ST. DAVID'S MARIST INANDA

ADVANCED PROGRAMME MATHEMATICS
COMPULSORY MODULE
CALCULUS & ALGEBRA

GRADE 12
PRELIMINARY EXAMINATION
31 AUGUST 2017

EXAMINER: MRS C KENNEDY	**MARKS: 200**
MODERATOR: MRS S RICHARD	**TIME: 2 hours.**

NAME:____

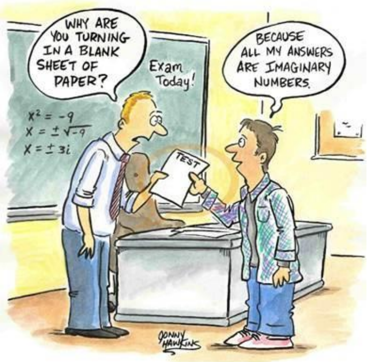
INSTRUCTIONS:

- This paper consists of 20 pages and a separate information sheet. Please check that your paper is complete.
- Please answer all questions in this booklet.
- A Casio FX 991 may be used, unless otherwise indicated.
- **CHECK THAT YOUR CALCULATOR IS IN RADIANS**
- All necessary calculations must be clearly shown.
- Work neatly. Do NOT answer in pencil.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
35	6	16	14	8	33	28	30	22	8	200

QUESTION 1 [35 marks]

a) Solve for p and q if $(3 + pi)^2 = 30i - 8q$ (7)



b) Solve for x in each of the following, rounding off to TWO decimal places where necessary:

i) $x|x+1| + 4 = x$ (7)

ii) $e^{2x} - 4e^{x} = 5$ (5)

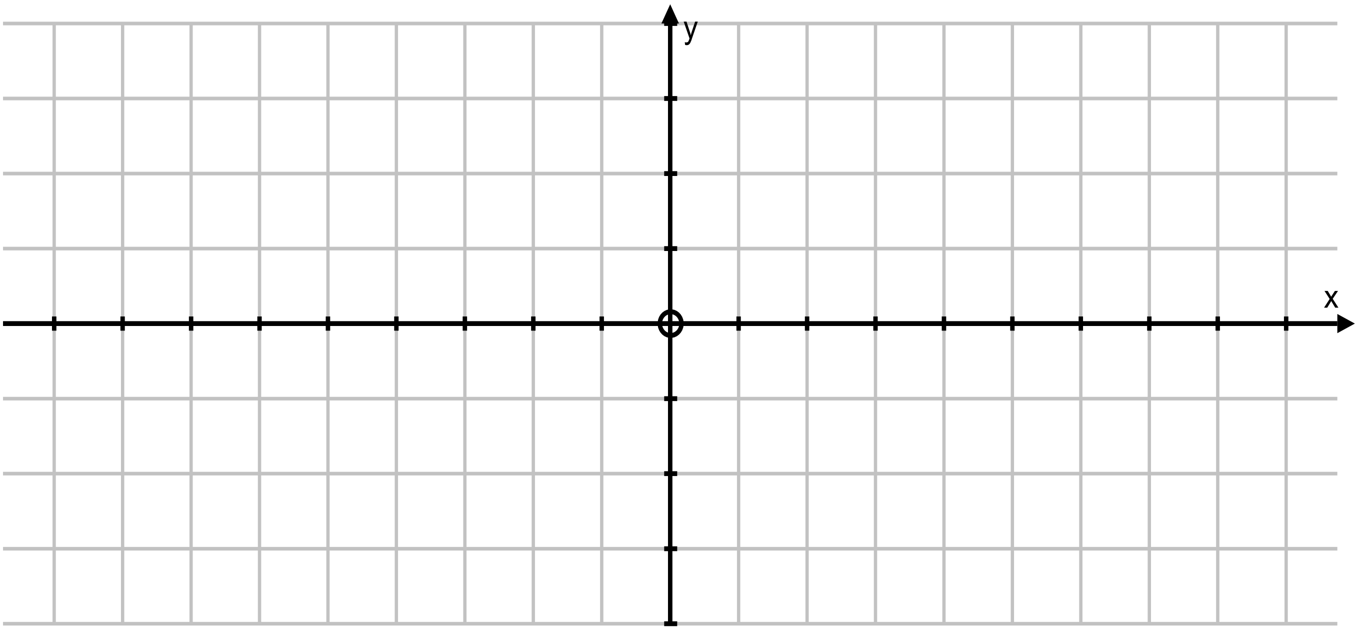
c) The points P(3 ; p) and Q(6 ; q) lie on the curve: $y = 3\ln x$.
Determine the gradient of the line PQ and give the answer in log form. (5)

d) Given $f(x) = 2\ln(x-1)$

i) Determine the equation of $f^{-1}(x)$, the inverse of $f(x)$. (4)

