ST STITHIANS GIRLS' COLLEGE

GRADE 12

ADVANCED PROGRAMME MATHEMATICS

July 2014

TIME: 2 hours

MARKS: 110 Marks

NAME: AP MEMO

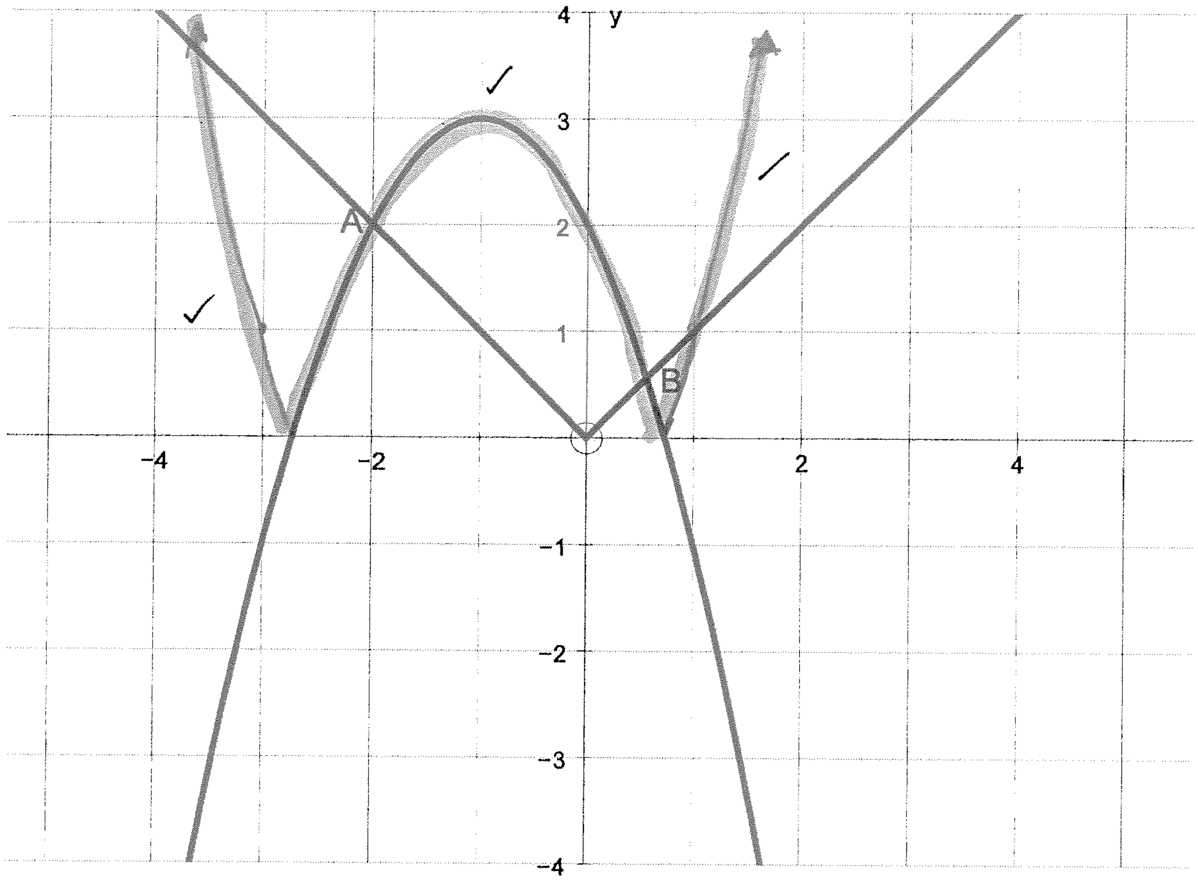
TEACHER: **Mr Schaerer** ☐ **Mr Ancillotti** ☐

ANSWER SHEET

Q1 /7	Q2 /10	Q3 /11	Q4 /16	Q5 /7	Q6 /12
Q7 /22	**Q8 /17**	**Q9 /8**			
			TOTAL	/110	%

QUESTION: 5

The sketch shows the graph of $g(x) = -x^2 - 2x + 2$ and $f(x) = |x|$, which intersect at A (-2; 2) and B.



(a) Determine the x - value of the point B (leave you answer in surd form)

$-x^2 - 2x + 2 = x$ (pos arm)

$\therefore -x^2 - 3x + 2 = 0$

$\therefore x = \frac{-3 + \sqrt{17}}{2}$

(4)

(b) Draw $h(x) = |g(x)|$ on the axes provided. (3)

[7]

Gr12 AP Prelims July 2014 /110

Q1) $3^n + 3^{n+1} + 3^{n+2}$

i) test $n=1$: $3 + 3^2 + 3^3 = 39$

$= 3(13)$

ie div by 3.

ii) assume true for $n=k$:

$3^k + 3^{k+1} + 3^{k+2} = 13p$

iii) Prove true for $n=k+1$

$3^{k+1} + 3^{k+2} + 3^{k+3}$

$= 3^k.3 + 3^{k+1}.3 + 3^{k+2}.3$

$= 3\left[3^k + 3^{k+1} + 3^{k+2}\right]$

$= 3\left[13p\right] = 13.(3p)$

ie mult of 13

$\therefore 3^n + 3^{n+1} + 3^{n+2}$ div by 13.

