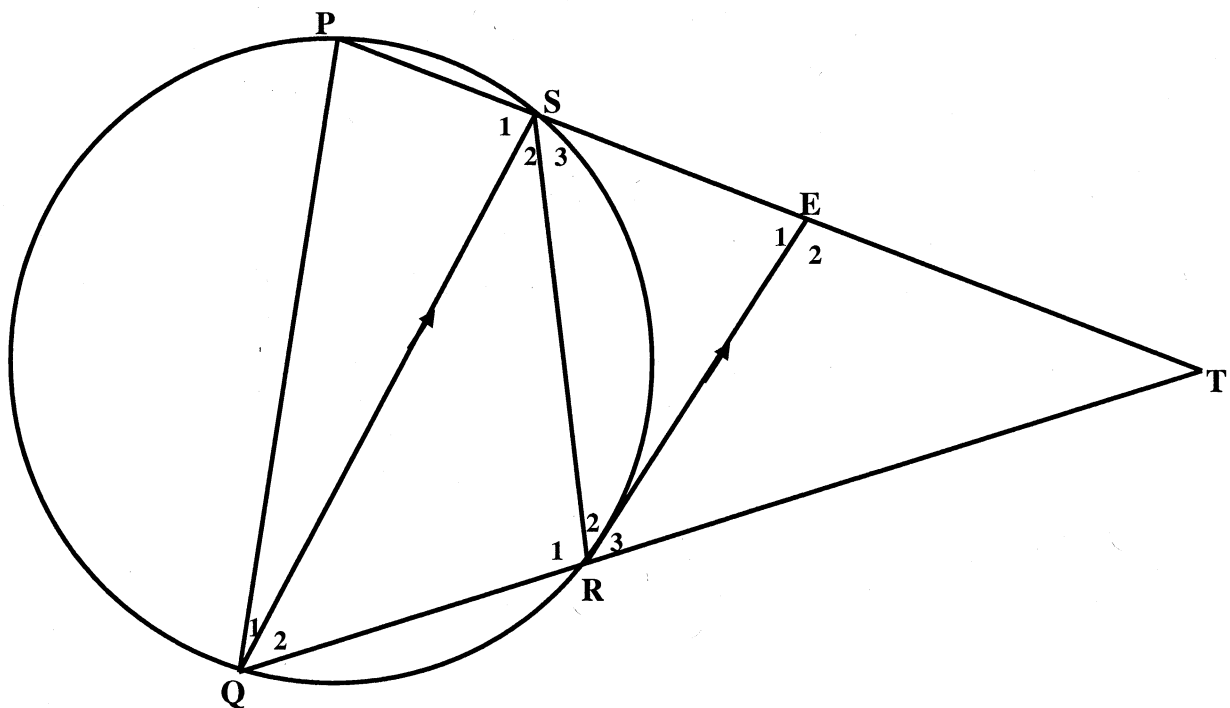
then  $\frac{AB}{DE} = \frac{AC}{DF}$





(7)

10.2 In the diagram below, PQRS is a cyclic quadrilateral. PS and QR are produced to meet at T. RE is a tangent to the circle at R, with E on PT and RE  $\parallel$  QS.



Prove that:

10.2.1  $QR = RS$  (4)

10.2.2  $\Delta RST \parallel \Delta PQT$  (4)

10.2.3  $\frac{PQ}{PT} = \frac{SE}{ET}$  (5)

[20]

**TOTAL: 150**



## INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1 \quad S_\infty = \frac{a}{1 - r} ;$$

$$-1 < r < 1 \quad F = \frac{x[(1+i)^n - 1]}{i} \quad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$