# MATHEMATICS: PAPER I

Time: 3 hours                                                                                           150 marks

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**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1.     This question paper consists of 8 pages and an Information Sheet of 2 pages (i–ii). Please check that your paper is complete.

2.     Read the questions carefully.

3.     Answer all the questions.

4.     Number your answers exactly as the questions are numbered.

5.     You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6.     Round off your answers to one decimal digit where necessary.

7.     All the necessary working details must be clearly shown.

8.     It is in your own interest to write legibly and to present your work neatly.

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**SECTION A**

**QUESTION 1**

(a)     Solve for $x$:

      (1)     $4 - \sqrt{2 - x} = 3x$             (6)

      (2)     $2(5)^{9-x} = 1\,250$             (3)

(b)     The graph of $f(x) = 3(x+1)(x-3)$, not drawn to scale, is sketched below.
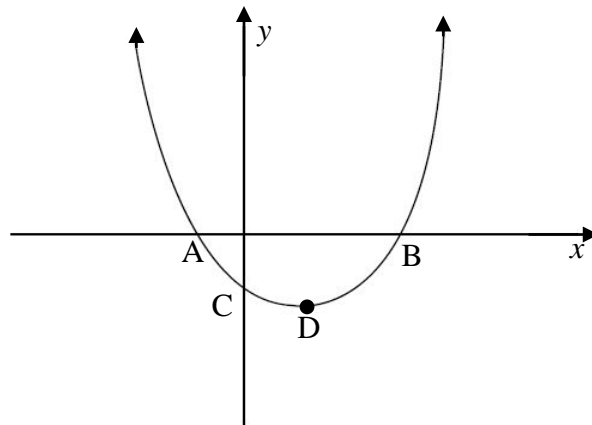


      (1)     Determine the coordinates of A, B, C, and D.           (6)

      (2)     Hence, or otherwise solve for $x$: $3(x+1)(x-3) \geq 0$.           (2)

(c)     If one root of the equation $x^2 + tx + 18 = 0$ is greater than the other root by 3, determine a possible value of $t$.           (5)

(d)     Solve for $x$ and $y$ simultaneously in the following set of equations:
$3x + y = 2$ and $y^2 = 2x^2 - 1$.           (7)

                                            **[29]**

**QUESTION 2**

An agricultural organisation assists farmers by arranging loans at very low interest rates.



[Source: <www.irrigation-uk.com>]

A farmer purchased irrigation equipment for R2 500 000. The organisation offered him a loan for the full amount at 6% per annum compounded quarterly.

(a)     Show that his quarterly instalment, if the loan repayment period was 5 years, is R145 614,34. Show all working.                                                             (4)

(b)     Calculate the outstanding balance of the loan after 4 years, i.e. immediately after the 16th payment.                                                                       (5)

(c)     Calculate the total amount paid towards interest charges over the 4-year period.     (3)

**[12]**

**QUESTION 3**

(a)     The first three terms of an infinite geometric series are 45, 15 and 5.

     (1)     Write the series in $\sum$-notion.                                                    (3)

     (2)     Why is the sum of the series a finite number?                                     (1)

     (3)     Determine the sum of the series.                                                  (2)

(b)     The sum to $n$ terms of an arithmetic series is given as:

$$S_n = \frac{n}{2}(7n+19).$$

     (1)     How many terms of the series must be added to give a sum of 4 878?             (5)

     (2)     Determine the 7th term of the series.                                             (4)

(c)    Twelve right circular cylinders are filled with a blue gel. The largest has a radius of 18 cm and a height of 64 cm. The second largest has a radius and height each equal to 90% of that of the first cylinder. The third largest has a radius and height each equal to 90% of that of the second cylinder. The process continues until the 12$^{th}$ largest cylinder.

