NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2017

MATHEMATICS: PAPER II

MARKING GUIDELINES

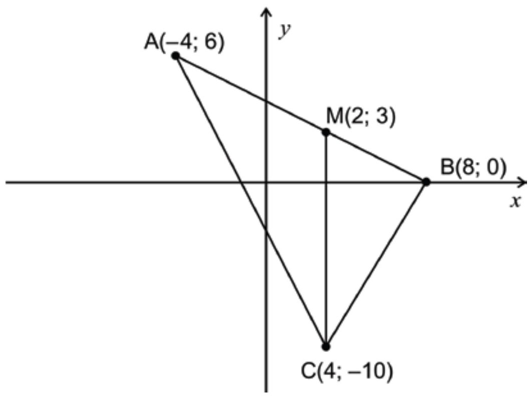
Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1



(a) $AB = \sqrt{(6-0)^2 + (-4-8)^2}$

$AB = \sqrt{180}$

OR

$6\sqrt{5}$

OR

13,42 (3)

(b) $M(2; 3)$ (1)

(c) $m_{AM} = \frac{6-3}{-4-2} = -\frac{1}{2}$ OR $m_{AM} = m_{AB} = -\frac{1}{2}$

$m_{MC} = \frac{-10-3}{4-(2)} = \frac{-13}{2}$

$-\frac{1}{2} \times \frac{-13}{2} \neq -1 \therefore \text{ not } 90°$ (3)

[7]

QUESTION 2

(a) $m = y_Q = \text{radius of } \odot Q = 8$

OR (15, 0) lies on OQ

$(15-15)^2 + (0-m)^2 = 64$

$\therefore m = 8$ (2)

(b) $PQ = 8 + 5 = 13$ units **OR** $PQ = \sqrt{(3-8)^2 + (3-15)^2}$

$= 13$ units (2)

(c) The coordinates of A $(0; y)$

$(x-3)^2 + (y-3)^2 = 25$

$(0-3)^2 + (y-3)^2 = 25$

$(y-3)^2 = 16 \quad \therefore y - 3 = \pm 4$

$y = 7$

A(0; 7)

Alternate

$x_A = 0$

$y_A = y_P + \sqrt{AP^2 - x_P^2}$

$y_A = 3 + \sqrt{5^2 - 3^2}$

$= 7$

A(0; 7) (4)

(d) $m_{PQ} = \frac{8-3}{15-3} = \frac{5}{12}$

$m_{AP} \frac{3-7}{3-0} = \frac{-4}{3}$

$m_{tan} = \frac{3}{4}$

$\therefore$ AP is not parallel to PQ since $m_{tan} \neq m_{PQ}$ (5)

[13]

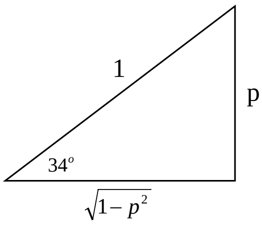
QUESTION 3

1(a) $x^2 + p^2 = 1$

$\therefore x = \sqrt{1 - p^2}$

$\cos 34°$

$= \sqrt{1 - p^2}$



(3)

(b) $\dfrac{\sin\theta - \sin^3\theta}{2\sin\theta\cos\theta}$

$\dfrac{\sin\theta(1-\sin^2\theta)}{2\sin\theta\cos\theta}$

$= \dfrac{\sin\theta\cos^2\theta}{2\sin\theta\cos\theta}$

$= \dfrac{\cos\theta}{2}$ (5)

(c) (1) $\sin^2\theta - \cos^2\theta + \sin\theta + 1 = 0$

$\sin^2\theta - (1 - \sin^2\theta) + \sin\theta + 1 = 0$

$2\sin^2\theta + \sin\theta = 0$ (1)

(2) $\sin\theta(2\sin\theta + 1) = 0$

$\sin\theta = -\dfrac{1}{2}$ Ref: $30°$

$\theta = 210° + k \cdot 360°$

$\theta = 330° + k \cdot 360°$

OR

$\sin\theta = 0$

$\theta = 0° + k \cdot 180°$ or $\theta = 0° + k \cdot 360°$

$\theta = 180° + k \cdot 360°$ (7)

