MEMORANDUM – JULY EXAM

A. P. MATHS – 2017

SECTION A

QUESTION 1.

a.
1. $\left|\frac{x+2}{3}\right| - 1 < 2$

$x+2 = 9$
$\underline{x = 7}$ ✓

$-x-2 = 9$
$\underline{x = -11}$ ✓

(4)

$-11 < x < 7$ ✓✓

2. $\ln(x+8) - \ln 7 = 3$

$\ln\left(\frac{x+8}{7}\right) = 3$ ✓

$\frac{x+8}{7} = e^3$ ✓✓

$\underline{x = 7e^3 - 8}$ ✓

(4)

3. $2e^{3x} - 7e^{2x} - 15e^{-x} = 0$

$2e^{6x} - 7e^{3x} - 15 = 0$ ✓✓

$e^{3x} = 5$ ✓ $e^{3x} \neq -3/2$ ✓

$3x = \ln 5$

$x = \frac{\ln 5}{3}$ ✓✓

(6)

b. $f(x) = \sqrt{x+3}$; $g(x) = \sqrt{x-2}$

$f(x) + g(x) = \sqrt{x+3} + \sqrt{x-2}$ ✓

∴ Domain $x \geq 2$ ✓✓

(3)

QUESTION 2:

$f(x) = 3 - 3/2 \ln\sqrt{x+4}$

a. $x = 3 - 3/2 \ln\sqrt{y+4}$ ✓

$3/2 \ln\sqrt{y+4} = 3 - x$

$\ln\sqrt{y+4} = 2/3(3-x)$ ✓

$(y+4)^{1/2} = e^{2/3(3-x)}$ ✓

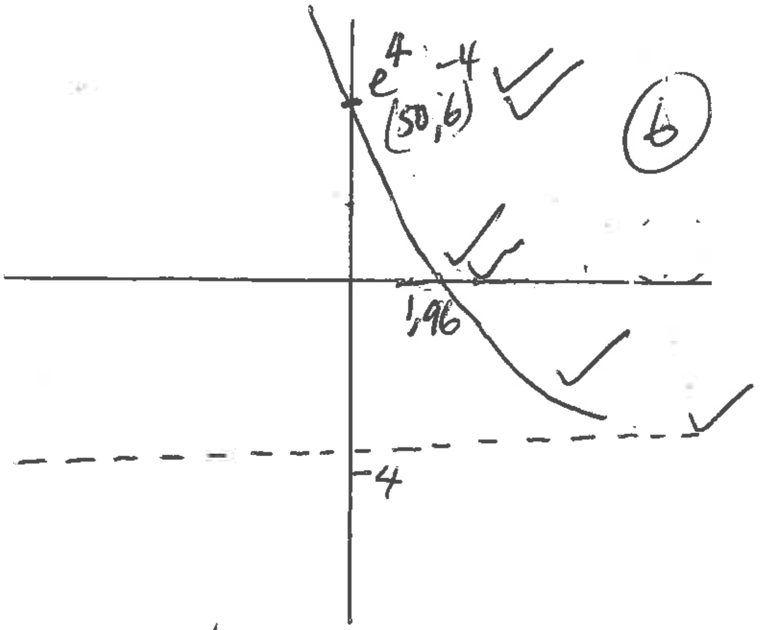
$y + 4 = e^{4/3(3-x)}$ ✓

(6)

∴ $f^{-1}(x) = e^{4/3(3-x)} - 4$ ✓✓

b) Sketch of $f^{-1}(x)$

$$f^{-1}(x) = e^{4/3(3-x)} - 4$$



(6)

c. $f^{-1}(x) \cdot f'(x) < 0$

$\underline{-4 < x < 1{,}96}$

(2)

QUESTION 3

a. $f(x) = 1 - 7/x^2$

1. $f'(x) = 14x^{-3}$

$x_1 = 2{,}5$

$$x_2 = 2{,}5 - \frac{1 - 7(2{,}5)^{-2}}{14(2{,}5)^{-3}}$$

(6)

$= \underline{2{,}633929}$

$$x_3 = 2{,}633929 - \frac{1 - 7(2{,}633929)^{-2}}{14(2{,}633929)^{-3}}$$

$= \underline{2{,}645672}$

2. $\underline{\alpha = 2{,}645751}$ (1)

