# Form 5 AP Paper 1 Trials MEMO

**Q1.** Assume for $n = k$

i.e. $5^{k+1} + 2^{k} = 3m \quad m \in N$

RTP $5^{k+2} + 2^{k+1}$ is divisible by 3

LHS $5^{k+2} + 2^{k+1}$

$= 5.5^{k+1} + 2.2^{k}$

$= 3.5^{k+1} + 2.5^{k+1} + 2.2^{k}$

$= 3(5^{k+1}) + 2(5^{k+1} + 2^{k})$

$= 3(5^{k+1}) + 2.3m$

$= 3[5^{k+1} + 2m]$

$\therefore$ it is divisible by 3.

(8)

**Q2.** $x^2 - 10$ factor of $x^4 + 3x^3 - 20x^2 - 30x + 100$

$= (x^2 - 10)(x^2 + 3x - 10)$

$0 = (x \pm \sqrt{10})(x+5)(x-2)$

$x = \pm\sqrt{10}$ or $-5$ or $2$

(8)

**Q3.** (a) $\dfrac{e^x}{e^x - 1} = 10 \Rightarrow e^x = 10e^x - 10$

$10 = 9e^x$

$\dfrac{10}{9} = e^x$

$x = \ln\left(\dfrac{10}{9}\right) =$

$x = 0,1054$

(5)

(b) $\ln\left(e^{3x-10}\right) = 2x$

$e^{3x-10} = e^{2x}$

$3x - 10 = 2x$

$x = 10$

(4)

(c) $(\ln x)^2 = \ln e^3 + \ln x^2$

$e^3 = 20,1$

$\frac{1}{e} = 0,3678$

let $\ln x = k$

$k^2 = 3\ln e + 2k$

(11)

$k^2 - 2k - 3\ln e = 0 \qquad \ln e = 1$

$k^2 - 2k - 3 = 0$

$(k-3)(k+1) = 0$

$k = 3$ or $-1$

$\therefore \ln x = 3 \qquad \ln x = -1$

$x = e^3 \qquad x = \frac{1}{e}$

## Question 4.

$(a+3i)(4-i) = 11+bi$

$4a - ai + 12i - 3i^2 = 11+bi$

$4a + 3 + i(12-a)$ | $11+bi$

$4a+3 = 11$ | $12-a = b$

$4a = 8$ | $12-2 = b$

$\underline{\underline{a = 2}}$ | $\underline{\underline{b = 10.}}$

(7)

## Question 5

$f(x) = x^2 - 3x$

$\Delta x = \frac{4-3}{n} = \frac{1}{n}$ $\qquad x_i = 3 + \frac{i}{n}$

$f(x_i) = \left(3+\frac{i}{n}\right)^2 - 3\left(3+\frac{i}{n}\right)$

$= \cancel{9} + \frac{6i}{n} + \frac{i^2}{n^2} - \cancel{9} - \frac{3i}{n}$

$= \frac{6i}{n} + \frac{i^2}{n^2} - \frac{3i}{n} = \frac{3i}{n} + \frac{i^2}{n^2}$

$\sum f(x_i) = \sum \frac{3i}{n} + \sum \frac{i^2}{n^2}$ ca

$= \frac{3}{n}\left(\frac{n^2}{2}+\frac{n}{2}\right) + \frac{1}{n^2}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)$ m m

$= \frac{3}{2}n + \frac{3}{2} + \frac{n}{3} + \frac{1}{2} + \frac{1}{6n}.$ a

$\frac{1}{n}\sum f(x_i) = \frac{3}{2} + \frac{3}{2n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$ ca m