(d) $\sin(2x + x)$

$= \sin 2x \cdot \cos x + \sin x \cdot \cos 2x$

$= 2\sin x \cdot \cos x \cdot \cos x + \sin x \cdot (1 - 2\sin^2 x)$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$ (6)

(e) (1) $a = 5$ and $b = 2$ (2)

(2) $q < -5$ or $q > 5$ (2)

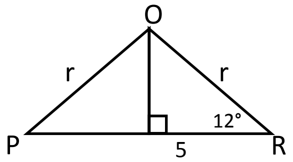
[26]

QUESTION 4

(a) (1) $\widehat{O}_2 = 204°$ (Angle at centre is twice the angle at circumference)

$\hat{O}_1 = 156°$ (Angles around a point)

(2)



$\hat{R}_1 = \frac{1}{2}(180° - 156°) = 12°$

$\cos 12° = \frac{5}{r}$

$r = \frac{5}{\cos 12°}$

$r = 5{,}11$

OR (3)

$10^2 = r^2 + r^2 - 2(r)(r)\cos 156°$

$100 = 2r^2(1 - \cos 156°)$

$\sqrt{\frac{100}{2(1-\cos 156°)}} = r$

$r = OP = 5{,}11$ units

OR

$\frac{OP}{\sin 12°} = \frac{10}{\sin 156°}$

$\therefore OP = 5{,}11$ (3)

(b) (1) $\hat{E}_1 = \hat{B}$ (tan chord theorem)

$\hat{D}_1 = \hat{E}_1$ (corresponding angles CD//AE)

$\therefore \hat{D}_1 = \hat{B}$ (4)

(2) $\hat{E}_3 = \hat{A}$ (tan chord theorem)

$\hat{C}_1 = \hat{E}_2$ (Alternate angles AE//CD) } for one of these statements

Therefore $\Delta ABE /// \Delta EDC$ (A,A,A)

NOTE: $\hat{B} = \hat{D}_1$ (proved)

(can be used as one of statements) (3)

(3) $\frac{AE}{EC} = \frac{BE}{DC}$; $\Delta ABE /// \Delta EDC$

$AE \cdot DC = BE \cdot EC$

but

$BE = 2EC$ (given)

therefore $2EC^2 = AE \cdot DC$ (4)

[17]

QUESTION 5

(a) $\frac{320}{2} = 160$

Median age = 35 years (refer to graph) (2)

(b) $\frac{3}{4} \times 320 = 240$

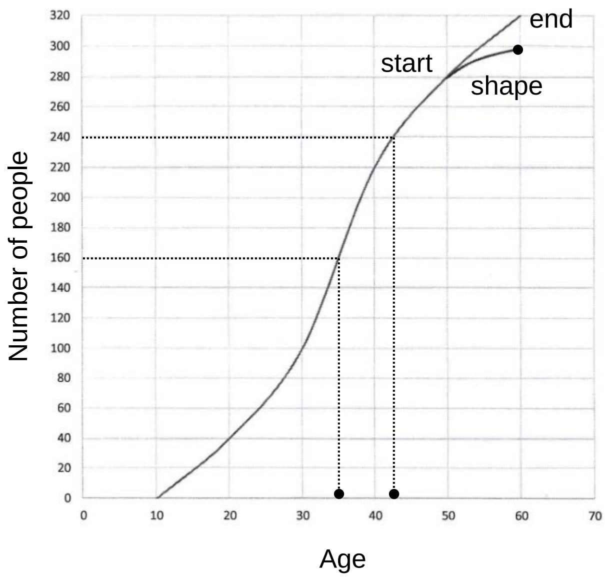
Minimum age is ± 42 (refer to graph) (No marks for 45) (2)

(c) There are 100 people younger than 30.