3. $e_3 = 2{,}645751 - 2{,}645672$

$= \underline{0{,}000079}$ (1)

b. 1. $\lim_{x \to -3} |x+1| + \frac{3}{x}$

$= |-3+1| + \frac{3}{-3}$

(3)

$= \underline{\underline{1}}$

2. $\lim_{x \to 3^-} \frac{x^2|x-3|}{x-3}$

$\lim_{x \to 3^-} \frac{-x^2(x-3)}{x-3}$

(4)

$= \underline{\underline{-9}}$

3. $\lim_{x \to -\infty} \sqrt[3]{\frac{x-3}{5-x}}$

$= -1$

(5)

QUESTION 4

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=\frac{2^n-1}{2^n}$$

Prove true for $n=1$ ✓

$LHS = \frac{1}{2}$ $RHS = \frac{1}{2}$ ✓

$\therefore LHS = RHS$.

Assume true for $n=k$ ✓

$$\frac{1}{2}+\cdots+\frac{1}{2^k}=\frac{2^k-1}{2^k}$$ ✓

Prove true for $n=k+1$ ✓

$$LHS=\frac{1}{2}+\cdots+\frac{1}{2^k}+\frac{1}{2^{k+1}}$$ ✓

$$=\frac{2^k-1}{2^k}+\frac{1}{2^{k+1}}$$ ✓

$$=\frac{2^{k+1}-2+1}{2^{k+1}}$$ ✓

$$=\frac{2^{k+1}-1}{2^{k+1}}$$ ✓

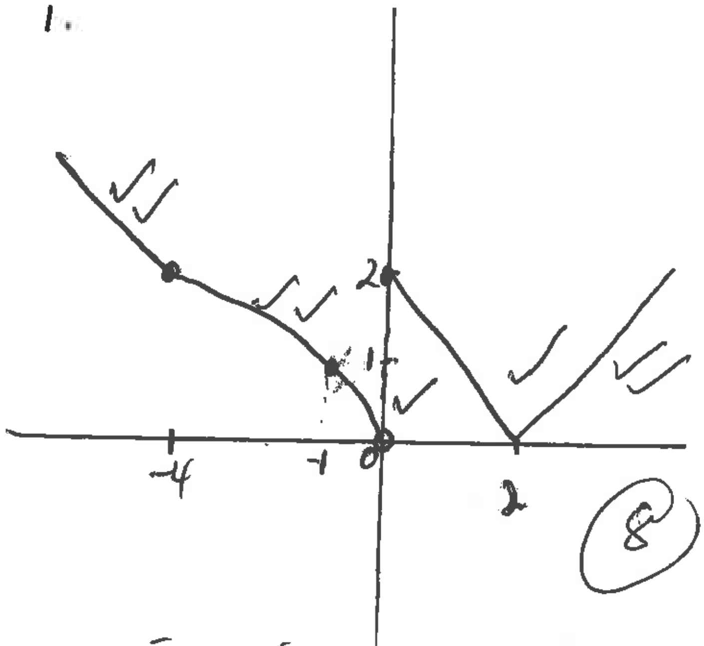
12

$\therefore$ By PMI; it is true $\forall n \in Z$.

QUESTION 5:

$$w(x)=\begin{cases}\frac{|x|}{2} & x\le -4\\ \sqrt{-x} & -4<x<0\\ |x-2| & x\geq 0\end{cases}$$

1.



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2. Discontinuous: ✓

$x=0$ jump discontinu ✓

2

3. Not differentiable.

$x=0$ ✓ $x=2$ ✓

$x=-4$ ✓

4

b. n^{th} derivative.

$(ax+b)^m$

$f'(x)=m(ax+b)^{m-1}\cdot a$ ✓

$f''(x)=m(m-1)(ax+b)^{m-2}\cdot a^2$ ✓

$f'''(x)=m(m-1)(m-2)(ax+b)^{m-3}\cdot a^3$ ✓

$$f^n(x)=\frac{m!}{(m-n)!}(ax-b)^{m-n}\cdot a^n$$ ✓

; $m\geq n$ ✓

8

Question 6:

a. $f(x)\cdot g(x) = y$

1. $\frac{dy}{dx} = f(x)\, g'(x) + g(x)\cdot f'(x)$

$= 3\cdot 4 + 5\cdot 2$

$= \underline{\underline{22}}$ (4)

