



NATIONAL SENIOR CERTIFICATE EXAMINATION  
SUPPLEMENTARY EXAMINATION MARCH 2016

**MATHEMATICS: PAPER I**

Time: 3 hours

150 marks

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**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 9 pages and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.
  2. Read the questions carefully.
  3. Answer all the questions.
  4. Number your answers exactly as the questions are numbered.
  5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
  6. Round off your answers to one decimal digit where necessary.
  7. All the necessary working details must be clearly shown.
  8. It is in your own interest to write legibly and to present your work neatly.
  9. Please hand in this question paper.
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**SECTION A**

**QUESTION 1**

(a) (1) Simplify  $\frac{2x^2 + 11x - 13}{x - 1}$ . (2)

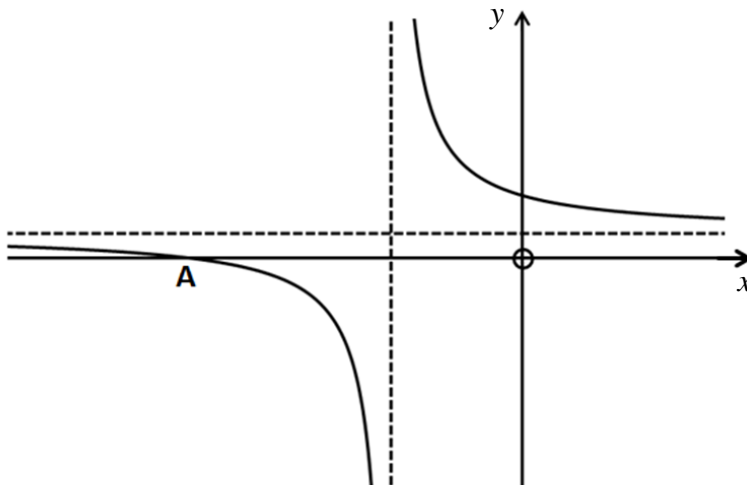
(2) Hence, solve for  $x$  if  $\frac{2x^2 + 11x - 13}{x - 1} = x^2$ .

Leave answers in simplest surd form. (3)

(b) (1) Simplify  $\frac{8^{3x} \cdot 4^{3x}}{2^{15x-2}}$ . (4)

(2) Hence solve for  $m$  if  $2^m = \frac{8^{3x} \cdot 4^{3x}}{2^{15x-2}}$ . (2)

(c) In the diagram below, the graph of  $h(x) = \frac{3}{x+2} + 1$  has been sketched.



(1) Write down the equations for the asymptotes of  $h(x)$ . (2)

(2) Write down the new equation for  $h(x)$  if  $h(x)$  is shifted horizontally so that point A is at the origin. (3)

**[16]**

**QUESTION 2**

The sequence  $3; p; 25$  is a quadratic sequence. The sequence of first differences is  $9; q; \dots$

(a) Calculate  $p$  and  $q$ . (4)

(b) Determine the  $n^{\text{th}}$  term of the quadratic sequence. (5)

**[9]**

**QUESTION 3**

- (a) In the first week of training, a student cycles 10 km. Thereafter, the distance he covers each week is 10% more than that of the previous week.
- (1) Determine the distance cycled by the student in the eighth week. (3)
- (2) Determine the total distance cycled by the student in the first 8 weeks. (3)
- (b) In an arithmetic sequence the fifth term has a value of 0 and the fourteenth term has a value of  $-36$ .
- (1) Calculate  $T_1$ . (5)
- (2) Find the value of  $p$  if  $T_{23} + T_{23-p} = -96$ . (4)
- (c)  $-4 - \frac{8}{3} - \frac{16}{9} - \dots$  is an infinite geometric series.
- (1) Explain why the series converges. (2)
- (2) Find the sum to infinity. (2)
- [19]**

**QUESTION 4**

A man wants to start a bakery so he takes out a loan of R301 964 from a bank, repayable over 10 years. The bank calculates the monthly repayments at an interest rate of 11% per annum compounded monthly. Payments start one month after the loan is received.