Percentage $\frac{100}{320} \times 100$

31, 25% of the people who attended the concert were younger than 30. (2)

(d) (1)



(3)

(2) Lower quartile has decreased. (1)

[10]

73 marks

SECTION B

QUESTION 6

(a) $r = -0,88$; it is a strong, negative relationship.

OR

It is very likely that the following is true: Students with higher incomes buy fewer loaves of bread. (3)

(b) $b = -0,00231$ (2)

(c) $y = -0,00231x + 31,74$

$y = -0,0031\ (7\ 500) + 31,74$

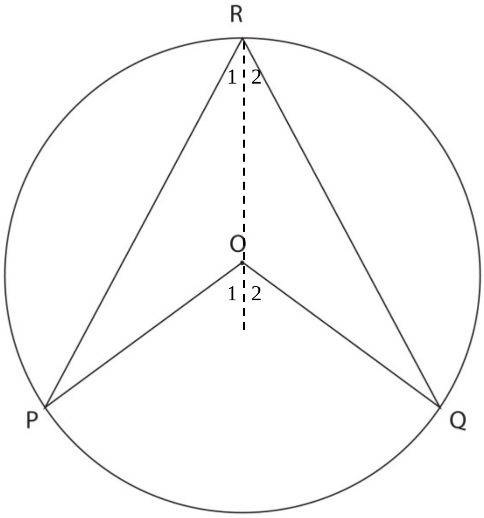
$y = 14,4 \approx 15$ or 14 (accept either)

This is a good approximation (interpolation) (4)

[9]

QUESTION 7

(a)



Construction

Draw a line through O and R

RTP: $\therefore \hat{O}_1 + \hat{O}_2 = 2\left(\hat{R}_1 + \hat{R}_2\right)$

Proof:

$\hat{O}_1 = \hat{R}_1 + P$ (ext angle of Δ)

but $\hat{R}_1 = P$ (isos Δ)

$\therefore \hat{O}_1 = 2\hat{R}_1$

Similarly $\hat{O}_2 = 2\hat{R}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\left(\hat{R}_1 + \hat{R}_2\right)$ (6)

(b) (1) $\hat{A}_1 = 45°$ (Angle at centre = two times angle at circumference)

$\hat{B}_1 = 43°$ (Given)

Therefore EB is not parallel to AC as alternate angles are not equal. (3)

(2) $\hat{C}_1 = \hat{B}_3$ (OB = OC)

$\therefore \hat{C}_1 = 45°$ (angles of Δ;)

$\hat{C}_2 = \hat{A}_2 = 12°$ (alternate angles OC//AD)

$F\hat{B}A = \hat{C}_1 + \hat{C}_2$ (tan chord)

$= 57°$

$\therefore F\hat{B}E = F\hat{B}A - \hat{B}_1$

$= 57° - 43°$

$= 14°$

Alternate:

$B\hat{A}D = \hat{A}_1 + \hat{A}_2$

$= 45° + 12°$

$= 57°$

$F\hat{B}O = 90°$ (radius $\perp$ tangent)

FB//OC (alternate angles)

$\therefore$ FB//AD

$F\hat{B}A = B\hat{A}D$ (alternate angles)

$F\hat{B}E = 57° - \hat{B}_1$

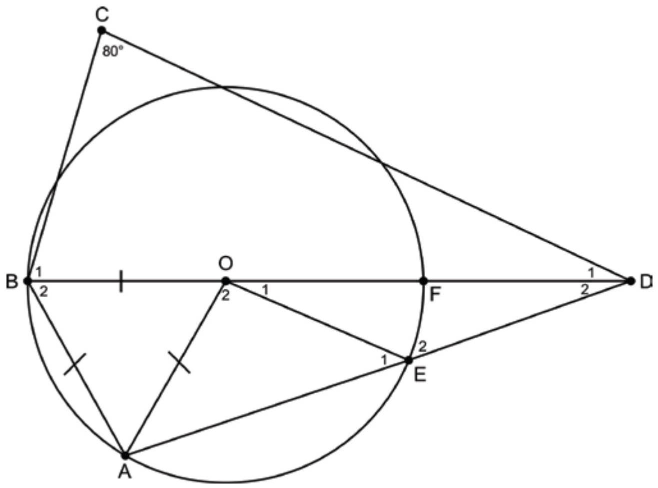
$= 57° - 43°$

$= 14°$ (8)

