

Beaulieu College



Mathematics Department

GRADE 12

ADVANCED PROGRAMME MATHEMATICS

Preliminary Examination Paper 1

ALGEBRA & CALCULUS

Time: 2 Hours

200 marks

Date: 27 July 2017

Examiner: Ms A Smith

Moderator: Mr J Ruiz-Mesa

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 9 pages and an Information Booklet of 2 pages (i-ii). Please check that your question paper is complete.
 2. Answer all the questions on the folio pages.
 3. Approved, non-programmable, non-graphical calculators may be used, unless otherwise indicated.
 4. Work neatly and show all the necessary steps in your calculations.
 5. Diagrams have not been drawn to scale.
 6. Trigonometric calculations should be done using radians and answers should be given in radians.
 7. Write your name on the question paper and folio pages.
 8. Round off your answers to two decimal digits, unless otherwise indicated.
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NAME: _____

MARKING GRID

Question	Algebra	Calculus
1	/26	
2	/28	
3		/26
4	/12	
5		/16
6		/17
7		/13
8	/18	
9		/34
10		/10
	/84	/116
	TOTAL	/200

Question 1

1.1 Solve for $x \in \mathbb{R}$, without the use of a calculator:

(a) $|x-2| > 2|x+2|$ (10)

(Hint: Use a diagram)

(b) $(\ln x)^2 + \ln x^2 = 15$ (6)

(c) $\sec x = 2 \quad x \in (0; 2\pi)$ (4)

1.2 Determine the domain and range of the graph of:

$$y = e^{\sqrt{x^2-4}} \quad (6)$$

[26]

Question 2

2.1 Simplify: $i^{10} \times i^3$ (2)

2.2 Determine the real values of a and b such that $(2+ai)(-3+4i) = 14+bi$ (7)

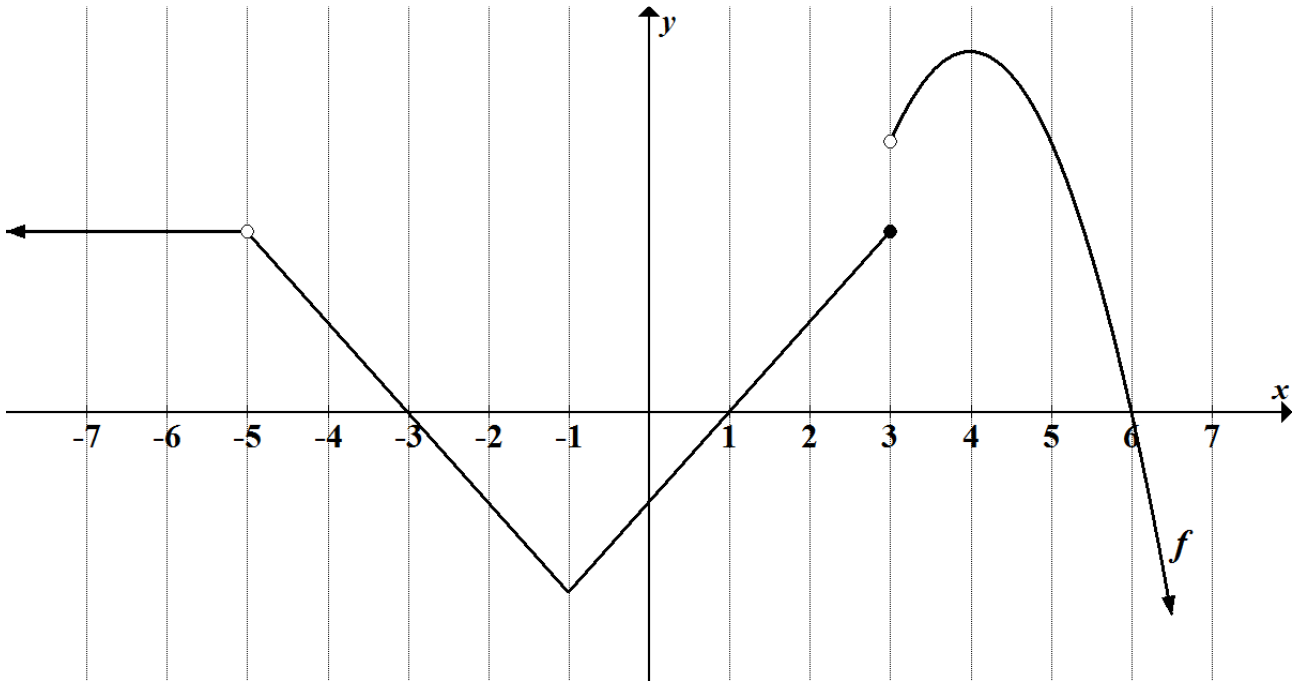
2.3 One solution to the equation $2x^3 + px^2 + qx - 75 = 0$ is $4+3i$.
Calculate the values of p and q respectively. (9)

