

**PRELIMINARY EXAMINATION 2017**

**GRADE 12 - ADVANCED PROGRAMME MATHEMATICS**

Time: 2 hours

Total: 200

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

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1. This question paper consists of 5 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. **Answers must be rounded off to two decimal places in SECTION A**
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

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**QUESTION 1: (all working must be clearly shown)**

a) Solve for  $x$  for the following equations and inequalities:

1)  $\left| \frac{x+2}{3} \right| - 1 < 2$  (4)

2)  $\ln(x + 8) - \ln 7 = 3$  (4)

3)  $2e^{5x} - 7e^{2x} - 15e^{-x} = 0$  (6)

b) If  $f(x) = \sqrt{x + 3}$  and  $g(x) = \sqrt{x - 2}$ .  
Write down the domain of  $f(x) + g(x)$  (3)  
**[17]**

**QUESTION 2**

Given  $f(x) = 3 - \frac{3}{2} \ln \sqrt{x + 4}$

a) Calculate the equation  $f^{-1}(x)$  (6)

b) Sketch the graph of  $f^{-1}(x)$ , clearly indicating the values of the intercepts with the axes correct to 1 decimal place, and asymptotes. (6)

c) Write down the values of  $x$  if  $f^{-1}(x) \cdot f'(x) < 0$  (2)  
**[14]**

**QUESTION 3:**

a) It is given that  $f(x) = 1 - \frac{7}{x^2}$ :

1) Use the Newton-Raphson method with a first approximation  $x_1 = 2.5$ , to find the next approximations  $x_2$  and  $x_3$  to a root of  $f(x) = 0$ . Give answer correct to 6 decimal places. (6)

2) The root of  $f(x) = 0$  for which  $x_1$ ,  $x_2$  and  $x_3$  are approximations is denoted by  $\alpha$ . Write down the value of  $\alpha$  (1)

3) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . Find  $e_3$ , giving your answers correct to 6 decimal places. (1)

b) Determine the following limits

1)  $\lim_{x \rightarrow -3} \left( |x + 1| + \frac{3}{x} \right)$  (3)

$$2) \lim_{x \rightarrow 3^-} \frac{x^2|x-3|}{x-3} \quad (4)$$

$$3) \lim_{x \rightarrow -\infty} \sqrt[3]{\frac{x-3}{5-x}} \quad (5)$$

[20]

**QUESTION 4:**

Prove that for all positive integers of  $n$  that:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = \frac{2^n - 1}{2^n} \quad [12]$$

**QUESTION 5:**

$$a) \text{ Given } w(x) = \begin{cases} \frac{|x|}{2}, & x \leq -4 \\ \sqrt{-x}, & -4 < x < 0 \\ |x - 2|, & x \geq 0 \end{cases}$$

1) Sketch  $w(x)$  (8)

2) Write down the value of  $x$  where the function is discontinuous and state the type of discontinuity. (2)

3) Write down the values of  $x$  where the function is not differentiable. (4)

b) Determine a formula for the  $n^{\text{th}}$  derivative of  $(ax + b)^m$  (8)

[22]

**QUESTION 6:**

a) If  $f(3) = 3$  and  $g(3) = 5$  and  $f'(3) = 2$  and  $g'(3) = 4$ , Find the derivative at  $x = 3$  for:

1)  $f(x) \cdot g(x)$  (4)

2)  $f(g(x))$  (4)

3)  $\frac{g(x)}{f(x)}$  (6)

b) Find  $\frac{dy}{dx}$  for the following:

1)  $\sin(\sin 2x)$  (4)

2)  $(x^2 + 3y^2)^5 = 2y$  (10)

[28]

**QUESTION 7:**

Given the graph of  $f(x) = \frac{x^3-1}{2(x^2-1)}$

- a) Find the coordinates of the stationary points and intercepts with the axis(8)
- b) Find equations of any asymptotes (6)
- c) Sketch the graph of  $f(x)$  (9)