[17]

QUESTION 8

(a)



Construction: OA

$\text{OA} = \text{OB}$ (radii)

$\hat{B}_2 = 60°$ (Equilateral triangle)

$B\hat{A}E = 100°$ (Opp angles of cyclic quad)

$\hat{D}_2 = 20°$ (Angles in a Δ) (6)

(b) $\hat{A} = 100°$ (Opposite angles of cyclic quad)

$\therefore B\hat{O}E = 200°$ (Angle at centre)

$\therefore \hat{O}_2 = 160°$

$\therefore \hat{O}_1 = 20°$ (angles on straight line)

$\therefore \text{EO} = \text{ED}$ (isos Δ)

OR

$B\hat{A}F = 90°$ (Δs in semi-circle)

$F\hat{A}E = 10°$

as $B\hat{A}E = 100°$ (Opposite angles of cyclic quad)

$\therefore \hat{O}_1 = 20°$ (Angle @ centre)

$\therefore \text{EO} = \text{ED}$ (isos Δ)

OR

$O\hat{A}E = 100° - 60°$

$= 40°$

$\therefore \hat{E}_1 = 40°$ (OA = OE radii)

$\therefore \hat{O}_1 = \hat{E}_1 - \hat{D}_2$ (Ext angle of Δ)

$= 20°$

$\therefore \text{EO} = \text{ED}$ (isos Δ) (4)

[10]

QUESTION 9

(a) Any two of the following:

$\hat{G}_1$ is a common angle

$\hat{B} = \hat{C}_2$ corresponding angles equal AB//CD

$\hat{A} = \hat{D}_2$ corresponding angles equal AB//CD (2)

(b) $\frac{GC}{GB} = \frac{5}{8}$

$\frac{GC}{GB} = \frac{CD}{AB}$ ($\Delta GCD /// \Delta GBA$)

$\therefore \frac{5}{8} = \frac{CD}{16}$

$CD = 10$ units (4)

(c) $\hat{E} = \hat{D}_2$ (alt angles CD//EF)

$\hat{F} = \hat{C}_2$ (alt angles CD//EF)

$CD = 10$ units $= EF$

$\Delta GCD \equiv \Delta GFE$ (A.S.A.)

$\therefore EG = GD$

OR

DC//FE (given)

and $DE = FE = 10$

$\therefore$ CDFE is a parallelogram

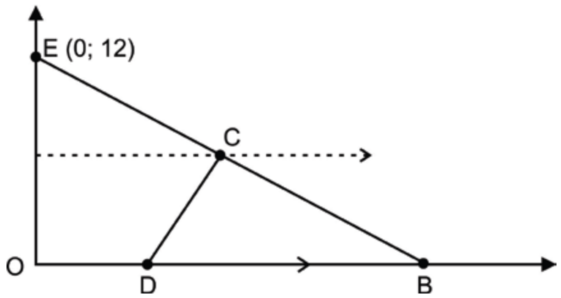
$\therefore$ Its diagonals bisect each other

$\therefore EG = GD$

(4)

[10]

QUESTION 10



$\tan C\hat{D}B = 0,35$

$C\hat{D}B = 19,29°$

Angle of inclination for line EC is:

$180° - (79,29 - 19,29)$

$= 120°$

Equation of line EC

$y = (\tan 120°)x + 12$ **OR** $y = (-\tan 60°)x + 12$

Therefore x coordinate of B is when $y = 0$

$0 = (\tan 120°)x + 12$

$x = \frac{-12}{\tan 120}$ **OR** $x = \frac{12}{\tan 60}$

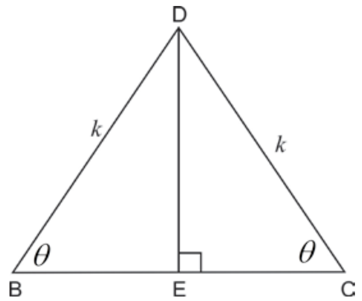
$= 6,93$ units $= 6,93$ units (6)