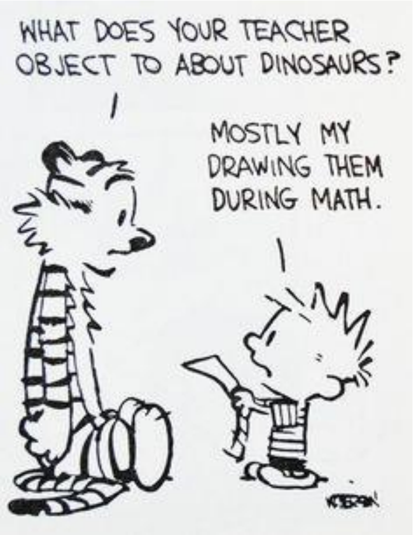
ii) Sketch the graphs of f(x) and $f^{-1}(x)$ on the same set of axes below. Clearly label all intersections with the axes and any asymptotes.

(7)



QUESTION 2 [6 marks]

Determine g(f(x)) if $f(g(x)) = \frac{1}{x+3} + x^2 + 6x + 9$ and $g(g(x)) = x + 6$ (6)



QUESTION 3 **[16 marks]**

a) Prove by Mathematical Induction that:

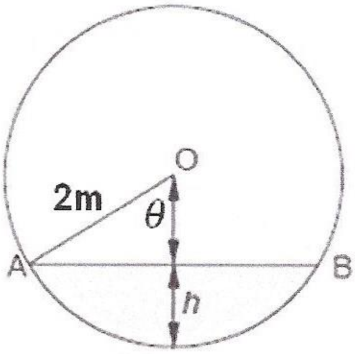
$\sum_{r=1}^{n} r(3r-1) = n^3 + n^2$ for all Natural numbers n. (12)

b) Hence, determine the value of:

$2 + 10 + 24 + \ldots. + 660$ (4)

QUESTION 4 [14 marks]

Petrol stations only run out of petrol when there is a major price increase. Generally, they manage their petrol levels in the tanks to ensure that they have sufficient stock. The petrol is stored in an underground cylindrical tank and the amount left in the tank is checked using a dipstick. Our problem is to calibrate the dipstick.



It is easy to measure the height, h, of the petrol left in the tank using a dipstick. However, the volume is proportional to the cross-sectional area.

a) Determine $\cos\theta$ in terms of h. (4)

b) Show that the cross sectional area of petrol in the tank is determined by:

$$A = 4\theta - 2\sin(2\theta) \quad (6)$$

c) What fraction of the tank is full when $\theta = \frac{\pi}{4}$? (4)

QUESTION 5 [8 marks]

The function f(x) is defined as follows:

$$f(x) = \begin{cases} \frac{a}{x} & \text{if} \quad x \geq 1 \\ b - 2x & \text{if} \quad x < 1 \end{cases}$$

Determine the value(s) of a and b if f(x) is differentiable at x = 1. (8)

QUESTION 6 [33 marks]

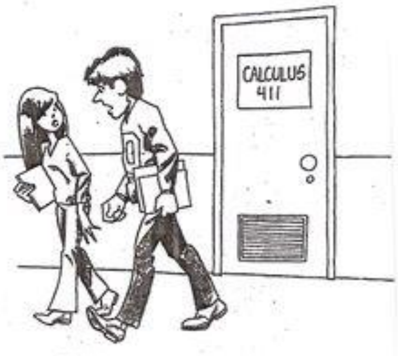
a) i) Prove that $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\cos A.\operatorname{cosec}^2 A$ (6)

ii) Hence, determine $\lim_{A \to 0}\left(\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1}\right)A^2$ (5)

b) Given: $y = \sqrt{4x^2 + 1}$

i) Show that $\frac{dy}{dx} = \frac{4x}{y}$ (6)

ii) Hence, or otherwise, show that $\frac{d^2y}{dx^2} = \frac{4}{y} - \frac{16x^2}{y^3}$ (6)



"I think I understood the part about bringing two sharp pencils to class every day."

c) If a drug is given to a patient and the percentage of the concentration of the drug in the blood stream t hours later is given by:

$$k(t) = \frac{5t}{t^2 + 1}$$

Determine when the concentration of the drug is increasing. (10)

QUESTION 7 [28 marks]

a) It is given that: $g(x) = \frac{3x^2 + x - 2}{x^2 + px - 2}$

i) For which value(s) of p will the graph of g have:

(1) one x-intercept. (5)

(2) an oblique asymptote. (2)