2.4 Decompose $\frac{2x-23}{2x^2+9x-5}$ into its partial fractions. (10)

[28]

Question 3

3.1 The function of f is represented by the graph below:



(No reasons are required in these questions)

Give in each of the following questions all the values of x for which:

- (a) the limit exists, but the function is not defined. (2)
- (b) the left- and right-hand limits both exist, but they are unequal. (2)
- (c) f is continuous, but not differentiable. (2)
- (d) $f'(x) = 0$ (3)
- (e) $f''(x) < 0$ (3)

3.2 A function is defined as follows:

$$f(x) = \begin{cases} x^2 + a & \text{if } x < 3 \\ bx - 14 & \text{if } x \geq 3 \end{cases}$$

Calculate the value(s) of a and b if f is differentiable at $x = 3$. (8)

3.3 If it is given that $f(x) = \ln(x - 2)$ and $g(x) = |x|$, sketch the graph of $y = g(f(x))$. (6)

[26]

Question 4

Use mathematical induction to prove that $5^n + 2 \times 11^n$ is always divisible by 3 for $n \in \mathbb{N}$. (12)

[12]

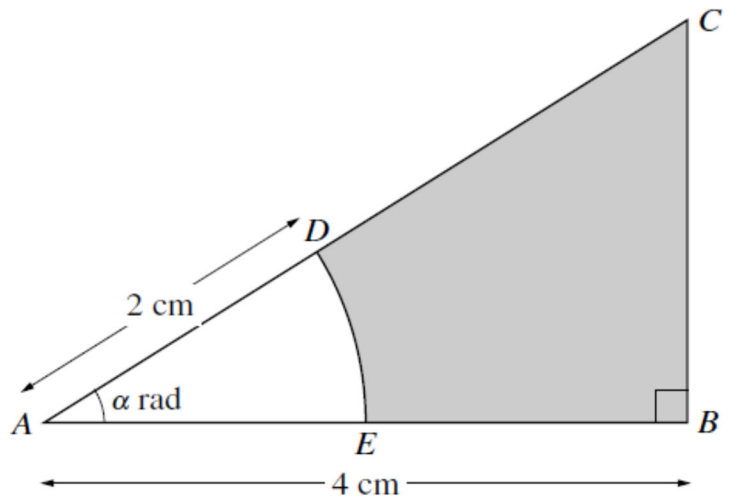
Question 5

The diagram shows $\triangle ABC$ in which

$AB \perp BC$. $AB = 4$ cm and

$\hat{CAB} = \alpha$ radians.

The arc DE with centre A and radius 2 cm meets AC at D and AB at E .



Determine, in terms of α :

5.1 the area of the shaded region. (8)

5.2 the perimeter of the shaded region. (8)

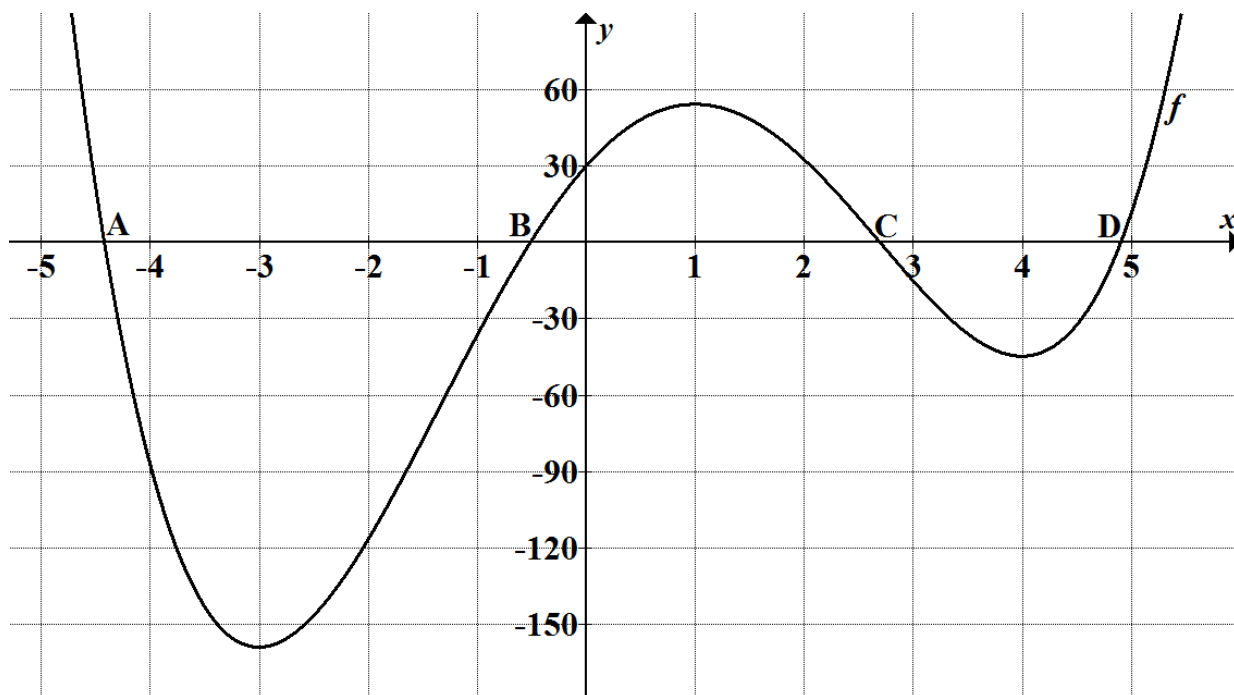
[16]

