GAUTENG PROVINCE
EDUCATION
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PREPARATORY EXAMINATION

2017

MARKING GUIDELINES

MATHEMATICS (FIRST PAPER) (10611)

14 pages

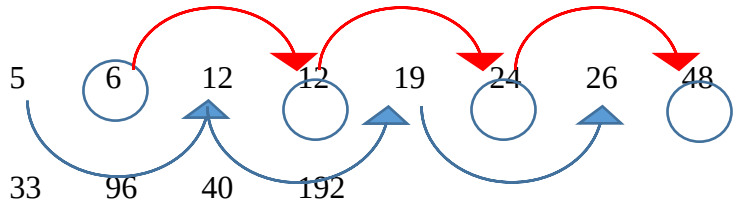
GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

MATHEMATICS
(First Paper)

MEMORANDUM

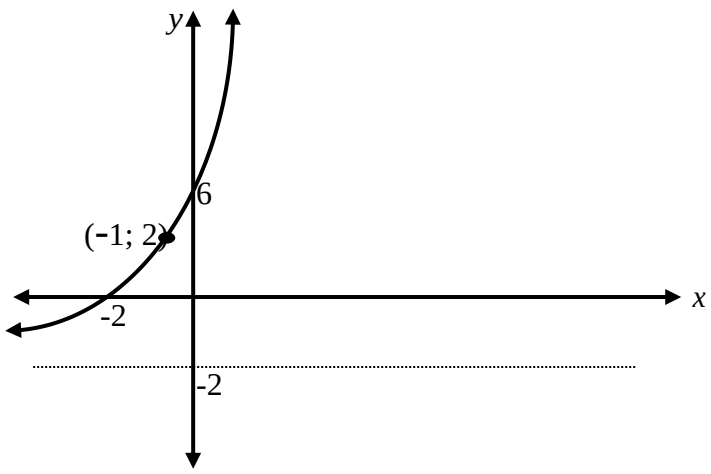
	QUESTION 1	
1.1		
1.1.1	$x(x-1)=12$ $x^2-x=12$ $x^2-x-12=0$ $(x-4)(x+3)=0$ $x=4$ **OR** $x=-3$	✓standard form ✓both factors ✓both answers (3)
1.1.2	$2x^2+3=8x$ $2x^2-8x+3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{8\pm\sqrt{(-8)^2-4(2)(3)}}{2(2)}$ $=\frac{8\pm\sqrt{64-24}}{4}$ $=\frac{8\pm\sqrt{40}}{4}$ $x=3{,}58$ **OR** $x=0{,}42$	✓standard form ✓substitution into correct formulae ✓✓each x answer (4)
1.1.3	$(2x+3)(3-x)>4$ $6x-2x^2+9-3x-4>0$ $-2x^2+3x+5>0$ $2x^2-3x-5<0$ $(2x-5)(x+1)<0$ $-1<x<\frac{5}{2}$	✓simplification ✓standard form $2x^2-3x-5<0$ ✓factors ✓critical values ✓inequalities (5)
1.1.4	$2^x-5.2^{x+1}=-144$ $2^x(1-5.2\ )=-144$ $2^x(-9)=-144$ $2^x=16$ $2^x=2^4$ $x=4$	✓common factor ✓simplification $2^x=2^4$ ✓answer (3)

1.2	$y+2x-3=0$ (1) $y=x^2+2x+3$ (2) From (1): $y=-2x+3$ (3) Substitute (3) into (2): $-2x+3=x^2+2x+3$ $x^2+4x=0$ $x(x+4)=0$ $x=0$ *or* $x=-4$ Substitute $x=0$ into (3): **OR** Substitute $x=-4$ into (3): $y=-2(0)+3$ $y=-2(-4)+3$ $y=3$ $y=11$	✓ $y=-2x+3$ ✓equating $-2x+3=x^2+2x+3$ ✓factorisation ✓both x-values ✓both y – values (5)
		[20]
	QUESTION 2	
2.1	$4a-a-2=6a+4-4a$ $a=6$	✓equating ✓answer (2)
2.2 2.2.1	 	✓ 33 ✓ 96 (2)
2.2.2	45 – Term odd 23rd term of the odd sequence: $T_{23}=5+(23-1)(7)$ $T_{23}=159$	✓45 – Term odd ✓ verifying T_{23} ✓ answer (3)

