



NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2016

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

FOR OFFICIAL USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13
18	10	9	18	12	9	13	11	10	10	10	14	6

TOTAL	/150
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SECTION A**QUESTION 1**

$$(a) \quad AB = \sqrt{(6-1)^2 + (-1+3)^2}$$

$$= \sqrt{29} \approx 5,4 \quad (2)$$

$$(b) \quad M = \left(\frac{-3+5}{2}; \frac{1+1}{2} \right)$$

$$M = (1; 1) \quad (2)$$

$$(c) \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{5}{2}$$

$$m_{AD} = \frac{6-1}{-1-5} = -\frac{5}{6} \quad (3)$$

$$(d) \quad \tan \theta = m_{AB} = \frac{5}{2}$$

$$\theta \approx 68^\circ$$

$$\tan (180^\circ - \alpha) = m_{AD} = -\frac{5}{6}$$

$$180^\circ - \alpha = 140^\circ$$

$$\alpha = 140^\circ$$

$$\hat{A} \approx 180^\circ - (68^\circ + 140^\circ)$$

$$\approx 72^\circ \quad (5)$$

(e) The diagonals of a parallelogram bisect each other
 $\therefore M = \text{Midpoint of AE}$

$$(1; 1) = \left(\frac{-1 + x_E}{2}; \frac{6 + y_E}{2} \right)$$

$$-1 + x_E = 2, x_E = 3$$

$$6 + y_E = 2, y_E = -4$$

$$E = (3; -4) \quad (4)$$

(f) $m_{AC} = \frac{6+4}{1+5} = \frac{5}{2} = m_{AB}$

$\therefore A, B$ and C lie on a straight line. (2)

[18]

QUESTION 2

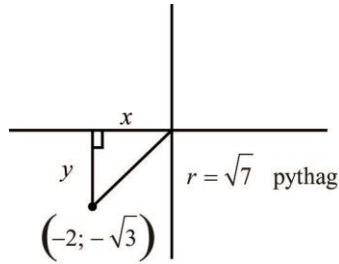
- (a) (1) $(\max) - (\min)$
 Range = $98 - 10$
 Range = 88 (2)
- (2) Median = 52 (1)
- (3) $IQR = Q_3 - Q_1$
 $IQR = 70 - 44$
 $IQR = 26$ (2)
- (b) Science, higher median, higher minimum. (2)
- (c) Mathematics, Range is 88% and the IQR is 30 compared to Science with a Range of 58 and IQR of 26. (2)
- (d) 25% or 0,25 (1)
- [10]**

QUESTION 3

- (a) $y = a + bx$
 $y = 51,6478 + 0,7776x$ (3)
- (b) $y = a + bx$
 $y = 51,6478 + 0,7776 (100)$
 $y = 129$ (2)
- (c) This prediction is not reliable as the maximum that could be obtained is 100. (1)
- (d) $r = 0,8$ which indicates a very strong positive correlation. (3)
- [9]**

QUESTION 4

(a) (1)



$$\sin \theta = \frac{-\sqrt{3}}{\sqrt{7}}$$

$$\cos \theta = \frac{-2}{\sqrt{7}}$$

(2)

$$\begin{aligned} (2) \quad \sin 2\theta &= 2\sin \theta \cos \theta \\ &= 2\left(\frac{-\sqrt{3}}{\sqrt{7}}\right)\left(\frac{-2}{\sqrt{7}}\right) \\ &= \frac{4\sqrt{3}}{7} \end{aligned}$$

(2)

$$\begin{aligned} (3) \quad \cos^2(90^\circ + \theta) &= (-\sin \theta)^2 \\ &= \sin^2 \theta \\ &= \left(\frac{-\sqrt{3}}{\sqrt{7}}\right)^2 \\ &= \frac{3}{7} \end{aligned}$$

(2)

$$\begin{aligned} (b) \quad \frac{\tan(180^\circ - \theta) \cdot \cos(-\theta) \cdot \sin 390^\circ}{\cos 300^\circ \cdot \sin \theta - \cos 450^\circ} &= \frac{(-\tan \theta)(\cos \theta) \cdot \sin 390^\circ}{\cos 300^\circ \cdot \sin \theta - \cos 450^\circ} \\ &= \frac{(-\tan \theta)(\cos \theta) \cdot \left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right) \sin \theta - 0} \\ &= \frac{\left(\frac{-\sin \theta}{\cos \theta}\right)(\cos \theta) \cdot \left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right) \sin \theta} = -1 \end{aligned}$$