QUESTION 6

- 6.1 Determine the derivative of $\frac{\cos 5x}{(3x-4)^6}$. (You do not need to simplify your answer.) (6)
- 6.2 Determine the equation of the tangent to the curve $xy^2 - x^2 = 3$ at the point $(1; -2)$. (11)
- [17]**
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QUESTION 7

The graph of $f(x) = x^4 - \frac{8x^3}{3} - 22x^2 + 48x + 30$ is shown, with stationary points at $x = -3$, $x = 1$ and $x = 4$. The graph has x -intercepts at the points indicated by A, B, C and D.



- 7.1 Without first solving the equation, state with clear justification which of the intercepts, A, B, C or D, will be found using Newton's method with an initial approximation of $x_0 = 3,9$. (3)

7.2 State any restrictions on the initial approximation of x . (2)

7.3 Given $x_0 = -1$, determine the x -intercept at B, correct to 6 decimal places. (8)

[13]

Question 8

Given: $g(x) = \frac{2x^2 + 7x - 15}{x + 2}$

8.1 Determine the equation of the vertical asymptote. (2)

8.2 Determine the equation of the oblique asymptote. (8)

8.3 Prove that the graph of g has no stationary points. (8)

[18]

Question 9

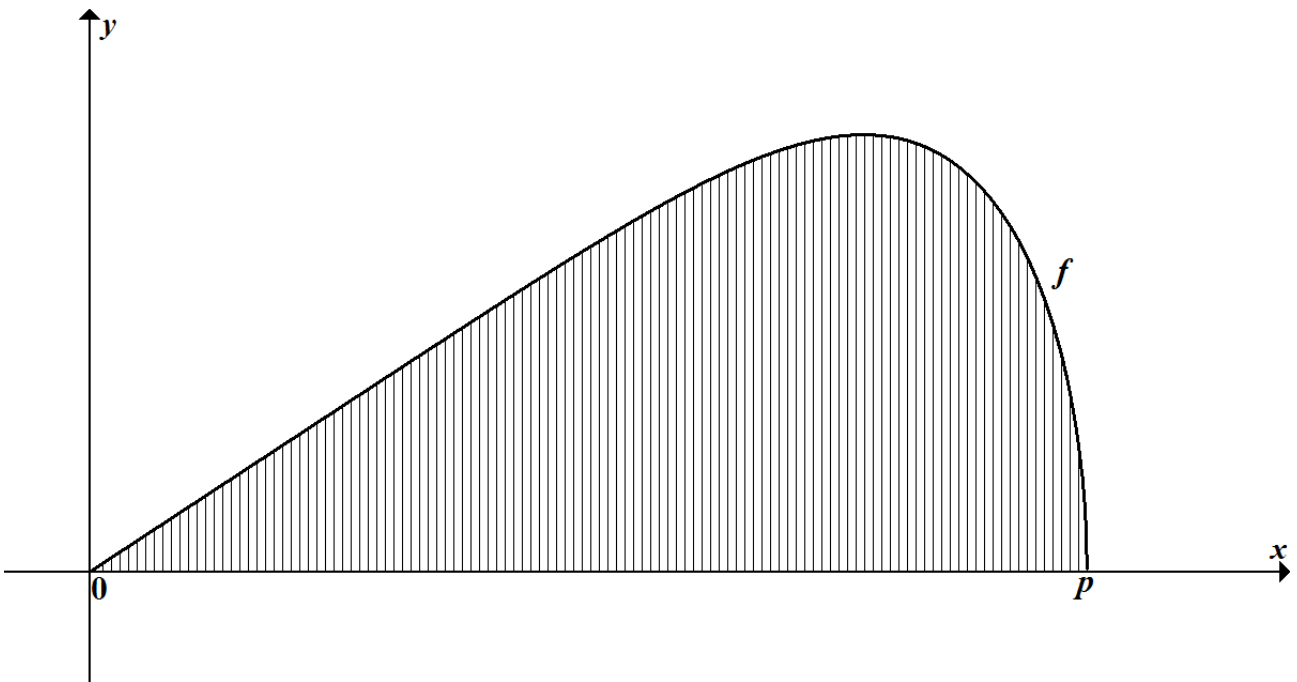
9.1 Integrate the following:

