ST BENEDICT'S COLLEGE

SUBJECT	**AP Mathematics Paper 1**		18 JULY 2017
GRADE	12	**MARKS**	200
EXAMINER	Mrs MH Povall	**MODERATORS**	Mrs H Rademeyer Mr N Benecke
NAME		**DURATION**	2.5 Hours
TEACHER			

READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Continuity and Differentiability	17	
2	Algebra	44	
3	Sectors and Segments	12	
4	Limits and function notation	13	
5	Newton's Method	10	
6	Differentiation	40	
7	Methods of Integration	30	
8	Graphs of rational functions	24	
9	Problem Solving	10	
TOTAL		200	

1. This question paper consists of 10 pages and a formula sheet is supplied.
2. Read the questions carefully.
3. All the necessary working details must be clearly shown.
4. Approved non-programmable calculators may be used except where otherwise stated.
5. It is in your own interest to write legibly and to present your work neatly.
6. Where necessary, round off to four decimal places.

QUESTION 1 **44 MARKS**

a) Given: $f(x) = 2^x$

Sketch the following curves on the same system of axes:

1) $f(x)$ (2)

2) $f(x-2)$ (2)

3) $f^{-1}(x)$ and state the equation of $f^{-1}(x)$ (3)

b) 1) Simplify $\frac{3+4i}{3-2i}$ without the use of a calculator. (4)

2) Solve for x:

$x^4 - 2x^3 - 4x^2 + 6x + 3 = 0$ if it is given that $1+\sqrt{2}$ is a zero of $f(x)$.

(7)

c) Solve for x:

1) $3 + 4x = 3\left|x - \frac{1}{2}\right|$ (6)

2) $\frac{6x+1}{(2x-3)} \geq 1$ (6)

3) $log_{0,1}(x - 20) + log2x = 1$ (6)

d) Resolve $\frac{4x^2-5x+3}{x^3-2x^2+x}$ into partial fractions. (8)

QUESTION 2 **44 MARKS**

a) Find the values of *a* and *b* if $f(x)$ is continuous for all values of x and

$$f(x)=\begin{cases} ax+b & \text{if } x<2 \\ a^2+6 & \text{if } x=2 \\ bx-a & \text{if } x>2 \end{cases}$$ (7)

b) Draw separate sketch graphs of each of the following five functions:

1) a function *f* such that *f* is continuous but not differentiable at $x=2$.

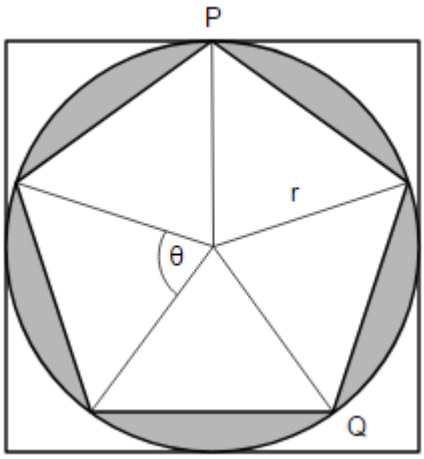
2) a function *g* such that $\lim\limits_{x\to 2} g(x)$ exists, but *g* is not continuous at $x=2$.

3) a function *h* such that $\lim\limits_{x\to 2} h(x)$ does not exist.

4) a function *k* such that $k'(2)=0$.

5) a function *p* such that $p''(2)=0$. (10)

QUESTION 3 **12 MARKS**



The pattern above is painted on a square piece of wood. It is designed in the form of a regular pentagon, which is inscribed in a circle as shown. The total shaded area measures 20cm².

Determine the following values, correct to one decimal:

a) the radius r (6)

b) the area of the piece of wood (3)

c) the length of the major arc PQ (3)

QUESTION 4 **13 MARKS**

a) Evaluate the following without using a calculator:

1) $\lim_{x\to 3} \frac{x^3-27}{3-x}$ (3)

2) $\lim_{x\to \infty} \frac{\sqrt{x^2+1}}{x}$ (5)

b) Given: $f(x) = \sqrt{x-4}$ and $g(x) = x^2 - 2$

1) Determine $f \circ g(x)$. (2)

2) Determine the domain of $f \circ g(x)$. (3)

QUESTION 5 **10 MARKS**

Given: $y = 1 + x$ and $y = 2tanx$

1) Create a function $f(x)$ that can be used to find the x – value of the point of intersection of the two graphs. (2)

2) Show that $f(x) = 0$ has a solution in the interval [0 ; 1] (3)

3) Calculate the x – value of the point of intersection of the two graphs in the interval [0 ; 1]. Round off to four decimal places. (5)

QUESTION 6 **40 MARKS**

a) Determine the gradient of the function $p(x) = \sqrt{3x}$ at any point by using first principles. (6)

b) Determine the following derivatives: Do not simplify your answers fully.