2. $f(g(x)) = y$

$\frac{dy}{dx} = f'(g(x))\cdot g'(x)$

$= f'(5)\cdot 4$ (4)

$= \underline{\underline{4f'(5)}}$

3 $\frac{g(x)}{f(x)} = y$

$\frac{dy}{dx} = \frac{f(x)\cdot g'(x) - g(x)\cdot f'(x)}{(f(x))^2}$

$= \frac{3\cdot 4 - 5\cdot 2}{9}$ (6)

$= \underline{\underline{2/9}}$

b. $y = \sin(\sin 2x)$

$f'(x) = \cos(\sin 2x)\cdot \cos 2x \cdot 2$ (4)

2. $5(x^2+3y^2)^4(2x + 6y\frac{dy}{dx}) = 2\frac{dy}{dx}$

$5(x^2+3y^2)^4\cdot 6y\frac{dy}{dx} - 2\frac{dy}{dx} = -10x(x^2+3y^2)^4$

(10)

$\therefore \frac{dy}{dx} = \frac{-10x(x^2+3y^2)^4}{30y(x^2+3y^2)^4 - 2}$

Question 7:

$f(x) = \frac{x^3-1}{2(x^2-1)} = \frac{\cancel{(x-1)}(x^2+x+1)}{2\cancel{(x-1)}(x+1)}$

$= \frac{x^2+x+1}{2(x+1)}$

a) y int: $x=0$ $y=1/2$ (8)

$f'(x) = \frac{2(x+1)(2x+1) - (x^2+x+1)2}{4(x+1)^2}$

$2(2x^2+3x+1) - (2x^2+2x+2) = 0$

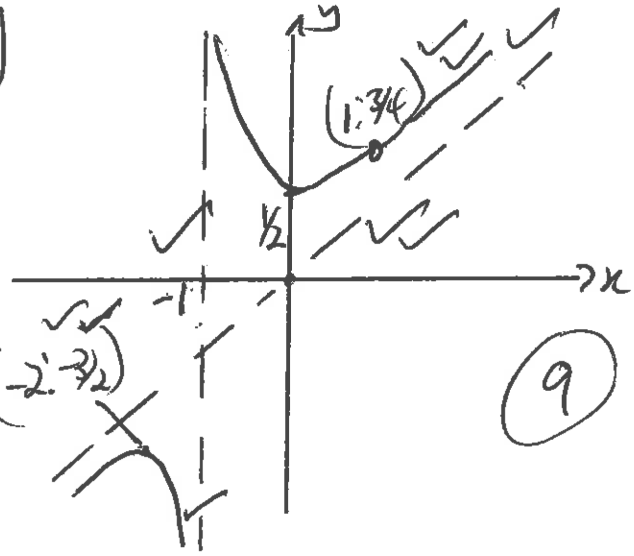
$2x^2+4x = 0$

$\underline{x=0}$ $\underline{x=-2}$

$y=1/2$ $\underline{y=-3/2}$

b. vertical Asymptote $\underline{x=-1}$

OA: $\underline{y = \frac{1}{2}x}$ (6)

c) 

(9)

QUESTION 8

a) 1. $\int (x+1)^{-n}\,dx = \dfrac{(x+1)^{-n+1}}{-n+1} + c$

(4)

2) $\int_1^2 \dfrac{x}{\sqrt{x^2+1}}\,dx$

let $u = x^2+1$

$du = 2x\,dx$

$= \int_2^5 u^{-\frac{1}{2}} \cdot \dfrac{du}{2}$

$= \dfrac{u^{\frac{1}{2}}}{\frac{1}{2}\times 2}\Big|_2^5$

$= \sqrt{5} - \sqrt{2}$

(8)

3. $\int \dfrac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

let $u = \sqrt{x}$

$du = \frac{1}{2} x^{-\frac{1}{2}}\,dx$

$2du = \dfrac{dx}{\sqrt{x}}$

$= \int \sin u \cdot 2du$

(8)