(5)

$$\begin{aligned} (c) \quad (1) \quad \cos(A - 45^\circ) &= \cos A \cdot \cos 45^\circ + \sin A \cdot \sin 45^\circ \\ &= \frac{\sqrt{2}}{2} \cos A + \frac{\sqrt{2}}{2} \sin A \\ &= \frac{\sqrt{2}}{2} (\cos A + \sin A) \\ \therefore \cos(A - 45^\circ) &= \frac{\sqrt{2}}{2} k \end{aligned}$$

(4)

$$\begin{aligned}
 (2) \quad & 1 + \sin 2A \\
 &= \sin^2 A + 2 \sin A \cos A + \cos^2 A \\
 &= (\cos A + \sin A)^2 \\
 &= k^2
 \end{aligned}$$

(3)
[18]

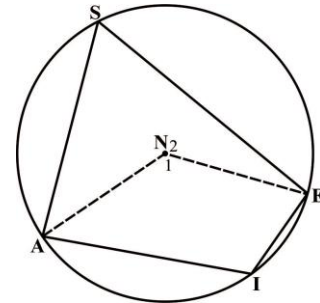
QUESTION 5

(a) $\hat{N}_1 = 2x$ *< at centre*

$\therefore \hat{N}_2 = 360^\circ - 2x$ *angles around a point*

$\therefore \hat{I} = 180^\circ - x$ *angle at centre*

$\therefore \hat{S} + \hat{I} = x + \hat{I} = 180^\circ$



(6)

(b) $\hat{I}_1 = x ; \hat{I}_2 = y$

$\hat{K}_1 = x$; $(GI = GK)$

$\hat{K}_2 = y$ $(JI = JK)$

$180^\circ = \hat{I}_1 + \hat{I}_2 + \hat{K}_1 + \hat{K}_2$ *opposite angles of a cyclic quadrilateral*

$180^\circ = x + y + x + y$

$180^\circ = 2(x + y)$

$\therefore x + y = 90^\circ$

\therefore GJ is a diameter

GJ subtends \angle of 90° at circumference

Alternate:

Since $GI = GK$, G lies on the perpendicular bisector of IK.

Since $IJ = JK$, J lies on the perpendicular bisector of IK.

\therefore GJ is the perpendicular bisector of IK.

Hence the centre lies on GJ, proving GJ is a diameter of the circle.

(6)
[12]

QUESTION 6

$\hat{O}LM = 90^\circ$ Tan \perp Radius

$\hat{O}_2 = 64^\circ$ Int. \angle s of $\triangle LOM$

$\hat{P} = \hat{L}_2$ OP = OL

$\hat{P} + \hat{L}_2 = 64^\circ$ ext. \angle of \triangle

$\therefore \hat{L}_2 = 32^\circ$

$\hat{L}_5 = \hat{P}$; tan / chord theorem

$\hat{P} = \hat{L}_2 = 32^\circ$

$\therefore \hat{L}_5 = 32^\circ$

$R\hat{L}O = 90^\circ$ tan \perp radius

$\therefore \hat{L}_1 = 90^\circ - 32^\circ = 58^\circ$

$\hat{Q}_1 = \hat{L}_1 = 58^\circ$ tan-chord theorem

$\hat{N} = 32^\circ$ tan-chord theorem

or \angle in same segment

or \angle at centre

NB:

Mark allocation

\hat{O}_2 (2)

\hat{L}_2 (3)

\hat{L}_5 (2)

\hat{Q}_1 (1)

\hat{N} (1)

(9)

[9]

76 marks

SECTION B

QUESTION 7

(a) (1) Sub $x = \frac{7}{5}$ and $y = -\frac{9}{5}$

$$\left(\frac{7}{5}\right)^2 + \left(-\frac{9}{5}\right)^2 - 4\left(\frac{7}{5}\right) + 2\left(-\frac{9}{5}\right) + 4 = 0$$

$\therefore \left(\frac{7}{5}; -\frac{9}{5}\right)$ is a point on circle (2)

(2) Eq. of tangent: $y = mx + c$ at point $\left(\frac{7}{5}; -\frac{9}{5}\right)$

$$x^2 - 4x + y^2 + 2y = -4$$

$$(x-2)^2 + (y+1)^2 = -4 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 = 1$$

\therefore Centre $(2; -1)$

$$m_{normal} = \frac{\frac{-9}{5} + 1}{\frac{7}{5} - 2}$$

$$m_{normal} = \frac{4}{3} \quad \therefore \quad m_{tan} = -\frac{3}{4}$$

$$y = -\frac{3}{4}x + c \quad \text{sub point } \left(\frac{7}{5}; -\frac{9}{5}\right)$$

$$c = -\frac{3}{4}$$

Eq. of tangent: $y = -\frac{3}{4}x - \frac{3}{4}$ or $4y = -3x - 3$ (8)