2.3	$T_8 = 33 = a + 7d$ (1) $T_{11} = 45 = a + 10d$ (2) From equation (2): $a = 45 - 10d$ (3) **OR** Substitute (3) into (1): $33 = 45 - 10d + 7d$ $-12 = -3d$ $d = 4$ Substitute into (2) $a = 45 - 10(4)$ $a = 5$ $T_{15} = 5 + 14(4)$ $T_{15} = 61$ **OR** $T_8 + 3d = T_{11}$ $33 + 3d = 45$ $3d = 12$ $d = 4$ $T_{15} = T_{11} + 4d$ $T_{15} = 45 + 4(4)$ $= 61$ $a + 10d = 45$ (2) $a + 7d = 33$ (1) (2) – (1): $3d = 12$ $d = 4$	✓equation 1 ✓equation 2 ✓value of d ✓value of a ✓$T_{15} = 61$ ✓$T_8 + 3d = T_{11}$ ✓substitution ✓value of d ✓$T_{15} = T_{11} + 4d$ ✓answer (5)
2.4	$2a = -4$ $a = -2$ $3a + b = 0$ $3(-2) + b = 0$ $b = 6$ $a + b + c = 6$ $-2 + 6 + c = 6$ $c = 2$ $T_n = -2n^2 + 6n + 2$	✓second difference ✓value of a ✓value of b ✓value of c (4)

2.5 2.5.1	$3;\ \frac{3}{2};\ \frac{3}{4};\ ...$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{3}{1-\frac{1}{2}}$ $= 6$	✓substitute into the correct formula ✓answer (2)
2.5.2	$S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$ $= 6\left(1-\left(\frac{1}{2}\right)^n\right)$	✓substitute into the correct formula ✓answer (2)
2.5.3	$A-B<\frac{1}{36}$ $6-6\left(1-\left(\frac{1}{2}\right)^n\right)<\frac{1}{36}$ $6\left(1-1+\left(\frac{1}{2}\right)^n\right)<\frac{1}{36}$ $1-1+\left(\frac{1}{2}\right)^n<\frac{1}{216}$ $\left(\frac{1}{2}\right)^n<\frac{1}{216}$ $n>\log_{\frac{1}{2}}\frac{1}{216}$ $n>7,75$ $\therefore n=8;\ 9;\ ...$	✓factorisation ✓$\frac{1}{216}$ ✓$\left(\frac{1}{2}\right)^n<\frac{1}{216}$ ✓$\log_{\frac{1}{2}}\frac{1}{216}$ ✓answer (5)
		[25]

	QUESTION 3	
3.1	$\left(1+i_{eff}\right)=\left(1+\frac{i_{nom}}{m}\right)^{m}$ $\left(1+i_{eff}\right)=\left(1+\frac{0,06}{4}\right)^{4}$ $i_{eff}=1,06-1$ $i_{eff}=6,14\%$	✓correct formula ✓correct substitution ✓answer (3)
3.2	$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P=\frac{2000\left[1-(1+0,02)^{-24}\right]}{0,02}$ $=R37\,827,85$ The gym borrowed R 37 827,85.	✓substitution of x into correct formula ✓ $n=24$ ✓ $i=0,02$ ✓ answer (4)
3.3	$F=\frac{x\left[(1+i)^{n}-1\right]}{i}$ $200000=\frac{x\left[\left(1+\frac{0,0625}{12}\right)^{57}-1\right]}{\frac{0,0625}{12}}$ $x=\frac{200000\times\frac{0,0625}{12}}{\left[\left(1+\frac{0,0625}{12}\right)^{57}-1\right]}$ $x=3022,73$ $\therefore$ The monthly payments are R 3 022,73 per month.	✓formula ✓ value of F ✓ $n=57$ ✓ $i=\frac{0,0625}{12}$ ✓making x the subject ✓answer (6)
		[13]