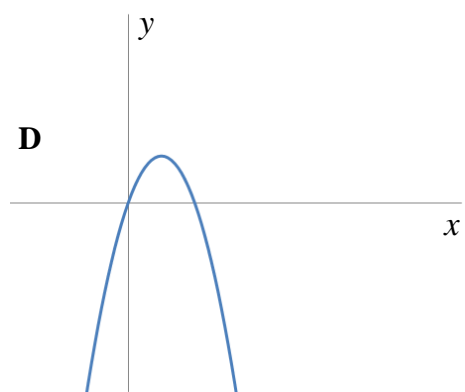
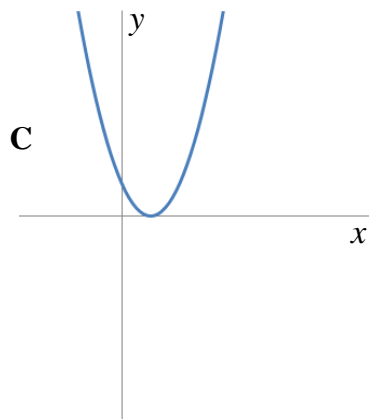
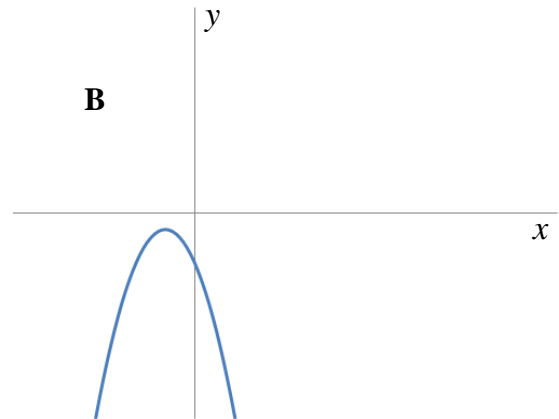
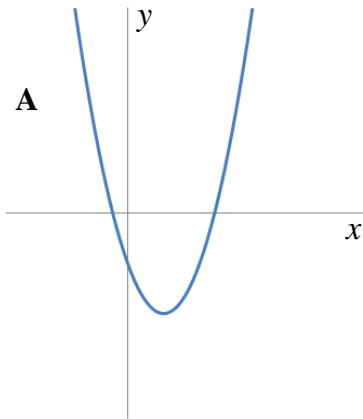
- (a) Calculate the monthly payment. (3)
- (b) What is the balance outstanding at the end of 7 years? (3)
- [6]**

**QUESTION 5**

- (a) Find  $f'(x)$  by first principles if  $f(x) = x^2 + \pi$ . (4)
- (b) Given  $f(x) = \frac{ax^2 + 5x + c}{x}$ ,  $f'(x) = \frac{x^2 - 3}{x^2}$  for all values of  $x$ ,  $x \neq 0$ . Determine the values of  $a$  and  $c$ . (7)
- [11]**

**QUESTION 6**

- (a) Solve for  $x$  if  $\sqrt{x - 2} + x = 4$ . (6)
- (b) Solve for  $x$  if  $x(x - 4) < 4 - x$  (4)
- (c) Study the diagrams below and then answer the questions that follow.



$y = a(x - p)(x - q)$ ,  $p \leq q$  represents the equation of the graphs in each situation above.

- (1) In which graph is  $p = q$ ? (1)
- (2) In which graph is  $pq = 0$ ? (1)
- (3) In which graph is  $a < 0$  and  $q > 0$ ? (1)
- (4) In which graph is  $p + q > 0$ ? (1)

**[14]**

**75 marks**

**SECTION B**

**QUESTION 7**

A young man and a young woman invest using different methods. They both start investing on the same day.

**A breakdown of the young man's investing habits**

- The young man puts in an initial deposit of R10 147.
- After 8 years he deposits R23 786.
- For the first 3 years the interest rate is 15% per annum compounded quarterly. Thereafter, it changes to 15% per annum compounded annually.

**A breakdown of the young woman's investing habits**

The young woman deposits R2 172 at the end of each year and the investment offers an interest rate of 15% per annum compounded annually.

- (a) At the end of 12 years, what is the value of the *young man's* investment? (5)
- (b) At the end of 12 years, what is the value of the *young woman's* investment? (3)
- (c) If the young woman continues to make annual payments of R2 172 and if the young man leaves his investment earning interest at 15% p.a. compounded annually, show that the young woman's investment will never exceed the young man's investment if her interest rate of 15% per annum compounded annually does not change. (5)

**[13]**

**QUESTION 8**

- (a) Consider the single digits:

**1      2      3      4      5**

- (1) How many five-digit numbers can be created using the digits above if the 2 and the 3 must be next to each other and if each digit can be used once only? (2)
- (2) How many three-digit even numbers can be created using the digits above, if each digit can be used once only? (Note: 023 is not a 3-digit number.) (3)
- (3) How many different numbers greater than 1 100 can be created with the numbers above if you can use each digit once only? (3)

(b) Below is a new board game called 'Sevens'.

- It is played with **one** regular six-sided dice and the board shown below.
- You move your counter from the 'START' position according to the number thrown on the dice.
- To finish the game you need to move past the number 20.