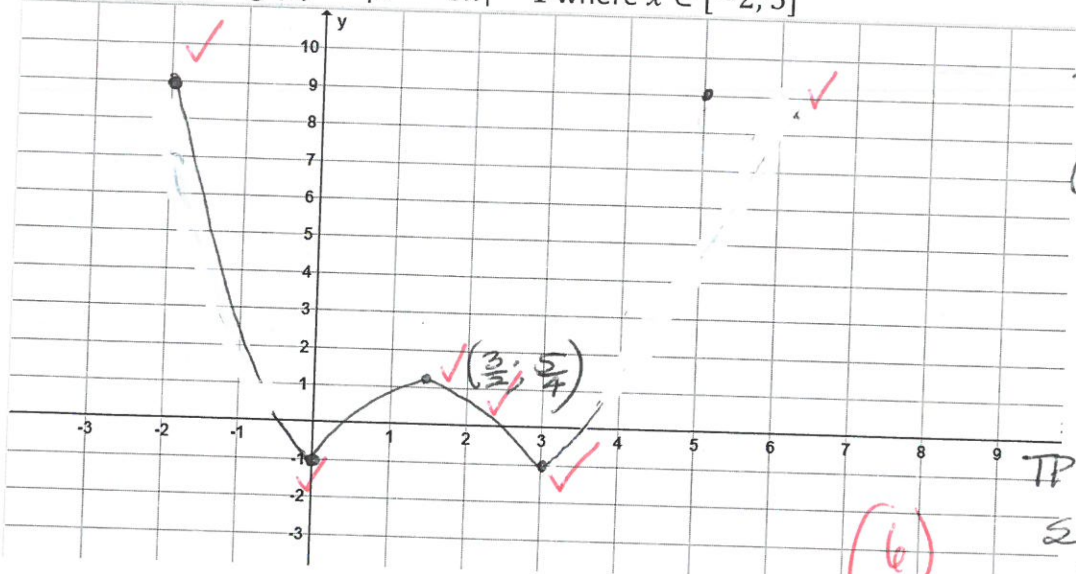
$\lim_{n\to\infty} \frac{1}{n}\sum f(x_i) = \frac{3}{2} + \frac{1}{3} = \frac{11}{6}$

area $= \underline{\underline{\frac{11}{6}}}$

(12)

3

Question 5(b) (i) $g(x) = |x^2 - 3x| - 1$ where $x \in [-2; 5]$

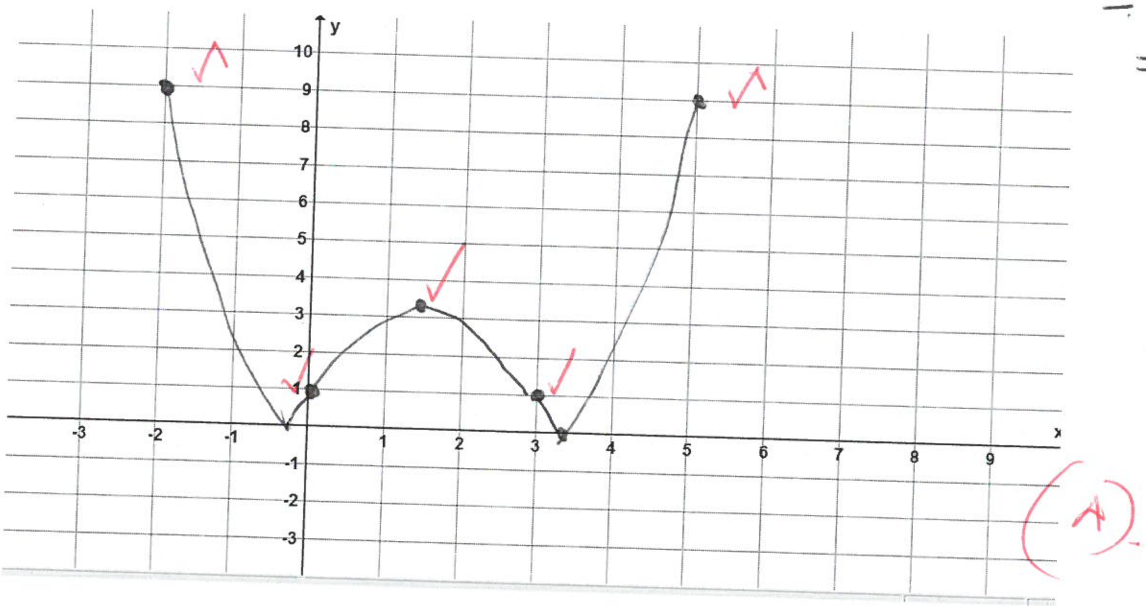



~~Sub $x = -2$~~
~~$g(-2)$~~
~~$= |4+6| - 1$~~
~~$=$~~
can be read off graph.

