ST BENEDICT'S COLLEGE

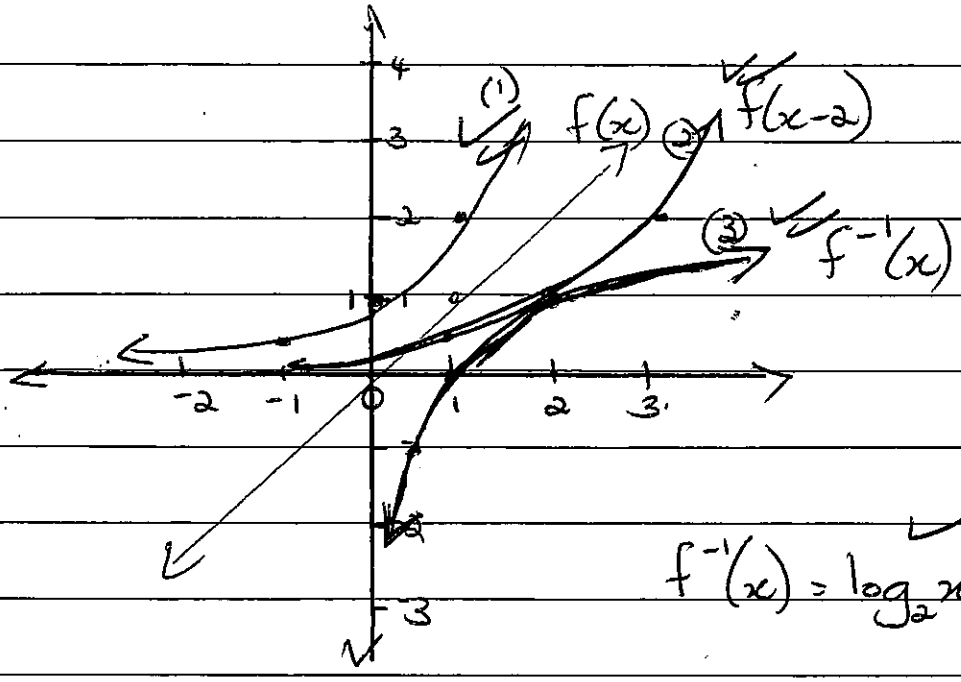
Answer Booklet

Name	AP Maths Paper 1
Grade & Homeroom class (eg: 12 F)	Prelim Memo 2017
Date	
Subject	
Teacher	

Book ..... of .....

QUESTION 1

a) i)



$f^{-1}(x) = \log_2 x$ (7)

(Qc, b swapped)

c) $3 + 4x = 3\left|x - \frac{1}{2}\right|$

ii) If $x - \frac{1}{2} \geq 0$ then $3 + 4x = 3(x - \frac{1}{2})$

$x \geq \frac{1}{2}$ $3 + 4x = 3x - \frac{3}{2}$

$x = -4\frac{1}{2}$ N/A

if $x - \frac{1}{2} < 0$ then $3 + 4x = 3(-x + \frac{1}{2})$

$x < \frac{1}{2}$ $3 + 4x = -3x + \frac{3}{2}$

$7x = -\frac{3}{2}$

$x = -\frac{3}{14}$ (6)

$\therefore x = -\frac{3}{14}$ (Can be done graphically)

3) $\log_{0,1}(x-20) + \log 2x = 1$ ✓

$-\log_{10}(x-20) + \log 2x = 1$

$\log \frac{2x}{x-20} = 1$ ✓

$\frac{2x}{x-20} = 10$ ✓✓

$2x = 10x - 200$

$-8x = -200$ ✓

$x = 25$ Check ✓

$x = 25$ ✓ (6)

2) $\frac{6x+1}{(2x-3)} \geq 1$

$\frac{6x+1}{2x-3} - 1 \geq 0$ ✓

$\frac{6x+1-2x+3}{(2x-3)} \geq 0$ ✓

$\frac{4x+4}{(2x-3)} \geq 0$ ✓

+ + 0 − − u/D + +

-1 $\quad$ $3/2$ ✓

$\therefore x \leq -1$ or $x > 3/2$ ✓ (6)

b) 1) $\frac{(3+4i)}{(3-2i)} \times \frac{(3+2i)}{(3+2i)}$ ✓

$= \frac{9+18i+8i^2}{9-4i^2}$ ✓

$= \frac{9+18i-8}{9+4}$ ✓

$= \frac{1}{13} + \frac{18}{13}i$ ✓ (4)