    (1)    Show that the sequence of volumes, written in decreasing order, is a geometric sequence. (3)

    (2)    Determine the volume of gel (in litres) that is required to fill all twelve cylinders. (4)

**Note:**  Volume of right cylinder = $\pi\,(\text{radius})^2 \times \text{height}$
1 litre = 1 000 cm$^3$

**[22]**

## QUESTION 4

The $n^{th}$ term of a quadratic sequence is given by $T_n = an^2 + bn + c$.
If $T_1 = -1$, $T_2 = 4$ and $T_3 = 11$, determine $a$, $b$ and $c$.

**[6]**

## QUESTION 5

(a)    Given $f(x) = \dfrac{1}{2}x^2$, determine $f'(x)$ from first principles. (5)

(b)    Differentiate with respect to $x$:  $y = \dfrac{2x^3 - \sqrt{x}}{x}$. Leave answer with positive exponents. (5)

**[10]**

79 marks

**SECTION B**

**QUESTION 6**

Given: $f(x) = 2x^2 - 20x + 47$ and $h(x) = b^x + q$.

(a) Rewrite the equation of $f$ in the form $f(x) = a(x+p)^2 + q$. (4)

(b) Given that $h(x) = b^x + q$ has a $y$-intercept of $(0; -2)$ and passes through $(1; -1)$, determine $b$ and $q$. (3)

(c) Sketch the graphs of $f$ and $h$ on the same set of axes.
Indicate on your graphs the coordinates of the turning point of $f$ and the $y$-intercept of $h$. Also indicate asymptote(s), if any, by means of dotted lines. (5)

(d) On your sketch, indicate by using the letter A, where $f(x) - h(x) = 0$. (1)

(e) Determine $h^{-1}$, the inverse of $h$, in the form $y = \text{........}$ (3)

(f) State the domain of $h^{-1}$. (2)

**[18]**

**QUESTION 7**

Given: $f(x) = \dfrac{-2x+2}{x+1}$

Express $f(x)$ in the form $f(x) = \dfrac{a}{x+p} + q$ and hence write down the equation of the asymptotes of $f(x)$.
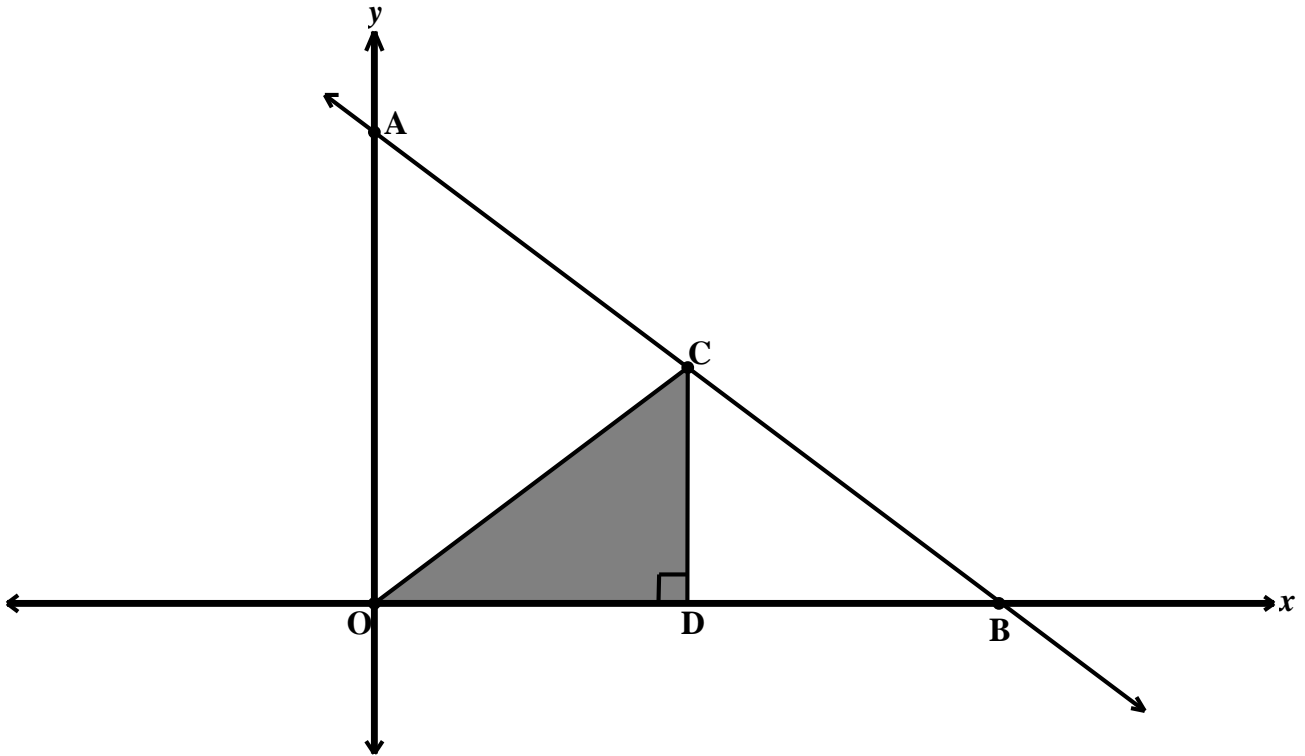
**[6]**