TP $x = \frac{3}{2}$

Sub.
$g\left(\frac{3}{2}\right) = \left|\frac{9}{4} - \frac{9}{2}\right| - 1$
$= \frac{9}{4} - 1$
$= \frac{5}{4}$

Question 5(b) (ii) $h(x) = |x^2 - 3x - 1|$ where $x \in [-2; 5]$



(4)

## Question 6

$$f(x) = \frac{1}{(5-3x)} = (5-3x)^{-1}$$

(a) $f'(x) = -1 \cdot (5-3x)^{-2} \cdot -3$

$f''(x) = -1 \cdot -2\,(5-3x)^{-3} \cdot -3, -3$

$f^{3}(x) = -1 \cdot -2 \cdot -3\,(5-3x)^{-4} \cdot -3 \cdot -3 \cdot -3$

(6)

(b) $f^{n}(x) = (-1)^{n} \cdot n!\,(5-3x)^{-(n+1)}(-3)^{n}$

(5)

## Question 7.

(a) $f'(x) = 0$ at

$x = -3$ and $x = 2$.

minimum. horizontal point of inflection

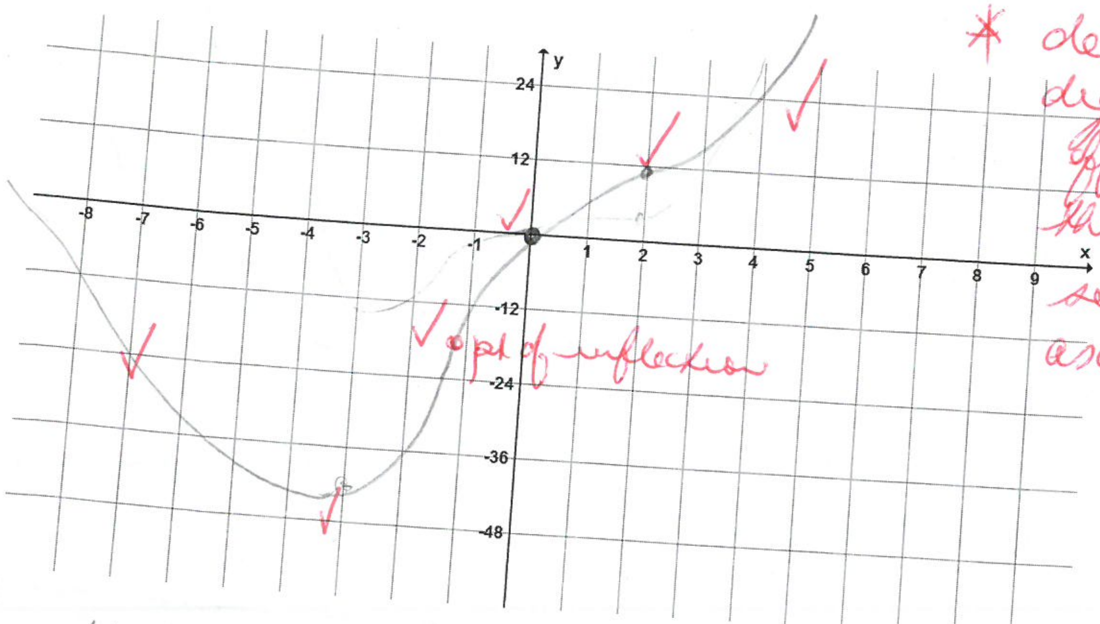
| $x$ | $<-3$ | $-3$ | $(-3;2)$ | $2$ | |
|---|---|---|---|---|---|
| m | $-$ | $0$ | $+$ | $0$ | $+$ |

$\begin{cases} f''(x) = 0 \text{ at } x = 2 \\ f''(x) = 0 \text{ at } x = -1{,}33 \end{cases}$

$f(x)$ passes thro $(0;0)$

(6)

Question 6(b)




* delete digits off this set of axes.

(6)

Sketch

$$y = \frac{x^{4}}{4} - \frac{7}{3}x^{3} - 4x^{2} + 12x$$

$x_1 = -11{,}15$

$x_2 = 0{,}91$

Question 8.

(a) $x < -4$ or $x \in (-4;4)$ or $x > 4$

or $x \in \mathbb{R}$ but $x \neq 4$ or $-4$ (2)

(b) (i) local maximum.