/7

2a $\lim_{x\to4} \dfrac{(\sqrt{x+5}-3)}{x-4} \times \dfrac{(\sqrt{x+5}+3)}{(\sqrt{x+5}+3)}$

$= \lim_{x\to4} \dfrac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$= \lim_{x\to4} \dfrac{1}{(\sqrt{x+5}+3)}$

$= \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

/6

b) $\lim_{x\to\infty} \dfrac{x^2+1}{2-3x-4x^2} \div \dfrac{x^2}{x^2}$

$= \lim_{x\to\infty} \dfrac{1 + \overset{=0}{1/x^2}}{\overset{=0}{2/x^2} - \overset{=0}{3/x} - 4}$

$= \dfrac{1}{-4} = -1/4$

/4

3a) $x = i$ $\therefore x = -i$

$\therefore (x-i)(x+i)$ is a factor

$\therefore x^2+1$ is a factor

$\therefore x^4 + 4x^3 + 3x^2 + 4x + 2 = 0$

$(x^2+1)(x^2+4x+2) = 0$

↓ QF

$\therefore x = \pm i$ $x = -2 \pm \sqrt{2}$

/6

b) $g(x) = \dfrac{2x}{(x-3)^2}$

$\dfrac{2x}{(x-3)^2} = \dfrac{A}{(x-3)} + \dfrac{B}{(x-3)^2}$

$2x = A(x-3) + B$ $(x=3)$

$\therefore B = 6$

Sub $x=4$: $8 = A + 6$

$\therefore A = 2$

$\therefore \dfrac{2x}{(x-3)^2} = \dfrac{2}{(x-3)} + \dfrac{6}{(x-3)^2}$ (5)

4a) LHS: $\sec x . \operatorname{cosec} x - \cot x$

$= \dfrac{1}{\cos x} . \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}$

$= \dfrac{1-\cos^2 x}{\cos x . \sin x}$

$= \dfrac{\sin^2 x}{\cos x . \sin x}$ (5)

$= \dfrac{\sin x}{\cos x} = \tan x$ = RHS

b) $f(x) = 1 - x^2$; $g(x) = \sin(4x)$

$f(g(x)) = 1 - [\sin(4x)]^2$

$= \cos^2(4x)$ (3)

(2)

4c) $A = \frac{1}{2}r^2\theta \quad ; \quad s = r\theta$

$\frac{1}{2}r^2\theta = 72 \quad ; \quad P = r\theta + 2r = 36$

$r\theta = 36 - 2r$

$\therefore \theta = \frac{36}{r} - 2$

$\therefore \frac{1}{2}r^2\left(\frac{36}{r} - 2\right) = 72 \quad (\times 2)$

$r^2\left(\frac{36}{r} - 2\right) = 144$

$36r - 2r^2 - 144 = 0 \quad (\div -2)$

$r^2 - 18r + 72 = 0$

$r = 12$ or $r = 6$

$\theta = \frac{36}{12} - 2 \qquad \theta = \frac{36}{6} - 2$

$= 1 \qquad = 4$ rads (8)

6)a) $f(x) = \begin{cases} 3 - ax^2 & \rightarrow x \geq 1 \\ -4x + 5 & \rightarrow x < 1 \end{cases}$

i) continuous $\therefore \lim_{x \to 1} f(x) = f(1)$

$\therefore 3 - a(1)^2 = -4(1) + 5$

$3 - a = 1$

$2 = a$ (3)

ii) diff $\rightarrow \lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

$\lim_{x \to 1^-} f'(x) = -4(1) \quad (a = 2)$

$= -4$

$\lim_{x \to 1^+} f'(x) = -4$ (4)

$\therefore$ differentiable

c) $y = x^3 - 5x - 2$

$dy/dx = 3x^2 - 5$

At A $\rightarrow$ choose $x \in (2;3)$

$x_{r+1} = x_r - \frac{(x^3 - 5x - 2)}{3x^2 - 5}$

let $x_1 = 2$:

$x_2 = 18/7$

$x_3 = 2{,}4268...$

$x = 2{,}41421$ (5dp) (5)

✓ max 1 no method.