[6]

QUESTION 11

(a)
$$\frac{BC}{\sin(180^o - 2\theta)} = \frac{k}{\sin\theta}$$
$$BC = \frac{k \cdot \sin 2\theta}{\sin\theta}$$
$$BC = \frac{2k \cdot \sin\theta\cos\theta}{\sin\theta}$$
$$BC = 2k \cdot \cos\theta$$

Alternate



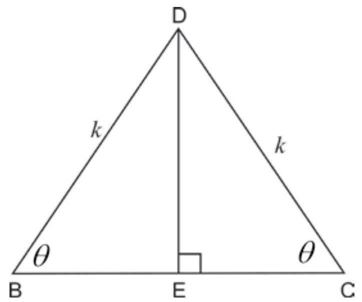
$$\cos\theta = \frac{\text{BE}}{k}$$
$$\text{BE} = k \cdot \cos\theta$$
$$\text{but BC} = 2\text{BE}$$
$$\therefore \text{BC} = 2k \cdot \cos\theta$$
(4)

(b) **Height of triangle BDC**

$$\frac{\text{h}}{k} = \sin\theta$$
$$\text{h} = k\sin\theta$$

Height of point A from the ground = radius of large semi-circle

$$\text{Height of A} = \frac{1}{2}\text{BC}$$
$$\text{Height of A} = \frac{1}{2} \cdot 2k\cos\theta$$
$$\text{Height of A} = k \cdot \cos\theta$$



$$\text{AD}^2 = \text{DE}^2 + \text{AE}^2 \text{ (Pythagoras)}$$
$$\frac{\text{DE}}{k} = \sin\theta$$
$$\text{DE} = k \cdot \sin\theta$$
$$\therefore \text{AD}^2 = (k \cdot \sin\theta)^2 + (k \cdot \cos\theta)^2$$
$$= k^2$$
$$\text{AD} = k$$

OR Alternate

Point D is equidistant from point B and point C and is the slant height of a cone.
Therefore AD $= k$.
Marks: slant height of cone.
AD $= k$ (6)

[10]

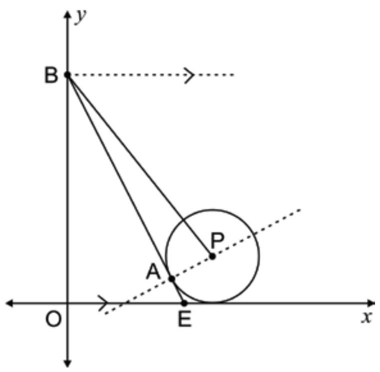
QUESTION 12

(a) $m_{BE} = -\frac{BO}{OE} = -2$

$PA \perp BE$ (rad $\perp$ tangent)

$\therefore m_{AP} = \frac{1}{2}$

Equation AP: $y - \left(\sqrt{5} - 1\right) = \frac{1}{2}\left(x - 10\right)$



(4)

(b) $y_P = \sqrt{5}$

$\therefore \sqrt{5} = \frac{1}{2}x + \sqrt{5} - 6$

$\therefore x = 12$

$\therefore P(12; \sqrt{5})$ (3)

(c) **MAIN**

Equation of BE: $y = -2x + c$

$\sqrt{5} - 1 = -2(10) + c$

$c = \sqrt{5} + 19$

$\therefore B(0; \sqrt{5} + 19)$

$m_{BP} = \frac{\sqrt{5} - (\sqrt{5} + 19)}{12 - 0}$

$= \frac{-19}{12}$

$\tan\theta = \frac{-19}{12}$ $\tan B\hat{E}O = 2$

$\therefore \theta = 122{,}3°$ $\therefore B\hat{E}O = 63{,}43°$

$\therefore A\hat{B}P = 63{,}43° - 57{,}72°$

$= 5{,}71°$

(8)

[15]

77 marks

Total: 150 marks