(a) $\int \sin 3x \cdot \sin 4x \, dx$ (6)

(b) $\int \frac{2x}{\sqrt[3]{5x+3}} \, dx$ (10)

(c) $\int \cot^3 2x \cdot \operatorname{cosec}^2 2x \, dx$ (8)

9.2 The graph of $f(x) = x\sqrt{\cos(x^3)}$ is shown below for the interval $x \in [0; p]$.



When the shaded area is rotated 360° about the x -axis, it forms a three dimensional shape, similar to the spinning top shown in the picture alongside.



Determine, in terms of p , the volume of the solid generated by rotating the shaded area about the x -axis.

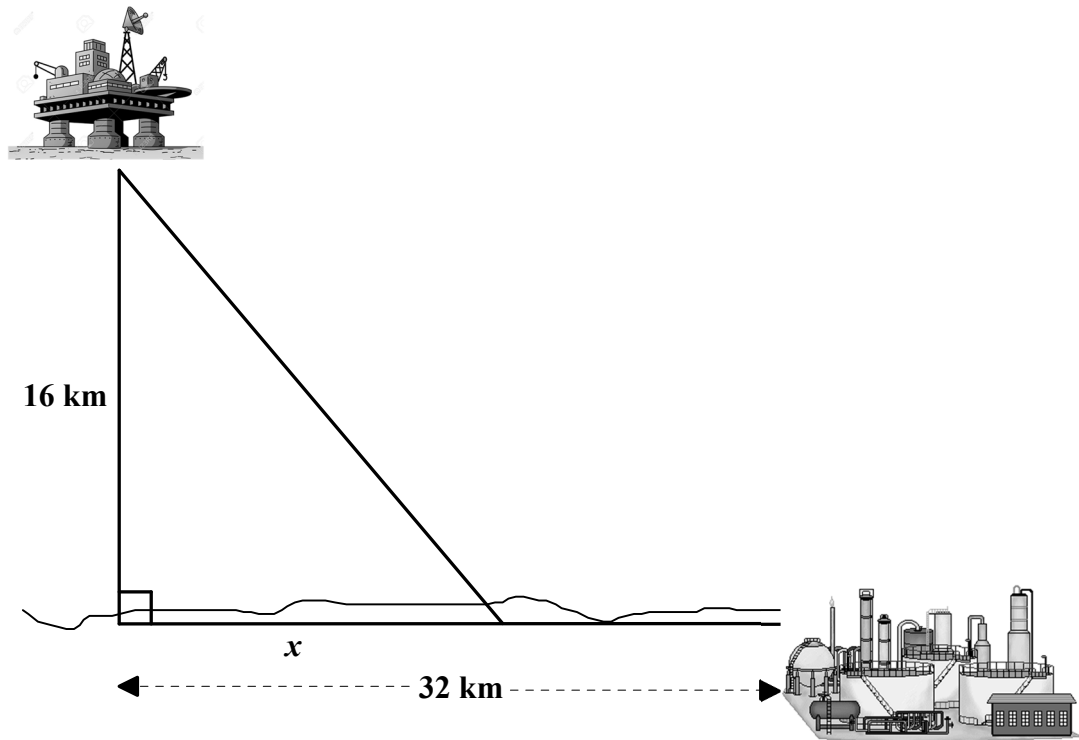
(10)

[34]

(Please turn over for Question 10.)

Question 10

An oil rig is 16 km out to sea and the refinery that the oil needs to go to is 32 km up the shore line from the oil rig. It costs R200 million per kilometre to place a pipeline in the water and R75 million per kilometre on land.



Set up an equation that you would use to determine how far down the shore line (the distance x) the pipeline should make landfall in order to minimise the cost.

You do not have to solve the equation to determine the value of x . (10)

[10]

Total: [200]