2) $x^4 - 2x^3 - 4x^2 + 6x + 3 = 0$

$1+\sqrt{2}$ is a zero ∴ $x - 1 - \sqrt{2}$ is a factor ✓

∴ $1-\sqrt{2}$ is a zero ∴ $x - 1 + \sqrt{2}$ is a factor ✓

∴ $(x-1-\sqrt{2})(x-1+\sqrt{2})$ is a factor ✓

∴ $x^2 - x + \sqrt{2}x - x + 1 - \sqrt{2} - \sqrt{2}x + \sqrt{2} - 2$ is a factor ✓

$x^2 - 2x - 1$ is a factor

∴ $(x^2 - 2x - 1)(x^2 - 3) = x^4 - 2x^3 - 4x^2 + 6x + 3$ ✓

$x = \pm\sqrt{3}$ or $x = 1 \pm \sqrt{2}$ ✓ (7)

d) $\dfrac{4x^2-5x+3}{x^3-2x^2+x} = \dfrac{4x^2-5x+3}{x(x^2-2x+1)}$

$\dfrac{4x^2-5x+3}{x(x-1)^2} = \dfrac{A}{x} + \dfrac{B}{(x-1)} + \dfrac{C}{(x-1)^2}$

$\therefore 4x^2-5x+3 = A(x-1)^2 + Bx(x-1) + Cx$

At $x=1$ $\quad 4-5+3 = C(1)$

$2 = C$

At $x=0$ $\quad 3 = A$

At $x=2$ $\quad 16-10+3 = 3(1) + B(2)(1) + 2(2)$

$9 = 3 + 2B + 4$

$+2 = 2B$

$B = 1$

$\therefore \dfrac{4x^2-5x+3}{x^3-2x^2+x} = \dfrac{3}{x} + \dfrac{1}{(x-1)} + \dfrac{2}{(x-1)^2}$

8

44 marks

QUESTION 2

a) $ax + b = a^2 + 6$ if $x = 2$

$a(2) + b = a^2 + 6$ ✓

$b = a^2 - 2a + 6$

$a^2 + 6 = bx - a$ if $x = 2$

$a^2 + 6 = 2b - a$ ✓

$\therefore 2b = a^2 + a + 6$

$\therefore 2a^2 - 4a + 12 = a^2 + a + 6$ ✓

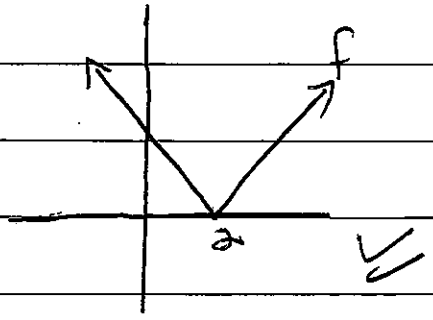
$a^2 - 5a + 6 = 0$

$(a - 2)(a - 3) = 0$ (7)

$a = 2$ ✓ or $a = 3$ ✓

$b = 6$ ✓ $b = 9$ ✓

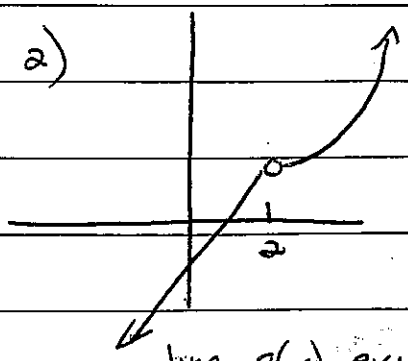
b) a)



✓✓

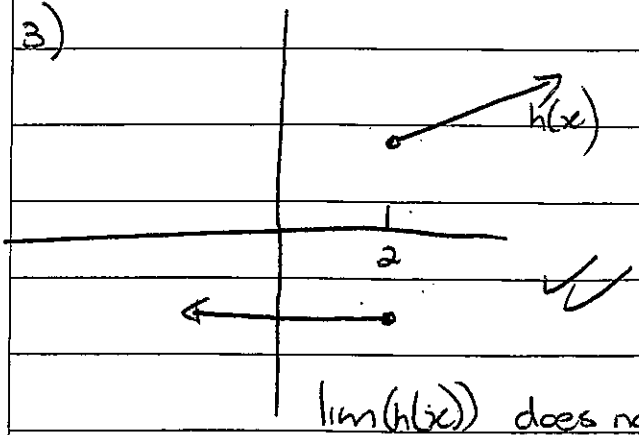
continuous but not diff at $x = 2$

a)