	QUESTION 4	
4.1 4.1.1	$f(-1) = 4.2^{-1+1} - 2$ $y = 4 - 2$ **OR** f(-1) = 2 $y = 2$	✓ $y = 2$ (1)
4.1.2	$f(x) = 4.2^{x+1} - 2$ x-intercept $f(x) = 0$ $4.2^{x+1} - 2 = 0$ $4.2^{x+1} = 2$ $2^{x+3} = 2^1$ $x + 3 = 1$ $x = -2$ $(-2; 0)$ **OR** $4.2^{x+1} - 2 = 0$ $4.2^{x+1} = 2$ $2^{x+1} = \frac{2}{4}$ $2^{x+1} = 2^{-1}$ $x + 1 = -1$ $x = -2$ $(-2; 0)$	✓ simplification $(2^{x+3} = 2^1)$ ✓ $x + 3 = 1$ ✓ answer ✓ simplification $(2^{x+1} = 2^{-1})$ ✓ $x + 1 = -1$ ✓ answer (3)
4.1.3	y-intercept $x = 0$ $y = 4.2^{0+1} - 2$ $y = 8 - 2$ $y = 6$ $(0; 6)$	✓substitute $x = 0$ ✓answer (2)
4.1.4	$y = -2$	✓ $y = -2$ (1)
4.2		✓asymptote ✓shape with any one correct coordinate ✓both intercepts with axes (3)

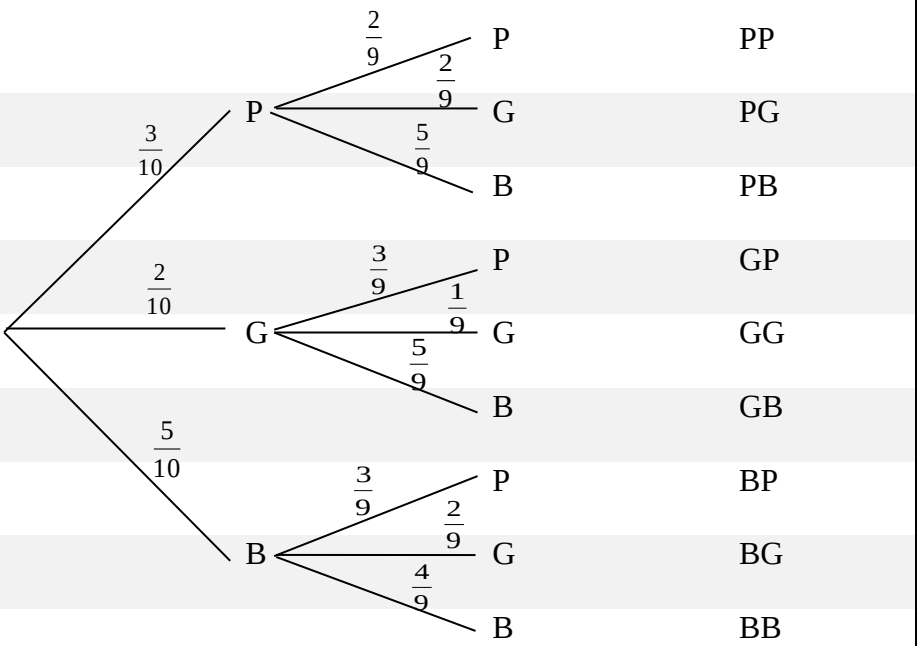
4.3	$g(x) = 4.2^{x-2+1} - 2$ $g(x) = 4.2^{x-1} - 2$ **OR** $g(x) = 2.2^{x} - 2$ **OR** $g(x) = 2^{x+1} - 2$	✓answer ✓answer ✓answer (1)
		[11]
	QUESTION 5	
5.1 5.1.1	(3; 4)	✓3 ✓4 (2)
5.1.2	$y \geq -4$ **OR** $y \in [-4; \infty)$	✓critical values ✓inequality ✓critical values ✓correct brackets (2)
5.1.3	**(**2; 3) (5; 0) $m = \frac{0-3}{5-2}$ $m = -1$	✓ substitute with correct coordinates ✓ answer (2)
5.1.4	$y = -(-1) + 5$ $y = 6$ $KM = 6$ $y = -(-1-3)^2 + 4$ $y = -12$ $LM = 12$ $KL = 12 + 6$ $= 18$	✓ KM = 6 **OR** $y = 6$ ✓ LM = 12 **OR** $y = -12$ ✓ KL = 18 (3)
5.1.5	$x \in (2; 5)$ **OR** $2 < x < 5$	✓ critical values ✓ correct brackets ✓ critical values ✓ correct inequality (2)