1) $D_x[\frac{x^2}{\sqrt{2x+3}}]$ (6)

2) $\mathrm{D}_x\left[(3x^4 - 10x)^{15}.\sqrt{4x^4 + 64}\right]$ (8)

3) $\frac{d}{dx}\ sin^2(3x - 4)$ (4)

4) $\frac{d}{dx}\ \tan\ (3x - 4)^2$ (4)

c) Determine the formula for the n-th derivative of $f(x) = \dfrac{1}{4x - 3}$. (5)

d) Find the gradient of the tangent to $y^2 - 5xy + 8x^2 = 7$ at the point $(1; 2)$. (7)

QUESTION 7 **30 MARKS**

Determine each of the following integrals:

a) $\int \left(\sqrt{x} + \frac{1}{5x}\right)^2 dx$ (6)

b) $\int (x+2)\sqrt{x^2+4x+5}\, dx$ (6)

c) $\int sec^2(3x-1).tan(3x-1)\, dx$ (5)

d) $\int sin^2 4x\, dx$ (6)

e) $\int x.sin4x\, dx$ (7)

QUESTION 8 **24 MARKS**

Match each of the following functions with the curves sketch alongside. Motivate your answer by referring to the asymptotes and the degrees of the expressions.

a) $f(x) = \dfrac{x-2}{x^2-4x+3}$

b) $f(x) = \dfrac{x^2-6x+9}{x-3}$

c) $f(x) = \dfrac{x^2-10}{x^2+4x+4}$

d) $f(x) = \dfrac{x-8}{x+4}$

e) $f(x) = \dfrac{1}{1+x^2}$

f) $f(x) = \dfrac{x^2+4x+4}{x+3}$ 6 x 4 = 24

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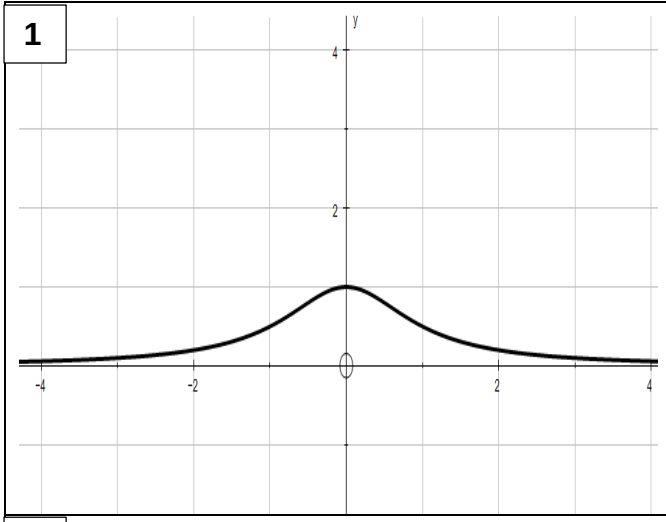

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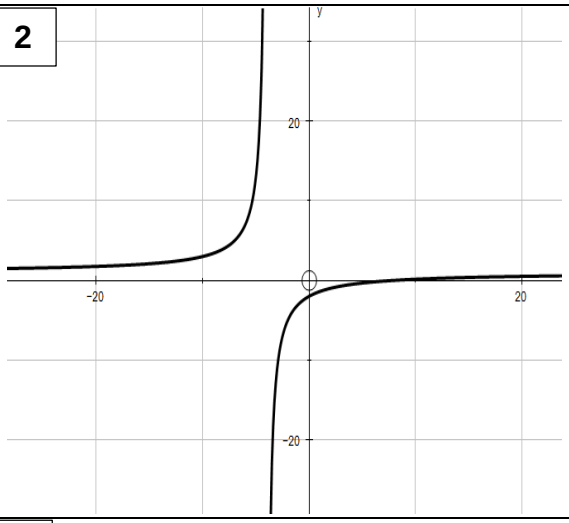