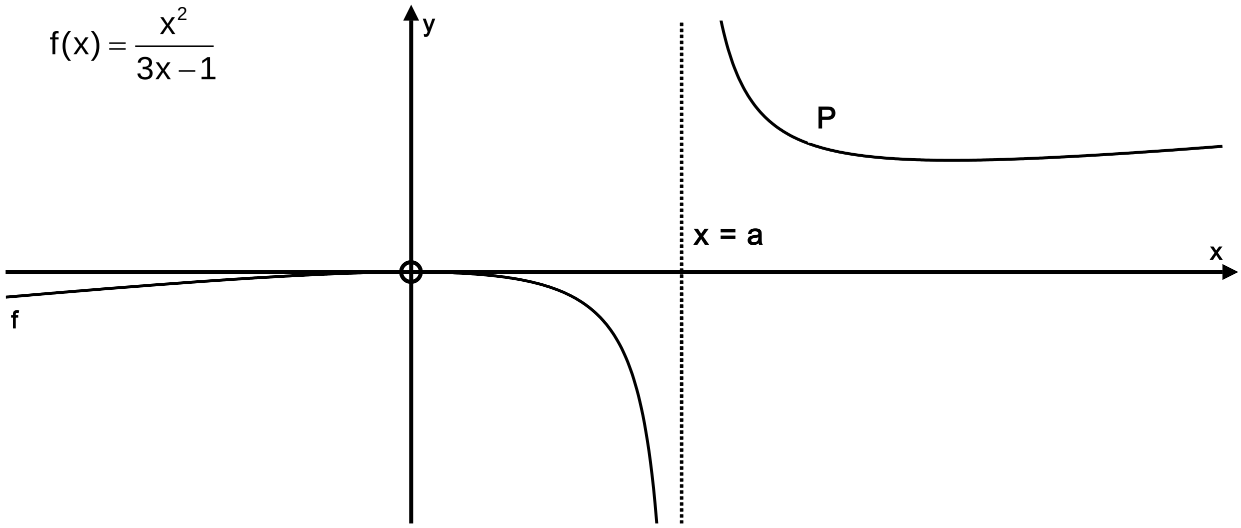
ii) If p = - 1,

(1) show that g is a hyperbola with a removable discontinuity at x = - 1. (3)

(2) hence, determine the equations of the asymptotes of g. (3)

b) The sketch shows part of the curve defined by:

$$f(x) = \frac{x^2}{3x - 1}$$



i) Write down the value of a. (1)

ii) If P is the turning point of f, show that the x co-ordinate of P is $\frac{2}{3}$. (6)

iii) If $f(x) = k$ has no real solutions, write down the possible values of k. (4)

iv) Determine the equation of the oblique asymptote of f. (4)

QUESTION 8 [30 marks]

a) Determine the following indefinite integrals:

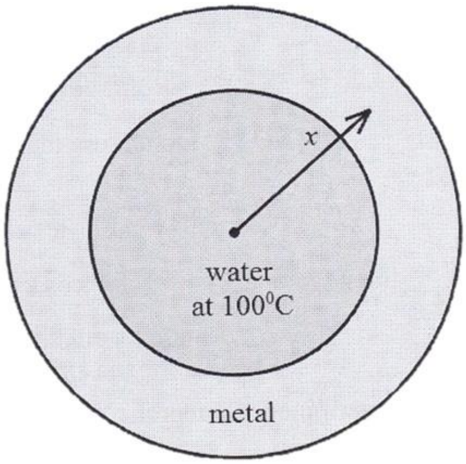
i) $\int 2\sin 5x.\cos 3x\, dx$ (4)

ii) $\int x^2 \sec^2(2x^3)\, dx$ (8)

b) Calculate a if: (8)

$$\int_0^a \frac{2}{\sqrt{x+4}}\, dx = 4$$

c) A metal tube has a cross-section as shown alongside.



Tubular Cross-Section

The outer radius is 4 cm and the inner radius is 2 cm.

Within the tube, water is maintained at a temperature of 100 ° C.

