

QUESTION 1

Mark did a survey on second-hand car prices. For a particular type of car, he surveyed the ages x (in years) and the prices Ry (in thousands) of 10 second-hand cars.

The data collected is summarized as follows:

$$\begin{aligned} \sum x &= 67 & \sum y &= 2745 & \sum xy &= 13472 & \sum x^2 &= 553 \\ \sum y^2 &= 1\,038\,765 \end{aligned}$$

(a) Determine \bar{x} and \bar{y}

(2)



(b) Hence determine the equation of the least squares regression line of y on x .

(8)

- (c) Estimate the initial value of a vehicle of this type. Comment on the validity of the prediction. (2)

[12]

QUESTION 2

Each year a large number of students write a Mathematics examination. Over a period of time it is found that the marks of the students follow a normal distribution with a mean 70 and a standard deviation 6.

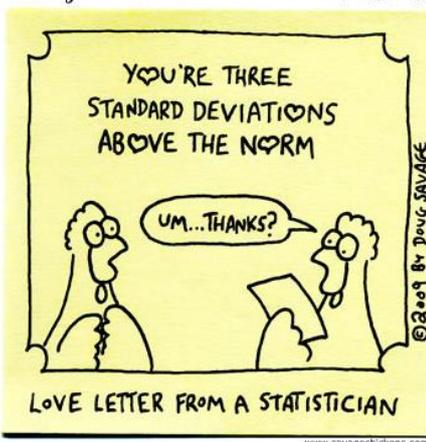
This year the examination contains questions on a new topic and the examiner believes that the marks are lower, on average. To test this belief, he calculates the mean mark of a random sample of 25 students and finds it to be 67.3.

Carry out a hypothesis test at the 5% level of significance to test whether the mean mark in the examination is lower this year.

(10)

Savage Chickens

by Doug Savage



[10]

QUESTION 3

Ballpoint pens are made in large quantities in a factory. They are tested in batches of 50 and it is known that 3% are faulty.

(a) From a test batch calculate:

(1) the probability that exactly 2 pens are faulty (3)

(2) the probability that at least one of the pens are faulty (4)

(b) In 200 batches of 50, how many pens would you expect to be faulty? (2)

© Randy Glasbergen
www.glasbergen.com



[9]

"I'm supposed to write an essay on what is meant by 'The pen is mightier than the sword'. First, I need to go to Google and find out what a pen is."

QUESTION 4

The probability mass function of the discrete random variable X , is given by

$$P(X = x) = a \left(\frac{2}{3}\right)^x \text{ for all } x \in \mathbb{N}_0$$

(a) Determine $P(x \leq 1)$ in terms of a . (3)

(b) Determine the value of the constant a . (5)

QUESTION 5

The lifetime of a lamp is divided according to the probability-density function:

$$f(t) = \begin{cases} k(-t^{\frac{3}{2}} + 8) & \text{as } 0 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}; k \text{ is a constant}$$

(a) Giving your answer as a fraction, determine the value of k

(6)

(b) Determine $P(0 < t < 2)$

(5)

[11]

QUESTION 6

(a) The 95% confidence interval for the mean length of life (in hours) of a particular brand of light bulb is (1023,3; 1101,7). This interval is based on results from a random sample of 36 light bulbs.

(1) Prove that the mean is 1062,50 and that the standard deviation is 120.

Show all working.

(8)

- (2) Find the 99% confidence interval for the mean length of life if this brand of light bulb, assuming that the length of life is normally distributed. (6)

- (b) A national safety council wants to estimate the proportion of car accidents that involve pedestrians.

How large a sample of accident records must be examined, to be 98% confident that the estimate does not differ from the true proportion by more than 0,03. The proportion estimated is 0,25.

(6)

[20]



"OH, GREAT, MY INSURANCE AGENT.
I WAS JUST TEXTING YOU."

QUESTION 7

- (a) Patrick wants to invest some of his monthly salary. He invests a certain amount of this every month for 18 months. For each month there is a probability of 0,25 that he will buy shares in a large company, there is a probability of 0,15 that he will buy shares in a small company and there is a probability of 0,6 that he will invest in a savings account.

Determine the probability that Patrick will buy shares in a small company in exactly 3 of these 18 months. (6)

- (b) A book club sends 6 paperback and 2 hardback books to Matthew. He chooses 4 of these books at random to take with him on holiday.

Determine the probability that he chooses exactly two paperback books.

(6)

- (c) The probability that Richard goes to the gym on any day is 0,85. On a day when he goes to the gym, the probability that he has a whey protein smoothie is 0,9. On a day when he does not go to the gym, the probability that he has a whey protein smoothie is 0,32.

Given that Richard had a whey protein smoothie, find the probability that he went to the gym that day. (4)



Filling in that application form was a severe first test

QUESTION 8

Given

$$P(A|B) = \frac{2}{5}; P(B|A) = \frac{1}{4} \text{ and } P(A \cup B) = \frac{11}{20}$$

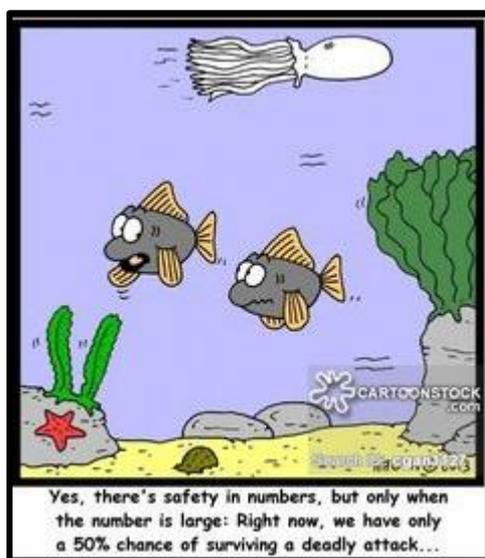
(a) Determine $P(A \cap B)$ in terms of $P(A)$ (3)

(b) Determine $P(A \cap B)$ in terms of $P(B)$ (3)

(c) Hence or otherwise determine $P(A)$ (6)

(d) Determine $P(A \cap B)$ (2)

[14]



[Total: 100 marks]