(ii) horizontal point of inflection

(iii) local minimum.

ans. (3)

(c) $h(x) = \dfrac{2x^3}{x^2-16}$

m m m.

$$h'(x) = \frac{(x^2-16).6x^2 - 2x^3(2x)}{(x^2-16)^2} = 0$$

m

$$6x^2(x^2-16) - 4x^4 = 0$$

$$6x^4 - 96x^2 - 4x^4 = 0$$

$$2x^4 - 96x^2 = 0$$

$$x^2(x^2 - 48) = 0$$

$x = 0$ or $x^2 = 48$

$x = \pm 4\sqrt{3}$

Sub. $x = -4\sqrt{3}$ $\qquad$ $x = 4\sqrt{3}$

$h(-4\sqrt{3}) = -20,78$ $\qquad$ $h(4\sqrt{3}) = 20,78$

$P(-4\sqrt{3}; -20,78)$ $\qquad$ $Q(0;0)$ $\qquad$ $R(4\sqrt{3}; 20,78)$

(12)

(d) Vertical asymptotes at $x = 4$ and $x = -4$

Oblique asymptote

$$\begin{array}{r|l} & 2x \\ x^2-16 & 2x^3 \\ & 2x^3 - 32x \\ \hline & 32x \end{array}$$

$y = 2x$

(8)

## Question 9.

$f(x) = x - 20 \qquad g(x) = (\cos x - 2)^3.$

$x - 20 = (\cos x - 2)^3$

$(\cos x - 2)^3 - x + 20 = 0.$

let $a_1 = 8$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}.$$

$h(x) = (\cos x - 2)^3 - x + 20$

$h'(x) = 3(\cos x - 2)^2 \cdot (-\sin x) - 1$

$= -3\sin x(\cos x - 2)^2 - 1$

$a_2 = 8{,}1448518$

$a_3 = 8{,}138404$

$a_4 = 8{,}138393$

$a_5 = 8{,}13839$

(sub $a_5$ into original. $h(8{,}1384) = -9{,}9588 \times 10^{-5}$).

$x = 8{,}1384$

(12)

## Question 10

$x^3 - xy^2 + y^3 = 7.$

(a) Sub. $(2;1)$ into Lhs $\quad 2^3 - 2.1^2 + 1^3$

$= 8 - 2 + 1$

$= 7 = \text{rhs}.$

(2)

(b) * $3x^2 - \left(1.y^2 + x\,2y\frac{dy}{dx}\right) + 3y^2.\frac{dy}{dx} = 0.$

(c) sub $(2;1)$

$12 - \left(1 + 2.2.\frac{dy}{dx}\right) + 3\frac{dy}{dx} = 0.$

$12 - 1 - 4\frac{dy}{dx} + \frac{3dy}{dx} = 0$

$11 = \frac{dy}{dx}$

$\frac{1^2 - 3.4}{3 - 4} = 11$

$y - 1 = 11(x - 2)$

$y = 11x - 21.$

(5)

* $3x^2 - y^2 - 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0.$

$\frac{dy}{dx}(3y^2 - 2xy) = y^2 - 3x^2$

$\frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy}.$

(8)

Question 11.

(a) $\int (3x^2-2)\sqrt{2x^3-4x+7}\,dx$.

let $u = 2x^3-4x+7$

$\frac{du}{dx} = 6x^2-4$

$du = 2(3x^2-2)\,dx$.

$\frac{du}{2} = (3x^2-2)\,dx$.

$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left(\frac{2u^{\frac{3}{2}}}{3}\right) + c$

$= \frac{u^{\frac{3}{2}}}{3} + c$

$= \frac{1}{3}(2x^3-4x+7)^{\frac{3}{2}} + c$

(10)

(b) $\int x\cos x\,dx$. by parts.

$f(x)$ $g'(x)$

$g'(x) = \cos x$

$g(x) = \sin x$.

$f'(x) = 1$

$= x.\sin x - \int \sin x\,dx + c$

$= x\sin x + \cos x + c.$

(9)

(c) $\int 2\sin 3x\cos 2x\,dx$.