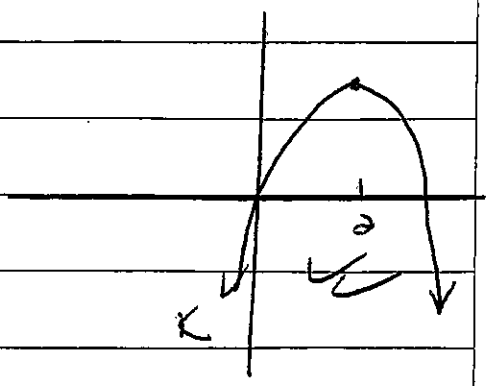
$\lim_{x \to 2} g(x)$ exists but not continuous at $x = 2$ ✓

3)

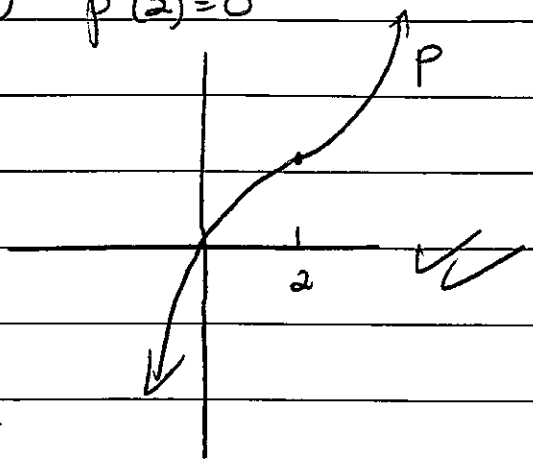


$\lim_{x \to 2} (h(x))$ does not exist

4) $K'(2) = 0$



5) $p''(2) = 0$



10

17 marks

QUESTION 3

a) $\theta = \frac{2\pi}{5}$ ✓

Area of circle − Area Pentagon = 20

$\pi r^2 - 5 \times \frac{1}{2} \times r \times r \times \sin\frac{2\pi}{5} = 20$

✓ ✓ ✓

$\pi r^2 - \frac{5}{2} r^2 \sin\frac{2\pi}{5} = 20$

$r^2\left(\pi - \frac{5}{2}\sin\frac{2\pi}{5}\right) = 20$

$r^2 = \frac{20}{\pi - \frac{5}{2}\sin\frac{2\pi}{5}}$ ✓

$r^2 = 26,1796$ ....

$r = 5,1166$ cm ✓ (6)

b) Area = $2r \times 2r$ ✓

$= 4r^2$ ✓

$= 4 \times 26,1796$

$= 104,7187$ cm^2 (3)

c) $s = r\theta$ ✓ ✓

$= 5,1166 \times 3 \times \frac{2\pi}{5}$ ✓

$= 19,2891$ cm (3)

12 marks

QUESTION 4

a) i) $\lim_{x \to 3} \dfrac{x^3 - 27}{3 - x}$ ✓

$= \dfrac{(x-3)(x^2+3x+9)}{(3-x)}$ ✓

$= \lim_{x \to 3}$

$= -(9 + 9 + 9)$ ✓

$= -27$ (3)

ii) $\lim_{x \to \infty} \dfrac{\sqrt{x^3+1}}{x}$

$= \lim_{x \to \infty} \sqrt{\dfrac{x^3+1}{x^2}}$ ✓✓

$= \lim_{x \to \infty} \sqrt{1 + \frac{1}{x^3}}$ ✓✓

$= \sqrt{1}$ ✓

$= 1$ (5)

b) i) $fog(x)$

$= f(x^3 - 2)$ ✓

$= \sqrt{x^3 - 2 - 4}$

$= \sqrt{x^3 - 6}$ ✓ (2)

ii) $x^3 - 6 \geq 0$ ✓

$(x - \sqrt{6})(x + \sqrt{6}) \geq 0$

$x \leq -\sqrt{6}$ or $x \geq \sqrt{6}$ ✓ (3)

13 marks

QUESTION 5

1) $1 + x = 2\tan x$ ✓✓ (2)

$1 + x - 2\tan x = 0$

2) At $x = 0$ $\quad 1 + (0) - 2\tan(0)$

$= 1$ ✓ > 0

At $x = 1$ $\quad 1 + 1 - 2\tan 1$

$= -1.1148$ ✓ < 0 (3)

$\therefore$ soln to eqn lies in int $[0;1]$

3) $x_{r+1} = x_r - \dfrac{f(x_r)}{f'(x_r)}$

$= x_r - \dfrac{1 + x_r - 2\tan x_r}{1 - 2\sec^2 x_r}$ ✓✓

Choose $x_0 = 0,5$ ✓

$x_1 = 0,7551\ldots$ ✓

$x_2 = 0,70919\ldots$

$x_3 = 0,706337$ (5)