**[25]**

**QUESTION 8:**

a) Evaluate the following integrals without the use of a calculator:

1)  $\int (x + 1)^{-n} \cdot dx$  (4)

2)  $\int_1^2 \frac{x}{\sqrt{x^2+1}} \cdot dx$  (8)

3)  $\int \frac{\sin\sqrt{x}}{\sqrt{x}} \cdot dx$  (8)

4)  $\int x \cdot \sin x \cdot \cos x \, dx$  (10)

c) Let R be the region in the first quadrant enclosed by  $y = \sqrt{x - 1}$ ,  
 $y = x - 7$  and x-axis.

1) Sketch the above graphs on same set of axes indicating the points of intersection between the two functions. (5)

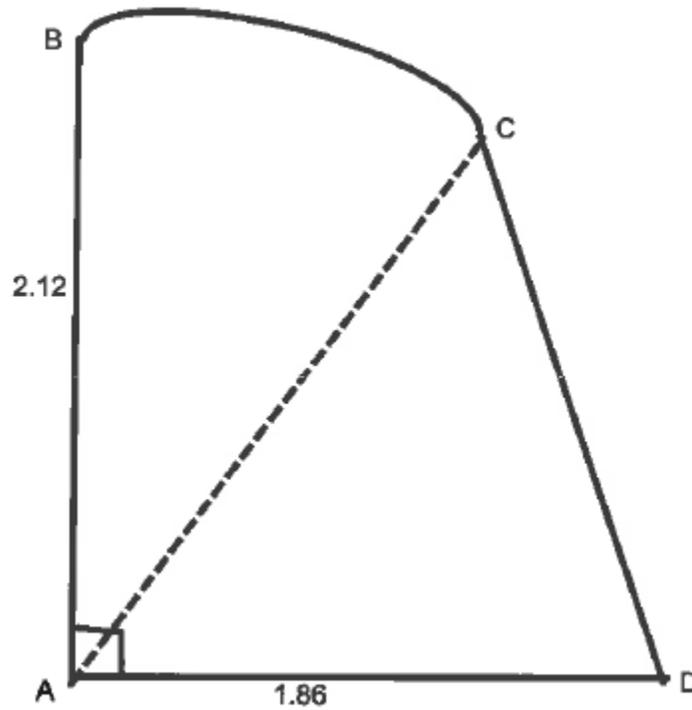
2) Determine the exact area of the region R. (8)

3) If R is rotated about x-axis, calculate the resulting exact volume. (10)

**[53]**

**QUESTION 9:**

The diagram below shows a cross section ABCD of a small shed. The straight line AB is vertical and has length 2.12m. The straight line AD is horizontal and has length 1.86m. The curve BC is an arc of a circle with centre A, and CD is a straight line.



Given the size of  $\widehat{BAC}$  is 0.65 radians, calculate:

- a) The length of the BC (2)
  - b) The area of the sector BAC (3)
  - c) The size of  $\widehat{DAC}$  (2)
  - d) The area of the cross section ABCD of the shed (4)
- [11]**



**PRELIMINARY EXAMINATION 2017**

**GRADE 12 - ADVANCED PROGRAMME MATHEMATICS**

Time: 1 hour

Total: 100

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

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1. This question paper consists of 4 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
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5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
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**QUESTION 1:**

- a) Suppose that you wake up for work in the dark and find that the lights don't work in your bedroom. In addition, your sock drawer is a mess and contains 12 black socks, 17 blue socks and 5 red socks that otherwise look alike.
- 1) What is the probability that you randomly select two black socks if you select exactly two. (4)
  - 2) Write down the probability distribution of selecting red socks if 6 socks are selected in total. (7)
- b) Sasha, an expert, uses a double-barrelled shotgun. She shoots at each target with the gun's first barrel and, only if she misses, does she then shoot at the target with the gun's second barrel.