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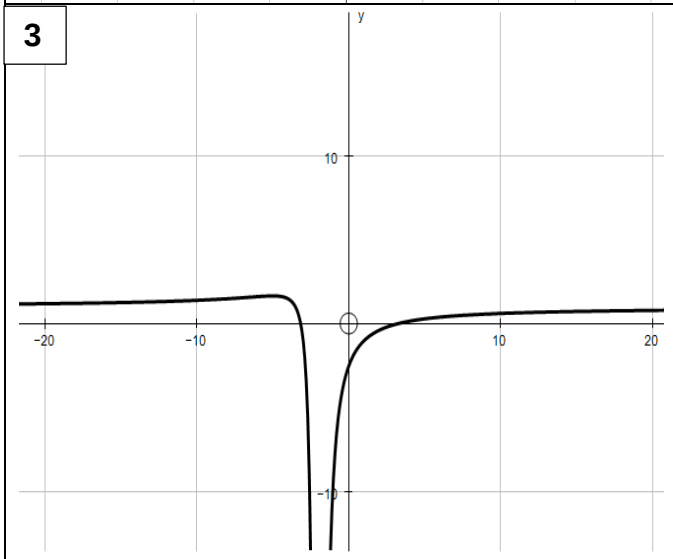

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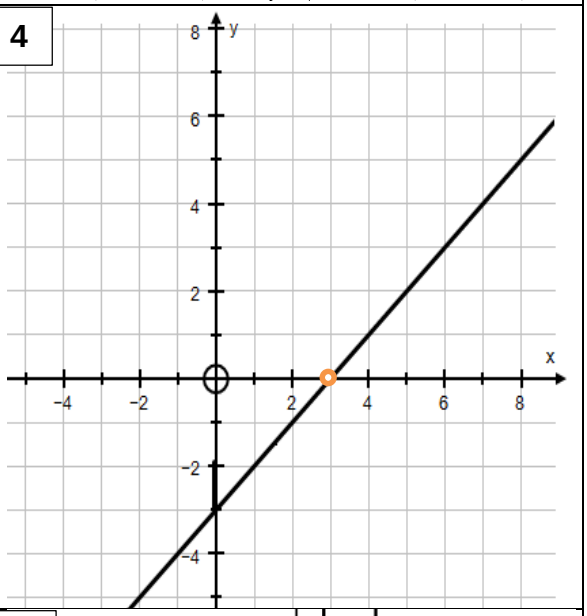

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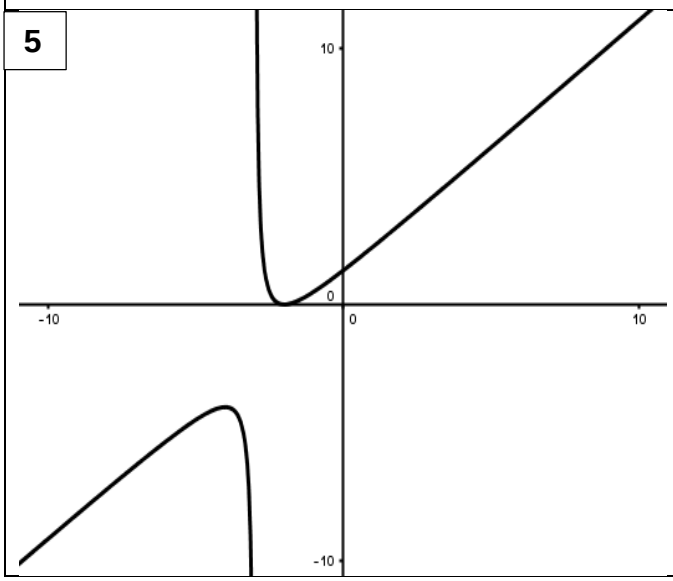

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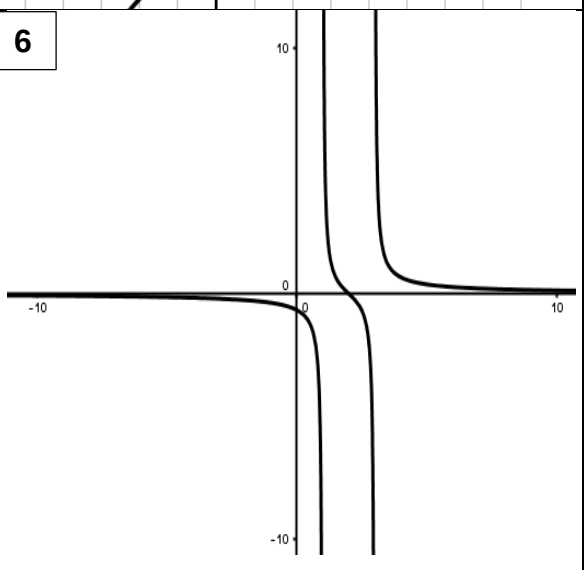

QUESTION 9 **10 MARKS**

If the perimeter of an isosceles triangle ABC is 18cm, calculate the maximum area of the triangle.