$x_4 = 0,70632$

$x_5 = 0,7063\ldots$

$\therefore x = 0,7063$ ✓ 10 marks

QUESTION 6

a) $P'(x) = \lim_{h \to 0} \frac{P(x+h) - P(x)}{h}$

$= \lim_{h \to 0} \frac{\left(\sqrt{3x+3h} - \sqrt{3x}\right)}{h} \times \frac{\left(\sqrt{3x+3h} + \sqrt{3x}\right)}{\left(\sqrt{3x+3h} + \sqrt{3x}\right)}$

$= \lim_{h \to 0} \frac{3x+3h-3x}{h\left(\sqrt{3x+3h} + \sqrt{3x}\right)}$

$= \lim_{h \to 0} \frac{3h}{h\left(\sqrt{3x+3h} + \sqrt{3x}\right)}$

$= \frac{3}{\sqrt{3x} + \sqrt{3x}}$

$= \frac{3}{2\sqrt{3}x}$

$= \frac{\sqrt{3}}{2x}$

(6)

b) i) $D_x\left[\frac{x^2}{\sqrt{2x+3}}\right]$ (Can be done as prod)

$= \frac{\sqrt{2x+3} \cdot 2x - x^2 \cdot \frac{1}{2}(2x+3)^{-1/2} \cdot 2}{(\sqrt{2x+3})^2}$

$= \frac{2x\sqrt{2x+3} - \frac{x^2}{\sqrt{2x+3}}}{2x+3}$

(6)

2) $D_x\left[(3x^4-10x)^{15}\cdot\sqrt{4x^4+64}\right]$

$= (3x^4-10x)^{15}\cdot\frac{1}{2}(4x^4+64)^{-\frac{1}{2}}\cdot 16x^3$

$+\sqrt{4x^4+64}\cdot 15(3x^4-10x)^{14}\cdot(12x^3-10)$

$= \frac{8x^3(3x^4-10x)^{15}}{\sqrt{4x^4+64}} + 15(12x^3-10)(3x^4-10x)^{14}\sqrt{4x^4+64}$ (8)

3) $\frac{d}{dx}\left(\sin^2(3x-4)\right)$

$= 2\sin(3x-4)\cdot\cos(3x-4)\cdot 3$

$= 6\sin(3x-4)\cos(3x-4)$

or $3\cdot\sin 2(3x-4)$ (4)

$= 3\sin(6x-8)$

4) $\frac{d}{dx}\left(\tan(3x-4)^2\right)$

$= \sec^2(3x-4)^2\cdot 2(3x-4)\cdot 3$

$= (18x-24)\sec^2(3x-4)^2$ (4)

c) $f(x) = (4x-3)^{-1}$ ✓

$f'(x) = -1\ (4x-3)^{-2} \cdot 4$ ✓

$f''(x) = -1 \cdot -2\ (4x-3)^{-3} \cdot 4 \cdot 4$ ✓

$f'''(x) = -1 \cdot -2 \cdot -3\ (4x-3)^{-4} \cdot 4 \cdot 4 \cdot 4$

etc

$f^n(x) = (-1)^n\, n!\, (4x-3)^{-(n+1)} \cdot 4^n$ ✓1 ✓1 ✓1 ✓1

(5)

d) $y^2 - 5xy + 8x^2 = 7$ ✓

$2y\frac{dy}{dx} - 5\left(x \cdot \frac{dy}{dx} + y \cdot (1)\right) + 16x = 0$ ✓ ✓

$2y\frac{dy}{dx} - 5x\frac{dy}{dx} - 5y + 16x = 0$

$\frac{dy}{dx} = \frac{-16x + 5y}{(2y - 5x)}$ ✓✓

At pt $(1;2)$ $m_{tan} = \frac{-16 + 5(2)}{(2(2) - 5(1))}$

$= \frac{-6}{-1}$

$= 6$ ✓

(7)

40 marks

Question 7

a) $\int \left(\sqrt{x} + \frac{1}{5x}\right)^2 dx$

$= \int \left(x + \frac{2\sqrt{x}}{5x} + \frac{1}{25x^2}\right) dx$

$= \int \left(x + \frac{2}{5}x^{-\frac{1}{2}} + \frac{1}{25}x^{-2}\right) dx$

$= \frac{1}{2}x^2 + \frac{4}{5}x^{\frac{1}{2}} - \frac{1}{25}x^{-1} + C$

$= \frac{1}{2}x^2 + \frac{4}{5}\sqrt{x} - \frac{1}{25x} + C$

−1 if no constant

(6)

b) $\int (x+2)\sqrt{x^2+4x+5}\ dx.$

let $u = x^2 + 4x + 5$

$\frac{du}{dx} = 2x + 4$

$du = 2(x+2)\,dx$

$\frac{1}{2}du = (x+2)\,dx$

$\Longrightarrow \frac{1}{2}\int \sqrt{u}\ du$

$= \frac{1}{2} \times \frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{1}{3}\sqrt{(x^2+4x+5)^3} + C$

