ST. DAVID'S MARIST INANDA

ADVANCED PROGRAMME MATHEMATICS
PAPER 2
(STATISTICS)

PRELIMINARY EXAMINATION

GRADE 12

31 AUGUST 2017

EXAMINER: MRS S RICHARD **MODERATOR: MR L VICENTE**	**MARKS: 100** **TIME: 1 hour**

NAME: Memo

INSTRUCTIONS:

- ✓ This paper consists of 15 pages and a separate 4 page information sheet. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphical calculator unless otherwise stated.
- ✓ Answers must be rounded off to **FOUR** decimal places, unless otherwise stated.
- ✓ It is in your interest to show all your working details.
- ✓ Work neatly. Do NOT answer in pencil.

QUESTION	Q1 [12]	Q2 [10]	Q3 [9]	Q4 [8]	Q5 [11]	Q6 [20]	Q7 [16]	Q8 [14]	TOTAL [100]
LEARNER'S MARKS									

QUESTION 1

Mark did a survey on second-hand car prices. For a particular type of car, he surveyed the ages x (in years) and the prices Ry (in thousands) of 10 second-hand cars.

The data collected is summarized as follows:
$\sum x = 67$ $\sum y = 2745$ $\sum xy = 13472$ $\sum x^2 = 553$
$\sum y^2 = 1038765$

(a) Determine $\bar{x}$ and $\bar{y}$ (2)

$\bar{x} = \frac{67}{10} = 6,7$ ✓

$\bar{y} = \frac{2745}{10} = 274,5$ ✓

(b) Hence determine the equation of the least squares regression line of y on x. (8)

$b = \frac{n\sum xy - \sum x \sum y}{n(\sum x^2) - (\sum x)^2}$

$= \frac{10(13472) - 67(2745)}{10(553) - 67^2}$ ✓

$= -\frac{49195}{1041}$

$= -47,26$ ✓

$y = a + bx$ subs $(\bar{x}; \bar{y})$

$274,5 = a - 47,26(6,7)$ ✓

$\therefore a = 591,14$ ✓

$\therefore y = 591,14 - 47,26x$ ✓✓

(c) Estimate the initial value of a vehicle of this type. Comment on the validity of the prediction.

(2)

591 142 not valid as model is for second hand cars (NOT new cars), extrapolation

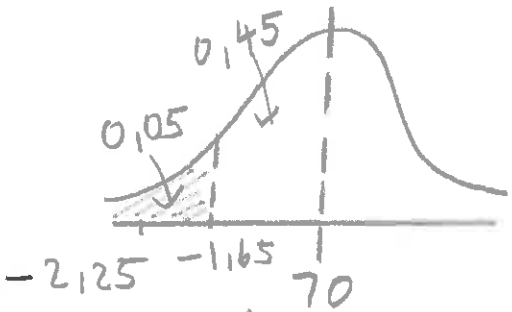
[12]

QUESTION 2

Each year a large number of students write a Mathematics examination. Over a period of time it is found that the marks of the students follow a normal distribution with a mean 70 and a standard deviation 6.

This year the examination contains questions on a new topic and the examiner believes that the marks are lower, on average. To test this belief, he calculates the mean mark of a random sample of 25 students and finds it to be 67.3.
Carry out a hypothesis test at the 5% level of significance to test whether the mean mark in the examination is lower this year.

(10)



one-tailed ✓

$\mu = 70$

$\sigma = 6$

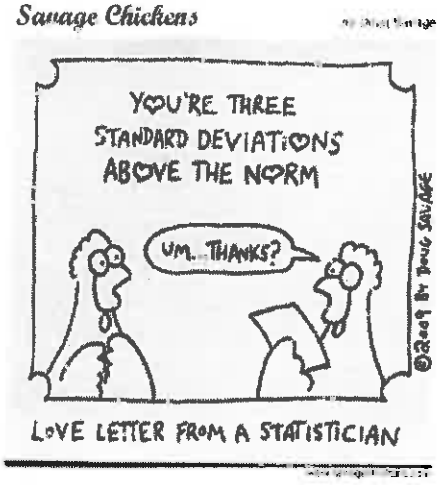
$H_0 : \mu = 70$ ✓

$H_1 : \mu < 70$ ✓

Critical value $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$= \dfrac{67,3 - 70}{\frac{6}{\sqrt{25}}}$ ✓ ✓ ✓

$= -2,125$ ✓



Critical value lies in rejection ✓ region, ∴ reject H_0 ✓ Accept H_1 at 5% level of significance that the mean mark is lower

[10]

QUESTION 3

Ballpoint pens are made in large quantities in a factory. They are tested in batches of 50 and it is known that 3% are faulty.