(b) Centre of circle = $(4; -3)$

Distance from $(2; -1)$ to the centre

$$= \sqrt{(4-2)^2 + (-3+1)^2}$$

$$= \sqrt{8}$$

$$\left. \begin{array}{l} \sqrt{8} > \sqrt{7} \\ \therefore \text{Point lies outside} \end{array} \right\} \quad (3)$$

[13]

QUESTION 8(a) In $\triangle OCA$: Let $OC = x$

$$\therefore EC = (x+2) = CA \quad \text{radii}$$

$$OA = 8 \text{ units}$$

$$(x+2)^2 = x^2 + 8^2 \quad \text{pythag}$$

$$\therefore x = 15$$

$$\therefore C(0; -15) \quad \text{and radius} = CP = 17$$

$$\text{Sub. into: } (x-a)^2 + (y-b)^2 = r^2$$

$$(x-0)^2 + (y+15)^2 = 17^2$$

$$x^2 + y^2 + 30y - 64 = 0$$

(8)

(b)
$$P\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$P\left(\frac{-8+0}{2}; \frac{0+0}{2}\right)$$

$$P(-4; 0)$$

$$\therefore Q(-4; y) \quad \text{sub } x = -4 \text{ in circle equation}$$

$$(-4)^2 + (y+15)^2 = 17^2$$

$$(y+15)^2 = 289 - 16$$

$$y + 15 \approx 16,5$$

$$y \approx 1,5$$

Height of the pillar is 1,5 metres

(3)

[11]

QUESTION 9

$$(a) \quad PT = \frac{5}{8}(144)$$

$$PT = 90 \text{ m}$$

$$\hat{TQP} = 51^\circ \quad \text{Ext. } \angle \text{ of } \triangle QPT$$

$$\frac{QT}{\sin 63^\circ} = \frac{90}{\sin 51^\circ}$$

$$QT \approx 103 \text{ m}$$

(4)

$$(b) \quad \text{In } \triangle QTS: TS = 54 \text{ m and } QT = 103 \text{ m}$$

$$QS^2 = TS^2 + QT^2 - 2.TS.QT.\cos 114^\circ$$

$$QS^2 = (54)^2 + (103)^2 - [2(54)(103)(\cos 114^\circ)]$$

$$QS = \sqrt{18049,53842}$$

$$QS = 134,3485706$$

$$\tan \hat{QSR} = \frac{70}{134,3485706}$$

$$\hat{QSR} \approx 28^\circ \text{ which is the angle of elevation of R from S}$$

(6)

[10]

QUESTION 10

$$\begin{aligned}
 \text{(a)} \quad & \frac{\sin 3\alpha}{\sin \alpha} \\
 &= \frac{\sin(2\alpha + \alpha)}{\sin \alpha} \\
 &= \frac{(\sin 2\alpha)(\cos \alpha) + (\cos 2\alpha)(\sin \alpha)}{\sin \alpha} \\
 &= \frac{(2 \sin \alpha \cdot \cos \alpha)(\cos \alpha) + (\cos^2 \alpha - \sin^2 \alpha)(\sin \alpha)}{\sin \alpha} \\
 &= \frac{\sin \alpha(2 \cos \alpha)(\cos \alpha) + (\cos^2 \alpha - \sin^2 \alpha)(\sin \alpha)}{\sin \alpha} \\
 &= (2 \cos \alpha)(\cos \alpha) + (\cos^2 \alpha - \sin^2 \alpha) \\
 &= 2 \cos^2 \alpha + \cos^2 \alpha - \sin^2 \alpha \\
 &= 3 \cos^2 \alpha - \sin^2 \alpha \\
 &= 3(1 - \sin^2 \alpha) - \sin^2 \alpha \\
 &= 3 - 3 \sin^2 \alpha - \sin^2 \alpha \\
 &= 3 - 4 \sin^2 \alpha \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\sin 3\alpha}{\sin \alpha} \\
 &= \frac{\sin 2\alpha \cos \alpha + \cos 2\alpha \cdot \sin \alpha}{\sin \alpha} \\
 &= \frac{2 \sin \alpha \cos^2 \alpha + (1 - 2 \sin^2 \alpha) \cdot \sin \alpha}{\sin \alpha} \\
 \text{OR} \\
 &= 2 \cos^2 \alpha + 1 - 2 \sin^2 \alpha \\
 &= 2 - 2 \sin^2 \alpha + 1 - 2 \sin^2 \alpha \\
 &= 3 - 4 \sin^2 \alpha \\
 &= \text{RHS}
 \end{aligned}$$