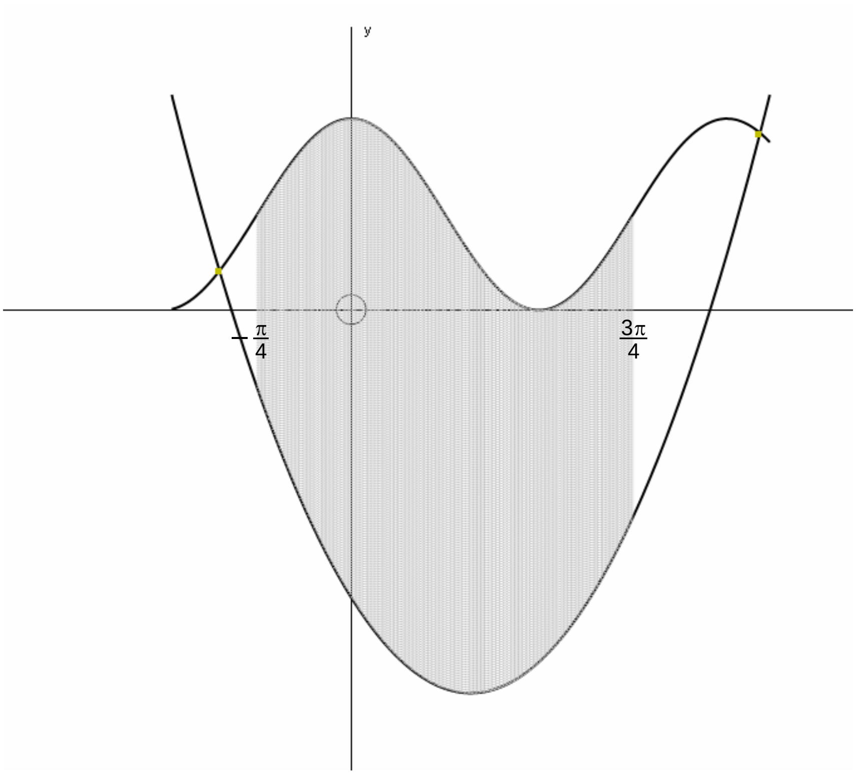
Within the metal the temperature drops from inside to outside according to

$\frac{dT}{dx} = -\frac{10}{x^2}$, where x is the distance from the central axis and $2 \leq x \leq 4$.

Determine the temperature of the outer surface of the tube. (10)

QUESTION 9 [22 marks]

In the diagram below, the shaded region is defined on the interval $\left[-\frac{\pi}{4};\frac{3\pi}{4}\right]$ and is bounded by the functions $h(x)=\cos 2x+1$ and $g(x)=x^2-2x-3$. The functions do not intersect at any point on this interval.

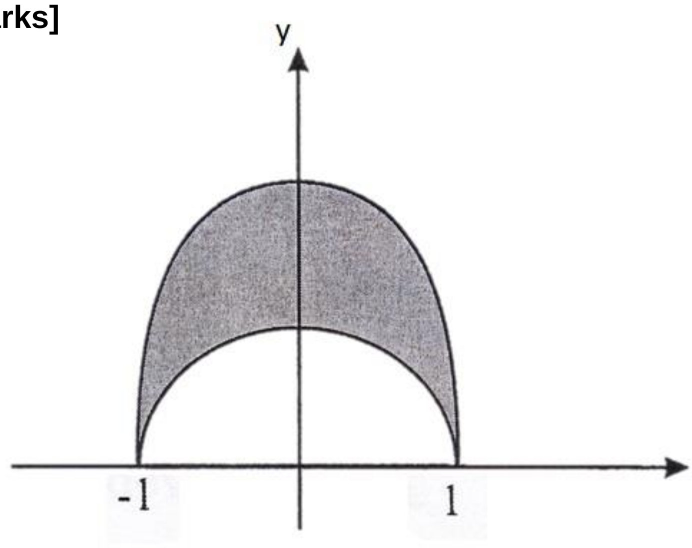


a) Show that the maximum distance between the two graphs on this interval can be found by solving the equation sin $2x = 1 - x$. (8)

b) Use Newton's method to write down a recursive equation that can be used to solve the equation in QUESTION 9a.
Hence, taking $x_0 = 0{,}5$ as an initial value, determine the answer correct to FIVE decimal places. (8)

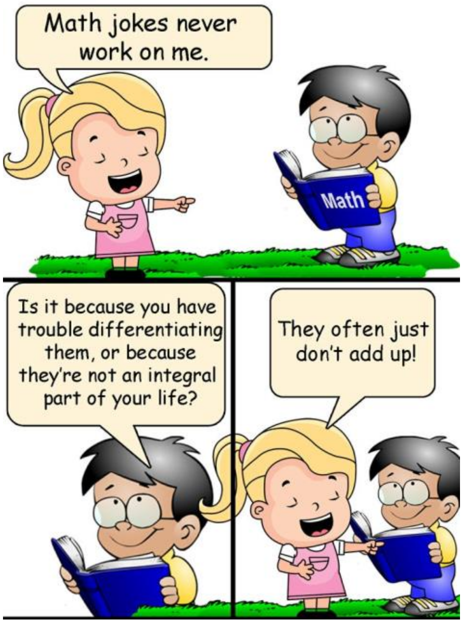
c) Now find the area of the shaded region. (6)

QUESTION 10 [8 marks]



The area enclosed between the parts of two curves $x^2 + y^2 = 1$ and $4x^2 + y^2 = 4$ is rotated by 2π radians about the x-axis.

Determine the volume of the solid formed.
(Leave your answer in terms of π) (8)



TOTAL: 200