START	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	END
-------	---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	-----

- (1) If you throw the dice twice, what is the probability of your counter landing on the number 7? (2)
- (2) If your counter is on the number 12, then what is the probability that you will throw the dice twice to finish the game without landing on the 14 or 17? (4)
- [14]**

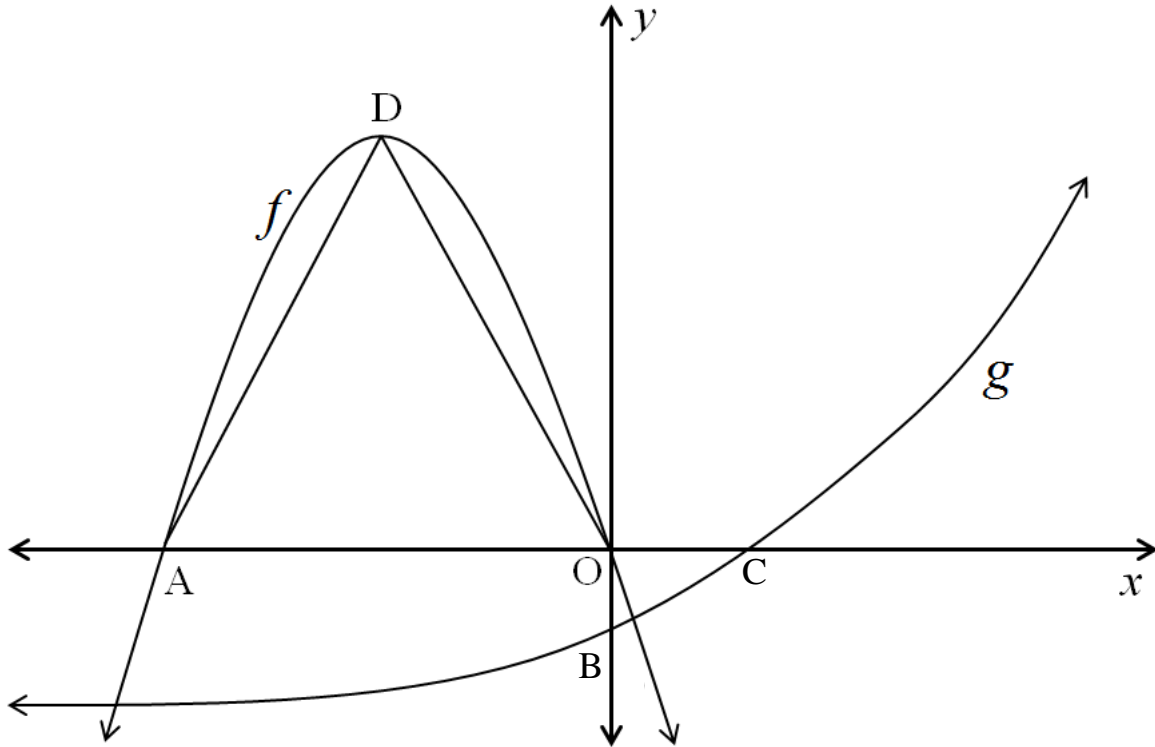
**QUESTION 9**

(a) If  $g(x) = \frac{x-6}{x-2}$  and  $h(x) = 3^x - 2$  then:

- (1) Determine  $h^{-1}(x)$ , the inverse of  $h(x)$  in the form  $y = \dots\dots\dots$  (3)
- (2) (i) Sketch the graph of  $h^{-1}(x)$ . (3)
- (ii) For what values of  $x$  is  $h^{-1}(x) > 0$ ? (1)
- (3) The graph of  $h(x)$  is translated vertically so that it passes through the point of intersection of the asymptotes of  $g(x)$ . Find the new equation of  $h$ . (4)

(b) In the diagram below:

- $f(x) = -x^2 - 4x$
- $g(x) = 2^x - 6$
- Point D is the turning point of  $f$
- Points A and C are the  $x$ -intercepts of  $f$  and  $g$
- Point B is the  $y$ -intercept of  $g$