7a) i) $g(x) = \cos(\sin x)$

$g'(x) = -[\sin(\sin x)] . \cos x$ (3)

ii) $y = x^3.(\cot x)^2$

$\frac{dy}{dx} = 3x^2.(\cot x)^2 + x^3.2(\cot x)(-\operatorname{cosec}^2 x)$

$= 3x^2\cot^2 x - 2x^3\cot x\operatorname{cosec}^2 x$ (4)

b) $x^2 - 4xy + 4y + 8 = 0$

$2x - 4\left(y + x\,dy/dx\right) + 4\frac{dy}{dx} = 0$

$2x - 4y - 4x\,dy/dx + 4\,dy/dx = 0$

$\frac{dy}{dx}(-4x + 4) = 4y - 2x$

$dy/dx = \frac{\cancel{2}(2y - x)}{\cancel{2}(-2x + 2)}$

$= \frac{2y - x}{-2x + 2}$ (5)

c: $f(x) = \dfrac{\sin 2x}{\cos 4x}$

i) $f(\pi/6) = \dfrac{\sin(\pi/3)}{\cos(2\pi/3)}$

$= \dfrac{\sqrt{3}/2}{-\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{-1/2}$

$= -\sqrt{3}$ (3)

ii) $f'(x) = \dfrac{\cos 2x.(2).\cos 4x - (-\sin 4x.4.\sin 2x)}{(\cos 4x)^2}$

$= \dfrac{2.\cos 4x \cos 2x + 4\sin 4x \sin 2x}{(\cos 4x)^2}$ (4)

iii) $m = f'(\pi/6)$

$= \dfrac{2.\cos(4.\frac{\pi}{6}).\cos(2.\frac{\pi}{6}) + 4\sin(4\frac{\pi}{6})\sin(2\frac{\pi}{6})}{[\cos 4(\frac{\pi}{6})]^2}$

$= 10$ ✓ ← calc

$\therefore y = 10x + c$ sub $(\pi/6; -\sqrt{3})$

$-\sqrt{3} = 10(\pi/6) + c$

$-\sqrt{3} - 5\pi/3 = c$

$\therefore y = 10x - \sqrt{3} - 5\pi/3$ ✓ (3)

8) a) $f(x) = \dfrac{x^2+4x+3}{x+2} = \dfrac{x(x+2)}{x+2} + \dfrac{2(x+2)}{(x+2)} - \dfrac{1}{x+2}$

$= x + 2 - \dfrac{1}{x+2}$ /2

b) i) $f'(x) = 1 - \left[\dfrac{-1}{(x+2)^2}\right]$

$= 1 + \dfrac{1}{(x+2)^2} = 0$ ✓ For tpt/stat

$(x+2)^2 = -1$

$\therefore$ no solution.

(no tpt/stat pt)

ii) xint:

$x^2+4x+3=0$

$(x+3)(x+1)=0$

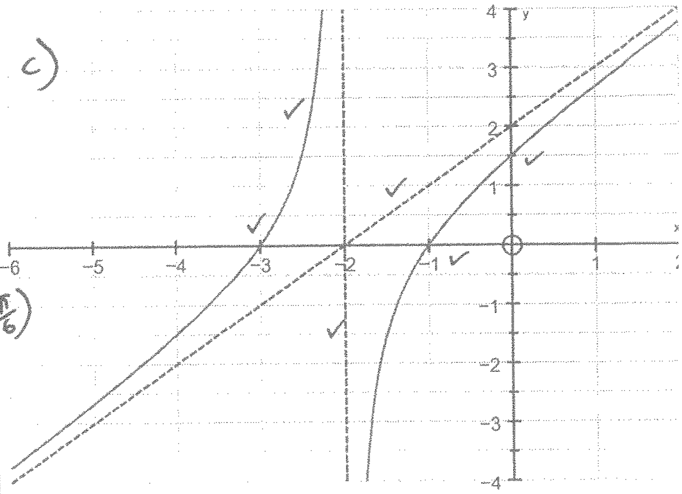
$x = -3$ or -1 (3)

yint:

$f(x) = 3/2$

iii) dol: $y = x+2$ (2)

c)



(6)

9a) $f(x) = 2\ln(x-1)$

f^{-1}: $x = 2\ln(y-1)$ ✓

$\frac{x}{2} = \ln_e(y-1)$

$e^{x/2} = y-1$ (2)

$\therefore y = e^{x/2}+1$ ✓

b) $(\ln x)^2 = \ln e^2 + \ln x$

$(\ln x)^2 = 2\ln e^1 + \ln x$ → k

$k^2 - k - 2 = 0$ ✓

$\therefore k = 2$ or -1 (6)

$\therefore \ln x = 2$ or $\ln x = -1$

$\therefore x = e^2$ ✓ $\therefore x = e^{-1}$ ✓

$\simeq 7{,}39$ ✓ $x = 1/e$

$\simeq 0{,}37$ ✓