(6)

−1 no constant

c) $\int \sin 5x \cdot \cos 2x \, dx$

$= \frac{1}{2} \int \sin(7x) + \sin 3x \, dx$ ✓

$= \frac{1}{2} \left[-\frac{1}{7}\cos 7x + \frac{1}{3}\cos 3x \right] + C$ ✓ ✓ leave

$= -\frac{1}{14}\cos 7x - \frac{1}{6}\cos 3x + C$ ✓ ✓ (5)

−1 no constant

c) $\int \sec^2(3x-1)\tan(3x-1) \, dx$

✓ ✓ ✓

$= \frac{1}{2}\tan^2(3x-1) \cdot \frac{1}{3} + C$

✓✓ (5)

$= \frac{1}{6}\tan^2(3x-1) + C$

−1 no constant

d) $\int \sin^2 4x \, dx$ ✓ → OR $\left(\frac{1}{2}(1-\cos 2x) = \sin^2 x\right)$ double angles

$= \int \sin 4x \cdot \sin 4x \, dx$

$= \frac{1}{2} \int \cos(4x-4x) - \cos(4x+4x) \, dx$

$= \frac{1}{2} \int (\cos 0 - \cos 8x) \, dx$ ✓ ✓

$= \frac{1}{2} \int (1 - \cos 8x) \, dx$ ✓ (6)

$= \frac{1}{2}\left[x - \frac{1}{8}\sin 8x\right] + C$

$= \frac{1}{2}x - \frac{1}{16}\sin 8x + C$ ✓ ✓

−1 no constant

e) $\int x \sin 4x \, dx$

$f: x$ ✓	$g: -\frac{1}{4}\cos 4x$
$f': 1$ ✓	$g': \sin 4x$ ✓

$= -\frac{1}{4} x \cos 4x - \int -\frac{1}{4} \cos 4x \, dx$ ✓

$= -\frac{x}{4} \cos 4x + \frac{1}{4} \sin 4x \cdot \frac{1}{4} + C$ ✓

$= -\frac{x}{4} \cos 4x + \frac{1}{16} \sin 4x + C$

(7)

−1 no constant

Max −2 for no constant

30 marks

Question 8

a) 6 Two vert asymptotes. $x=1$; $x=3$; deg of den higher

b) 4 Remov. discon at $x=\pm 3$ → Str line $y=x-3$

c) 3 One vert asym $x=-2$; Deg num + den same ∴ horiz asymp $y=1$

d) 2 Vert asym $x=-4$ linear → hyperbolic curve

e) 1 No vert asymp $x^2+1>0$ ∴ y is always +ve
No x int Horizontal asym $y=0$

f) 5
↳ Vert asym at $x=-3$
Oblique asym $y=x+1$
One cut x-axis $x=-2$

$$x+3 \overline{)\, x^2+4x+4}$$
$$x^2+3x$$
$$x+4$$
$$x+3$$
$$1$$

✓ ans 3 reasoning in each case

24 marks

QUESTION 9

$BC = 18 - 2x$

$DC = 9 - x$ ✓

$(9-x)^2 + y^2 = x^2$ ✓

$81 - 18x + x^2 + y^2 = x^2$

$y^2 = 18x - 81$

$y = \sqrt{18x - 81}$ ✓

$\text{Area} = \frac{1}{2}(18-2x) \times y$ ✓

$= (9-x)\sqrt{18x-81}$

Max Area $A' = 0$ ✓

$(9-x) \cdot \frac{1}{2}(18x-81)^{-\frac{1}{2}} \cdot 18 + \sqrt{18x-81}(-1) = 0$ ✓ ✓

$\frac{9(9-x)}{\sqrt{18x-81}} - \sqrt{18x-81} = 0$

$81 - 9x - (18x - 81) = 0$

$-27x = -162$ ✓

$x = 6$ (10)

$y = \sqrt{18(6) - 81}$

$y = \sqrt{27}$ ✓

$\text{Area} = \frac{1}{2}(18 - 2(6)) \times \sqrt{27}$

$= 3\sqrt{27}$ ✓

$= 9\sqrt{3}$ units^2

$= 15{,}59$ cm^3