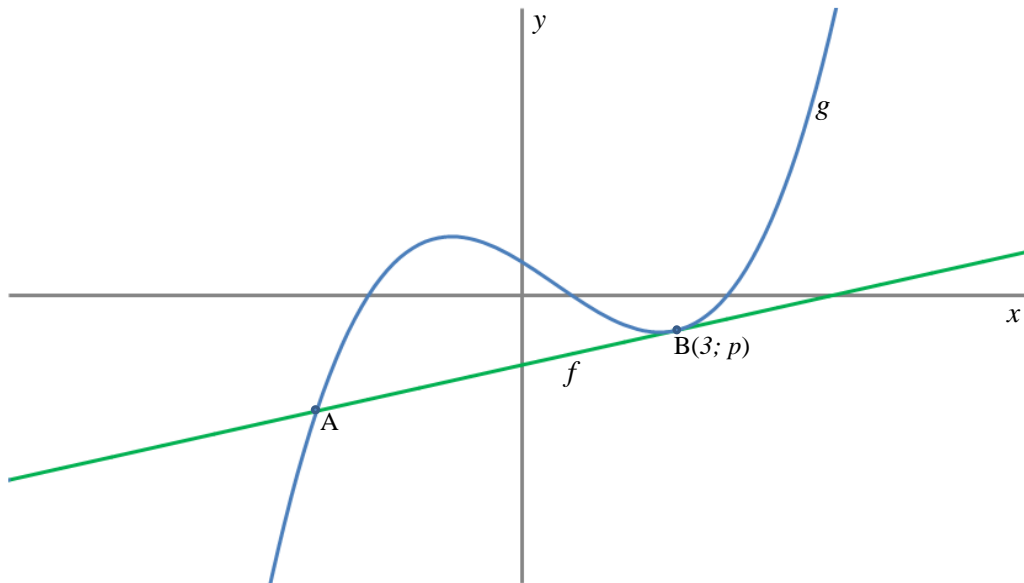


- (1) Determine the area of  $\triangle AOD$ . (6)
  - (2) For what values of  $k$  will  $-x^2 - 4x = 2^x + k$  have two real roots that are opposite in sign? (2)
  - (3) For what values of  $p$  will  $-(x-p)^2 - 4(x-p) = 2^x - 6$  have two real negative roots? (4)
- [23]**

**QUESTION 10**

(a) In the diagram below, the graphs of  $f(x)$  and  $g(x)$  are sketched.

- $g(x) = x^3 - 2x^2 + kx + 12$
- $f(x) = 4x + t$  is a tangent to  $g(x)$  at point  $B(3; p)$  and it intersects the graph at  $g$  again at A



Determine the coordinates of point A. (9)

(b) Given the following information:

- $f(x)$  has exactly one real root
- $f(4) = 30$  and  $f(0) = -40$
- $f'(4) = 0$
- $f''(4) = 0$

Sketch a possible graph of  $f(x)$  and label two points on the graph. (6)

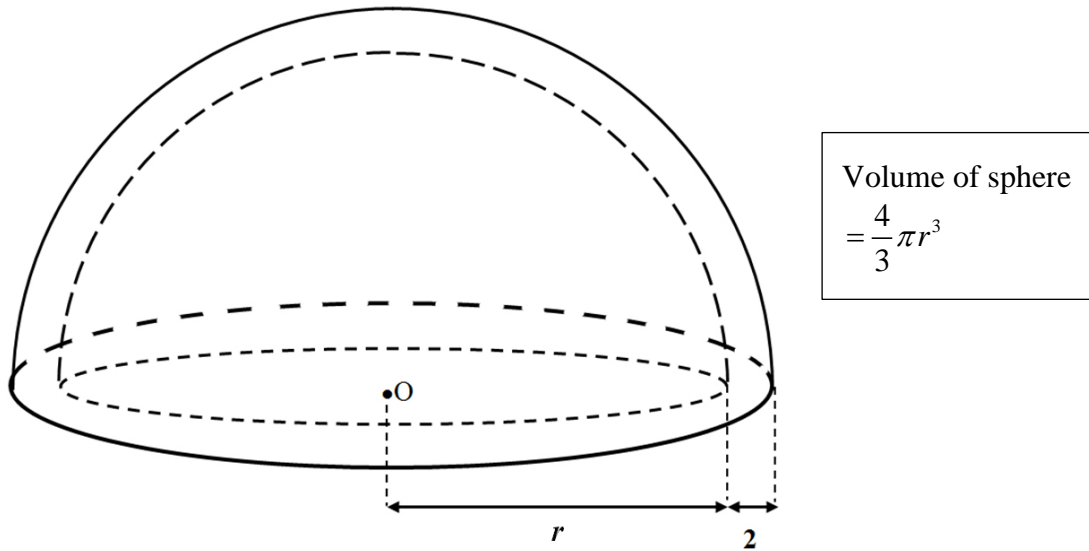
**[15]**



**QUESTION 11**

In the diagram, two hemispheres of different size have the same centre.

The radius of the larger hemisphere is 2 more than the radius of the smaller hemisphere.  
The region between the two hemispheres is made out of rubber and the inner hemisphere is hollow.



(a) Show that the volume of rubber is given by  $4\pi r^2 + 8\pi r + \frac{16\pi}{3}$ . (4)

(b) The rate of change of volume of the rubber with respect to  $r$  when  $r = p$  is  $\frac{88\pi}{3}$  cm<sup>3</sup>/cm. Determine  $p$ . (6)

[10]

**75 marks**

**Total: 150 marks**