The probability that she hits a target with a shot using her gun's first barrel is 0.85. The conditional probability that she hits a target with a shot using her gun's second barrel, given that she has missed the target with a shot using her gun's first barrel is 0.80. Assume that Sasha's shooting is independent from target to target.

- 1) What is the probability that Sasha hits the target? (3)
- 2) Determine the probability that in a series of shots at 15 targets, Sasha hits at least 14 targets. (6)

**[20]**

**QUESTION 2:**

- a) 7 players play in a chess league. At the end of the season medals are presented to those that came first, second and third. How many different possible results are possible? (4)
- b) There are 9 dots randomly placed on a circle.



How many triangles can be formed within the circle?

(5)  
**[9]**

**QUESTION 3:**

a) A random variable  $X$  has density function

$$f(x) = \begin{cases} cx^2 & 1 \leq x \leq 2 \\ cx & 2 < x < 3 \\ 0 & \text{otherwise} \end{cases} \quad \text{where } c \text{ is a constant.}$$

Find:

1)  $c$  (5)

2)  $P(\frac{1}{2} \leq X < \frac{5}{2})$  (5)

b) Given the following summary data,

$$\sum x = 367 \quad \sum y = 270 \quad \sum x^2 = 33\,845 \quad \sum y^2 = 12\,976$$

$$\sum xy = 17\,135 \quad n = 6$$

Calculate the equation of the line of best fit and comment on the data.

(10)

**[20]**

**QUESTION 4:**

a) Becky owns a taxi. Each weekday morning she collect Steve from his home and takes him to the train station. A record of the times,  $x$  minutes, for a random sample of 65 such taxi journeys is summarized by:

$$\sum x = 1326 \quad \sum (x - \bar{x})^2 = 400.24$$

1) Construct a 98% confidence interval for the mean (7)

2) Comment on Becky's claim that the mean journey time is more than 20 minutes (2)

b) During 2012, a tabloid newspaper invited its readers to take part in a telephone poll regarding whether the bank holiday at the beginning of May should be moved to October. Of the 2450 who phoned in, 1029 thought that the bank holiday should be moved.

1) Calculate an approximate 99% confidence interval for the proportion in the corresponding population who are not in favour of the move. (7)

2) Give a reason in context why it could be misleading to use the confidence interval above to draw conclusions about general public opinion on this issue. (1)

**[17]**

### **QUESTION 5**

- a) The top 5% of applicants (as measured by GRE scores) will receive scholarships. If  $GRE \sim N(500, 100^2)$ , how high does your GRE score have to be to qualify for a scholarship? (5)
- b) A new tax law is expected to benefit “middle income” families those with incomes between R10 000 and R16 000. If Family income  $\sim N(13\ 000, 1000^2)$ , what percentage of the population will benefit from the law? (5)
- c) The heights of male freshmen entering a large state university are normally distributed with a mean of 168 *cm* ( $\mu = 168$ ). It is know that about 2% of the freshmen are taller than 172 *cm*. What is the standard deviation of distribution of heights? (5)

**[15]**

### **QUESTION 6:**

- a) What two assumptions must be met when one is using the z-test to test differences between two means? (2)
- b) The diameter of pin in a small motor is supposed to be 5 *mm*. If the pin is either too small or too large, the motor will not perform properly. The manufacturer measures the diameter in a sample of 60 motors to determine whether the mean diameter has moved away from the target. They found the mean diameter of the sample to be 4.951 *mm*. From past studies it is known that the standard deviation is 0.2 *mm*. At  $\alpha = 4\%$ , do you have enough evidence to conclude that the mean diameter is not 5 *mm*? (8)
- c) The mean weekly earnings of a sample of 30 construction workers was R2 789, with a standard deviation of R173, and the mean weekly earnings of a sample of 28 manufacturing workers was R2 658, with a standard deviation of R165. Is it reasonable to say that the construction workers earn more than the manufacturing workers, at  $\alpha = 0.03$ . (9)

**[19]**