$= -2\cos u + c$

$= -2\cos\sqrt{x} + c$

4 $\int x \sin x \cos x\,dx$

$= \int x \dfrac{\sin 2x}{2}\,dx$

$= \frac{1}{2}\int x \sin 2x\,dx$

$f(x) = x \quad g'(x) = \sin 2x$

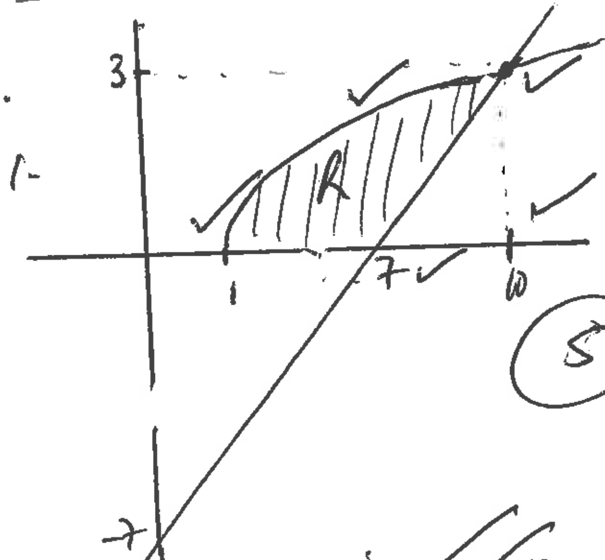
$f'(x) = 1 \quad g(x) = -\dfrac{\cos 2x}{2}$

$= \frac{1}{2}\left[-\dfrac{x\cos 2x}{2} + \int \dfrac{\cos 2x}{2}\,dx\right]$

(10)

$= \frac{1}{2}\left[-\dfrac{x\cos 2x}{2} + \dfrac{\sin 2x}{4}\right] + c$

C.



(5)

2. Area $= \int_1^7 (x-1)^{\frac{1}{2}}\,dx + \int_7^{10} (x-1)^{\frac{1}{2}} - \dfrac{9}{2}$

$= \dfrac{(x-1)^{3/2}}{3/2}\Big|_1^{10} - \dfrac{9}{2}$

(8)

$= \dfrac{27}{2}$

3. Volume

$$= \pi \left[\int_1^{10} (\sqrt{x-1})^2 - \int_7^{10} (x-7)^2 \, dx \right]$$

$$= \pi \left[\left(\frac{x^2}{2} - x \right) \Big|_1^{10} - \left(\frac{x^3}{3} - \frac{14x^2}{2} + 49x \right) \Big|_7^{10} \right]$$

$$= \pi \left[\frac{81}{2} - 9 \right]$$

$$= \frac{63\pi}{2}$$

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QUESTION 9

a) $BC = 2{,}12(0{,}65) = 1{,}378$ 2

b) Area of sector BAC

$$= \frac{(2{,}12)^2 \cdot 0{,}65}{2} = 1{,}46$$ 2

c) $D\hat{A}C = 0{,}9208$ 2

d) Area of $\triangle ADC =$

$$\frac{1}{2} \cdot (1{,}86)(2{,}12) \sin 0{,}9208$$

$$= 1{,}569$$ 4

$\therefore$ Total Area $= 1{,}5696 + 1{,}46$

$$= 3{,}03$$

STATS - MEMO - 2017

QUESTION 1:

a1. $\frac{{}^{12}C_2 \, {}^{22}C_0}{{}^{34}C_2} = \frac{2}{17}$ (0,1176) (4)

2. $P(X = \text{red}) = \frac{{}^{5}C_x \, {}^{29}C_{6-x}}{{}^{34}C_6}$ (7)

for $x \in [0;1;2;3;4;5]$

$P(X=0) = 0,3532$
$P(X=1) = 0,4415$
$P(X=2) = 0,1766$
$P(X=3) = 0,0272$
$P(X=4) = 0,0015$
$P(X=5) = 0,00002$

b 1. P(hit target) (3)
$= 0,85 + 0,15 \times 0,8 = 0,97$

2. $P(X \geq 14) = {}^{15}C_{14} (0,97)^{14} (0,03)^1$
$+ {}^{15}C_{15} (0,97)^{15} (0,03)^0$
$= 0,9270$ (6)

QUESTION 2:

a) ${}^{7}P_3 = 210$ (4)

b. ${}^{9}C_3 = 84$ (5)