(6)

$$\begin{aligned}
 \text{(b)} \quad & 3 - 4 \sin^2 \alpha = 2 \\
 & 4 \sin^2 \alpha = 1 \\
 & \sin \alpha = \frac{1}{2} \text{ OR } -\frac{1}{2} \\
 & \alpha = 30^\circ + 360k; \quad k \in \mathbb{Z}
 \end{aligned}$$

OR

$$\alpha = 150^\circ + 360k; \quad k \in \mathbb{Z}$$

(4)

[10]

QUESTION 11

(a) $\hat{S}\hat{W}T = 90^\circ$ \angle in semi-circle
 $\hat{V}\hat{U}S = 90^\circ$ given
 \therefore WVUT is a cyclic quad. Converse of Ext angle of cyclic quadrilateral (4)

(b) $U\hat{W}T = \hat{S}$ tan-chord theorem
 $U\hat{W}T = \hat{V}_1$ \angle s in same segment
 $\therefore \hat{V}_1 = \hat{S}$
 \therefore VU is a tangent ; converse of tan/chord theorem (6)
[10]

QUESTION 12(a) In ΔSWU

$$\frac{ST}{18} = \frac{8}{12} \quad \text{line parallel to one side of triangle}$$

$$\therefore ST = \frac{8 \times 18}{12} = 12$$

In ΔXUS

$$\frac{XW}{20} = \frac{12}{18} \quad \text{line parallel to one side of triangle}$$

$$\therefore XW = \frac{20 \times 12}{18} = \frac{40}{3} \quad (6)$$

(b) Determine Area ΔSXU :

$$\frac{VT}{20} = \frac{12}{20} \quad (\Delta VUT \sim \Delta WUS)$$

$$\therefore VT = 12$$

$$(12)^2 = (12)^2 + (18)^2 - 2(12)(18) \cos \hat{U}$$

$$\cos \hat{U} = \frac{3}{4}$$

$$\sin \hat{U} = \sqrt{1 - \cos^2 \hat{U}} = \frac{\sqrt{7}}{4}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \left(33 \frac{1}{3} \right) (30) \left(\frac{\sqrt{7}}{4} \right) \\ &= 125\sqrt{7} \end{aligned} \quad (8)$$

[14]

QUESTION 13

- (a) $OR = ON = p$ radii
 $\therefore \hat{ORN} = \hat{N}_1 = \alpha$ \angle opp = sides
 $\therefore \hat{RON} = 180^\circ - 2\alpha$ int. \angle s of Δ

Using the Cosine Rule:

$$(RN)^2 = p^2 + p^2 - [2 \cdot p \cdot p \cdot \cos(180^\circ - 2\alpha)]$$

$$(RN)^2 = 2p^2 - 2p^2(-\cos 2\alpha)$$

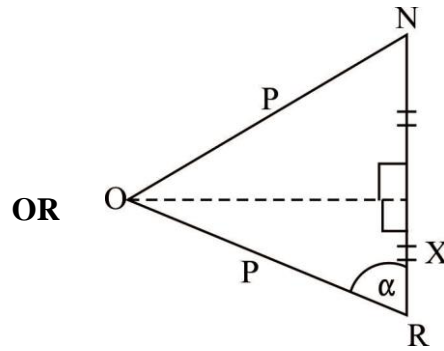
$$(RN)^2 = 2p^2(1 + \cos 2\alpha)$$

$$(RN)^2 = 2p^2[1 + (2\cos^2 \alpha - 1)]$$

$$(RN)^2 = 2p^2(2\cos^2 \alpha)$$

$$(RN)^2 = 4p^2 \cos^2 \alpha$$

$$RN = 2p \cos \alpha$$



$$\cos \alpha = \frac{x}{p}$$

$$x = p \cos \alpha \tag{3}$$

$$\therefore RN = 2p \cos \alpha$$

- (b) $RN = 20 \cos 45^\circ$
 $= 10\sqrt{2}$

Perpendicular height = circumference of sphere = 25 m

$$\text{Volume of pyramid} = \frac{1}{3}(10\sqrt{2} \times 10\sqrt{2})(25)$$

$$\text{Volume of pyramid} = 1\,667 \text{ m}^3$$

$$\begin{aligned} \text{Volume of hemisphere} &= \frac{2}{3} \times \pi \times 25^3 \\ &= 3\,2724,92 \end{aligned}$$

$$\therefore \text{volume of water} = 31\,058 \text{ m}^3 \tag{3}$$

74 marks

Total: 150 marks