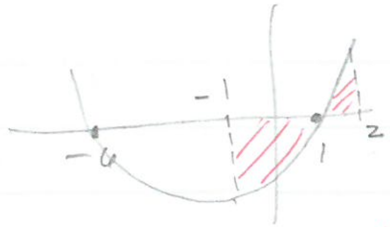
$= \int \cancel{2}.\frac{1}{\cancel{2}}\left[\sin 5x + \sin x\right]dx$

$= -\frac{\cos 5x}{5} - \cos x + c.$

(6)

## Question 12

$y = x^2 + 5x - 6.$
$(x+6)(x-1)$




(a) $\displaystyle\int_{-1}^{2} (x^2 + 5x - 6)\, dx.$

$= \left[\frac{x^3}{3} + \frac{5x^2}{2} - 6x\right]_{-1}^{2}$

$= \left(\frac{8}{3} + 10 - 12\right) - \left(-\frac{1}{3} + \frac{5}{2} + 6\right)$

$= \left(\frac{2}{3}\right) - \left(\frac{49}{6}\right) = -\frac{15}{2}$

(8)

(b) area

$= \left|\displaystyle\int_{-1}^{1} (x^2 + 5x - 6)\, dx\right| + \displaystyle\int_{1}^{2} (x^2 + 5x - 6)\, dx.$

$= \left|\left[\frac{x^3}{3} + \frac{5x^2}{2} - 6x\right]_{-1}^{1}\right| + \left[\frac{x^3}{3} + \frac{5x^2}{2} - 6x\right]_{1}^{2}$

$\left|\left(\frac{1}{3} + \frac{5}{2} - 6\right) - \left(-\frac{1}{3} + \frac{5}{2} + 6\right)\right| + \left[\left(\frac{8}{3} + 10 - 12\right) - \left(\frac{1}{3} + \frac{5}{2} - 6\right)\right]$

$\left|-\frac{19}{6} - \left(\frac{49}{6}\right)\right| + \left[\frac{2}{3} - \left(-\frac{19}{6}\right)\right.$

$= \frac{68}{6} + \frac{23}{6}.$

$= \frac{91}{6}\ u^2.$

(6)

Question 13

Shaded region $= \pi \int_4^6 (x+4)^2\,dx + \pi \int_2^4 (x^2-2x)^2\,dx$

$\int (x+4)^2 = \int_4^6 x^2 + 8x + 16$ $\qquad \int (x^2-2x)^2 = \int_2^4 x^4 - 4x^3 + 4x^2.$

$$\pi\left\{\left[\frac{x^3}{3} + \frac{8x^2}{2} + 16x\right]_4^6 + \left[\frac{x^5}{5} - \frac{\cancel{4}x^4}{\cancel{4}} + \frac{4x^3}{3}\right]_2^4\right\}$$

$$= \pi\left[\left(\frac{6^3}{3} + \frac{8.6^2}{2} + 16.6\right) - \left(\frac{4^3}{3} + \frac{8.4^2}{2} + 16.4\right) + \boxed{\frac{4^5}{5} - 4^4 + \frac{4.4^3}{3} - \left(\frac{32}{5} - 16 + \frac{32}{3}\right)}\right]$$

$$V = \pi\left(\frac{2936}{15}\right) u^3.$$

$\pi\left(\frac{488}{3} + \frac{496}{15}\right)$

$\pi\ \frac{2936}{15}\ u^3.$

(11)

Question 14.

(a) area shaded = area sector − area $\triangle RQO$

$= \frac{1}{2} r^2\theta - \frac{1}{2} QO.\, r.\, \sin\theta$ ①

$= \frac{1}{2} r^2\theta - \frac{1}{2} r^2 \cos\theta \sin\theta$

$= \frac{1}{2} r^2 (\theta - \cos\theta \sin\theta)$

$\frac{QO}{r} = \cos\theta$

$QO = r\cos\theta.$

Sub in ①.

(6)

(b) area $= \frac{1}{2} \cdot 10^2 \left(\frac{\pi}{3} - \cos\frac{\pi}{3} \sin\frac{\pi}{3}\right)$

$= 50\left(\frac{\pi}{3} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}\right)$

$= \frac{50\pi}{3} - \frac{25\sqrt{3}}{2}$

(4)

$= 30{,}7$ cm