5.2 5.2.1	$y=\left(\frac{4}{3}\right)^x$ $x=\left(\frac{4}{3}\right)^y$ $y=\log_{\frac{4}{3}} x$ $g(x)=\log_{\frac{4}{3}} x$	✓ interchange x and y ✓ $g(x)=\log_{\frac{4}{3}} x$ (2)
5.2.2	$h(x)=\left(\frac{4}{3}\right)^{-x}$ $=\left(\frac{3}{4}\right)^x$	✓ $-x$ ✓answer (2)
5.2.3 a)	$x=0$	✓answer (1)
b)	$0<x\leq 1$ **OR** $x\in(0;1]$	✓critical values ✓inequalities ✓ critical values ✓ correct brackets (2)
		[18]

	QUESTION 6	
6.1	$f'(x)=\frac{4}{x^2}$ $\therefore x^2 \geq 0$ $\therefore \frac{4}{x^2} > 0$	✓ $f'(x)=\frac{4}{x^2}$ ✓ $x^2 \geq 0$ ✓ $\frac{4}{x^2} > 0$ (3)
6.2	$m=-4$ *Perpendicular lines* $m=\frac{1}{4}$ $f'(x)=\frac{4}{x^2}$ $\frac{1}{4}=\frac{4}{x^2}$ $16=x^2$ $\therefore x=-4 \qquad y=-\frac{4}{-4}+7$ $y=8$ Substitute $(-4;8)$ $y=\frac{1}{4}x+c$ **OR** $y-8=\frac{1}{4}(x+4)$ $8=\frac{1}{4}(-4)+c$ $y=\frac{1}{4}x+9$ $c=9$ $y=\frac{1}{4}x+9$	✓ $m=\frac{1}{4}$ ✓ $\frac{1}{4}=\frac{4}{x^2}$ ✓selecting $x=-4$ ✓ $y=8$ ✓answer (5)
		[8]

	QUESTION 7	
7.1 7.1.1	$f(x+h)=1-4(x+h)^2$ $=1-4x^2-8xh-4h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{1-4x^2-8xh-4h^2-\left(1-4x^2\right)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $=-8x$	✓ $f(x+h)=$ $1-4x^2-8xh-4h^2$ ✓correct substitution into correct formula ✓simplify $\left(-8xh-4h^2\right)$ ✓common factor h ✓ $-8x$ (5)
7.1.2	$f'(2)=-8(2)$ $=-16$	✓substitute $x=2$ into $f'(x)$ ✓ answer (2)
7.2 7.2.1	$y=x^2-4x+4$ $\dfrac{dy}{dx}=2x-4$	✓expansion ✓$2x$ ✓-4 (3)
7.2.2	$f(x)=x^{\frac{2}{3}}+\dfrac{1}{4}x^{-4}$ $f'(x)=\dfrac{2}{3}x^{\frac{-1}{3}}-x^{-5}$	✓ $x^{\frac{2}{3}}$ ✓ $\dfrac{1}{4}x^{-4}$ ✓ $\dfrac{2}{3}x^{\frac{-1}{3}}$ ✓ $-x^{-5}$ (4)
		[14]