QUESTION 3:

a). $\int_1^2 cx^2 \, dx + \int_2^3 cx \, dx = 1$

1. $\left. \frac{cx^3}{3} \right|_1^2 + \left. \frac{cx^2}{2} \right|_2^3 = 1$

$c\left(\frac{8}{3} - \frac{1}{3}\right) + c\left(\frac{9}{2} - 2\right) = 1$

$c = \frac{6}{29}$ (5)

2 $P\left(\frac{1}{2} \leq X \leq \frac{5}{2}\right)$

$= \left. \frac{6}{29} \cdot \frac{x^3}{3} \right|_1^2 + \left. \frac{6}{29} \cdot \frac{x^2}{2} \right|_2^{5/2}$

$= \frac{6}{29}\left(\frac{7}{3} + \frac{9}{8}\right)$ (5)

$= \frac{83}{116}$ (0,7155)

b) $\bar{x} = \frac{367}{6}$ $\bar{y} = \frac{270}{6} = 45$

$b = \frac{6.(17135) - (367).(270)}{6.(33845) - (367)^2}$

$= \frac{3720}{68381}$ (0,0544)

10

$\bar{y} = a + b\bar{x}$

$45 = a + 0{,}0544\left(\frac{367}{6}\right)$

$a = 41{,}6725$

$\therefore$ $y = 41{,}6725 + 0{,}0544x$

- As x increases, y increases.

QUESTION 4

a) $\mu = \frac{1326}{65} \pm z.\frac{\delta}{n}$ 7

$\delta = \sqrt{\frac{400{,}24}{64}} = 2{,}5007$

$\mu = 20{,}4 \pm 2{,}33 . \frac{2{,}5007}{\sqrt{65}}$

$= [19{,}6774 ; 21{,}1225]$

2) According to sample provided Beck's claim is incorrect 2
since the interval lies 19,6774 to 21,1225
hence It can be less than 20.

b) 1. $P = \frac{1421}{2450} \pm 2{,}58\sqrt{\frac{\frac{1421}{2450} \cdot \frac{1029}{2450}}{250}}$

$= [0{,}5543 ; 0{,}6057]$ 7

2. - readers of newspapers likely to have similar backgrounds/opinion
- self selected phone interviews.

QUESTION 5

a) $P(X > a) = 0{,}05$

$P\left(z > \frac{a - 500}{100}\right) = 0{,}05$

$\frac{a - 500}{100} = 1{,}645$ 5

$a = 664{,}5$

b. $P(10000 < X < 16000)$

$P\left(\frac{10000 - 13000}{1000} < z < \frac{16000 - 13000}{1000}\right)$

$P(-3 < z < 3)$ 5

$= 0{,}49865 \times 2$

$= 0{,}9973$

5c. $P(X < 154) = 0{,}07$

$P\left(Z < \frac{154-168}{\sigma}\right) = 0{,}07$

$\frac{154-168}{\sigma} = -1{,}48$

5

$\sigma = 9{,}4595$

QUESTION 6

a) - samples should be independent of one another.

- population from which the samples are taken should follow normal distribution

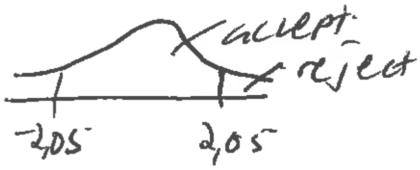
- etc.

2

b) $H_0: \mu = 5$

$H_1: \mu \neq 5$

critical value $= \pm 2{,}05$



Test statistic $= \frac{4.951 - 5}{0{,}2/\sqrt{60}}$

$= -1{,}8978$

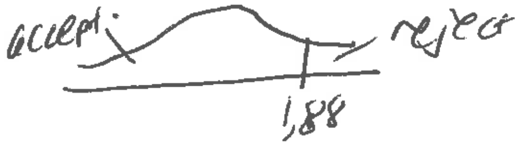
8

- enough evidence at 4% sig level to accept H_0 ∴ reject the claim

c) $H_0: \mu_c = \mu_m$

$H_1: \mu_c > \mu_m$

critical value $= 1{,}88$



test statistic $= \frac{2784 - 2658}{\sqrt{\frac{173^2}{30} + \frac{165^2}{28}}}$

$= 2{,}9515$

9

- no enough evidence to accept H_0 at 0,03 sig level ∴ we accept the claim that construction workers earn more than manufacturing workers.