**QUESTION 8**

Given: $f(x) = 2x + \dfrac{1}{2x}$

Determine the equation of the tangent to the curve of $f$, at the point $\left(-1; -\dfrac{5}{2}\right)$.

**[6]**

**QUESTION 9**

In the diagram, C($x$; $y$) is a variable point on the line segment AB, where A and B are the points (0; 9) and (12; 0) respectively. COD is a right-angled triangle with D on the $x$-axis.



Determine the value of $x$ for which $\Delta$COD will have a maximum area.

**[8]**

**QUESTION 10**

Given: $f(x) = ax^3 + bx^2 - 3$ and $f''(-2) = 0$. If it is further given that the point $(-3; 6)$ lies on the graph of $f$:

(a)     Show that $a = \dfrac{1}{3}$ and $b = 2$.                                                    (7)

(b)     Determine the values of $x$ for which:

(1)     $\dfrac{f'(x)}{x} \geq 0$                                                                      (4)

(2)     the graph of $f$ is concave up.                                            (3)

**[14]**

**QUESTION 11**

(a)     Six-lettered "words" are made up using the letters of STATED. For example, TSDTAE is one such word.

       (1)     How many of these words begin with T?     (2)

       (2)     How many words can be made altogether?     (2)

       (3)     What is the probability that one of the words made, begins with T?     (1)

(b)     A shelf (A) contains 10 books on fiction and 6 on non-fiction. Another shelf (B) has 8 books on fiction and 12 on non-fiction. Joel selects a bookshelf randomly and then selects a book randomly from that shelf.

       (1)     What is the probability that he picks a book on fiction?     (4)

       (2)     What is the probability that he does not pick a non-fiction book from shelf A?     (3)

**[12]**

## QUESTION 12
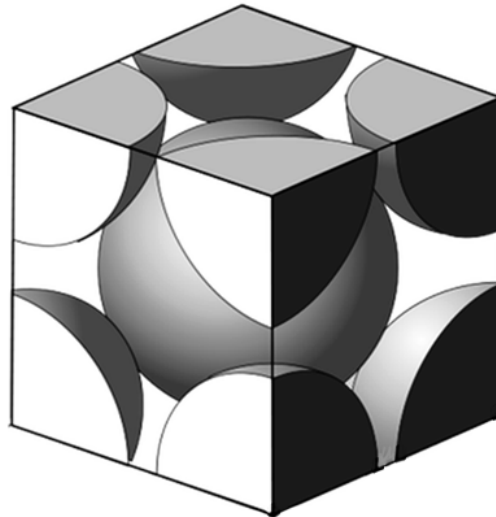


Two identical solid spheres each having radius *x,* are given. One of the spheres is cut into three places to form eight identical pieces. These are placed in a hollow cube as shown in the diagram. The second sphere touches all eight pieces.

The volume of liquid used to fill up the unoccupied space in the cube is 116,8 cm$^3$.
Determine *x*, correct to one decimal place.

**Useful formula:**

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3$$

**[7]**

**71 marks**

**Total: 150 marks**