	QUESTION 8	
8.1	$f(x) = (x+2)(x-1)(x-3)$ $= x^3 - 2x^2 - 5x + 6$	✓factors $(x+2)(x-1)(x-3)$ ✓product of any two factors ✓product of any binomial and trinomial $x^3 + x^2 - 2x - 3x^2 - 3x + 6$ (3)
8.2	$f'(x) = 3x^2 - 4x - 5$ $0 = 3x^2 - 4x - 5$ $x = \frac{4 \pm \sqrt{16+60}}{6}$ $x = 2{,}12$ **OR** $x = -0{,}7$ $y = 8{,}21$ **OR** $y = -4{,}06$ $A(-0{,}79;8{,}21)$ $B(2{,}12;-4.06)$	✓ $f'(x) = 0$ ✓substitute into quadratic formula ✓x-values ✓y-values (4)
8.3	$y = x^3 - 2x^2 - 5x + 6 - (2x)$ $= x^3 - 2x^2 - 7x + 6$ $\frac{dy}{dx} = 3x^2 - 4x - 7$ $= (3x-7)(x+1)$ $for\,\max$ $0 = (3x-7)(x+1)$ $x = \frac{7}{3}$ **OR** $x = -1$ $y = (-1)^3 - 2(-1)^2 - 7(-1) + 6$ $PQ = 10$	✓simplification $x^3 - 2x^2 - 7x + 6$ ✓ differentiation ✓ factors ✓ $\frac{dy}{dx} = 0$ ✓x-values ✓ answer (6)
8.4	$f'(x) = 3x^2 - 4x - 5$ $f''(x) = 6x - 4$ $6x - 4 < 0$ $x < \frac{2}{3}$ $k = \frac{2}{3}$	✓ $f''(x)$ ✓ $f''(x) < 0$ ✓answer (3)
		[16]

	QUESTION 9	
9.1	BC = $50 - 2x$	✓answer (1)
9.2	Area = AB × BC $= x(50 - 2x)$ $= 50x - 2x^2$ Max $A'(x) = 50 - 4x$ $0 = 50 - 4x$ $x = 12{,}5m$	✓ $x(50 - 2x)$ ✓simplification $(50x - 2x^2)$ ✓differentiate ✓answer (4)
		[5]
	QUESTION 10	
10.1		✓first branches ✓ $\frac{2}{9}; \frac{2}{9}; \frac{5}{9}$ ✓ $\frac{3}{9}; \frac{1}{9}; \frac{5}{9}$ ✓ $\frac{3}{9}; \frac{2}{9}; \frac{4}{9}$ ✓all outcomes (5)
10.2 10.2.1	P(B, B) = $\frac{5}{10} \times \frac{4}{9}$ $= 0{,}22$ **OR** $\frac{2}{9}$	✓ $\frac{5}{10} \times \frac{4}{9}$ ✓answer (2)
10.2.2	P(G,P) = $\frac{3}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{3}{9}$ $= 0{,}13$ **OR** $\frac{12}{90}$ **OR** $\frac{2}{15}$	✓ $\frac{3}{10} \times \frac{2}{9}$ **OR** $\frac{6}{90}$ ✓ $\frac{2}{10} \times \frac{3}{9}$ **OR** $\frac{6}{90}$ ✓answer (3)
		[10]

	QUESTION 11	
11.1	Letters : $26 - 6 = 20$ Digits: $0 - 9 = 10$ $20^4 \times 10^4$ $= 1\,600\,000\,000$	✓ 20^4 ✓ 10^4 ✓ answer (3)
11.2 11.2.1	$P'(A) = 1 - P(A)$ $0,45 = 1 - P(A)$ $P(A) = 0,55$ $P(A \text{ or } B) = P(A) + P(B)$ $= 0,55 + 0,35$ $= 0,9$	✓ P(A) ✓ substitute in correct formula ✓ answer (3)
11.2.2	$P(A) \times P(B)$ $= 0,55 \times 0,35$ $= 0,1975$ **OR** $\frac{77}{400}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$ $= 0,55 + 0,35 - 0,1975$ $= 0,71$ **OR** $\frac{283}{400}$ **OR** $\frac{71}{100}$	✓ $0,55 \times 0,35$ ✓ 0,1975 **OR** $\frac{77}{400}$ ✓ substitute in correct formula ✓ answer (4)
		[10]
		TOTAL: 150