(a) From a test batch calculate :

(1) the probability that exactly 2 pens are faulty (3)

$P(X = 2) = \binom{50}{2}(0,03)^2(0,97)^{48}$

$= 0,2555$

(2) the probability that at least one of the pens are faulty (4)

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - \binom{50}{0}(0,03)^0(0,97)^{50}$

$= 0,7819$

(b) In 200 batches of 50, how many pens would you expect to be faulty? (2)

$200 \times 50 \times 0,03 = 300$ faulty

"I'm supposed to write an essay on what is meant by 'The pen is mightier than the sword'. First, I need to go to Google and find out what a pen is."

[9]

QUESTION 4

The probability mass function of the discrete random variable X, is given by

$$P(X = x) = a\left(\frac{2}{3}\right)^x \quad for\ all\ x \in \mathbb{N}_0$$

(a) Determine $P(x \leq 1)$ in terms of a. (3)

$$P(x \leq 1) = P(x=0) + P(x=1)$$
$$= a\left(\tfrac{2}{3}\right)^0 + a\left(\tfrac{2}{3}\right)^1$$
$$= a + \frac{2}{3}a$$
$$= \frac{5}{3}a$$

(b) Determine the value of the constant a. (5)

$$\sum_{x=0}^{\infty} P(X=x) = \frac{a}{1-r} = \frac{a}{1-\frac{2}{3}} = 1$$

$$\therefore\ a = \frac{1}{3}$$

[8]

QUESTION 5

The lifetime of a lamp is divided according to the probability-density function:

$$f(t) = \begin{cases} k\left(-t^{\frac{3}{2}} + 8\right) & as\ 0 \le t \le 4 \\ 0 & otherwise \end{cases}$$; k is a constant

(a) Giving your answer as a fraction, determine the value of k (6)

$$\int k\left(-t^{3/2} + 8\right) dt = 1 \checkmark$$

$$\int_0^4 -kt^{3/2} + 8k\,dt = 1$$

$$-\frac{2}{5}kt^{5/2} \checkmark + 8kt \checkmark \Big]_0^4 = 1$$

$$-\frac{2}{5}k(4)^{5/2} + 8k(4) - \left(-\frac{2}{5}k(0)^{5/2} + 8k(0)\right) = 1 \checkmark$$

$$-\frac{64}{5}k + 32k = 1 \checkmark$$

$$1 = \frac{96}{5}k$$

$$\frac{5}{96} = k \checkmark$$

(b) Determine $P(0 < t < 2)$ (5)

$$\int_0^2 -\frac{5}{96}t^{3/2} + 8\left(\frac{5}{16}\right) dt \checkmark \quad \text{can go in calculator!}$$

$$= -\frac{5}{96} \cdot \frac{2}{5}t^{5/2} + \frac{5}{12}t \Big]_0^2 \checkmark\checkmark$$

$$= 0{,}7155 \checkmark\checkmark$$

[11]

QUESTION 6

(a) The 95% confidence interval for the mean length of life (in hours) of a particular brand of light bulb is (1023,3; 1101,7). This interval is based on results from a random sample of 36 light bulbs.

(1) Prove that the mean is 1062,50 and that the standard deviation is 120. Show all working. (8)

$$\bar{x} - 1,96\frac{\sigma}{\sqrt{36}} = 1023,3$$

$$\bar{x} + 1,96\frac{\sigma}{\sqrt{36}} = 1101,7$$

$$2\bar{x} = 1101,7 + 1023,3$$

$$\bar{x} = 1062,50 \text{ given}$$

$$\bar{x} + 1,96\frac{\sigma}{6} = 1101,7$$

$$1062,50 + 1,96\frac{\sigma}{6} = 1101,7$$

$$\sigma = 120 \text{ given}$$

(2) Find the 99% confidence interval for the mean length of life if this brand of light bulb, assuming that the length of life is normally distributed. (6)

$$1062,50 - 2,58 \cdot \frac{120}{6} < \mu < 1062,50 + 2,58 \frac{120}{6}$$

$$1010,90 < \mu < 1114,10$$

(b) A national safety council wants to estimate the proportion of car accidents that involve pedestrians.
How large a sample of accident records must be examined, to be 98% confident that the estimate does not differ from the true proportion by more than 0,03. The proprtion estimated is 0,25.

(6)

$p - 0,25 < 2,575$

$0,03 < 2,575 \sqrt{\frac{0,25 \cdot 0,75}{n}}$

$\frac{b}{515} < \frac{0,25 \cdot 0,75}{n}$

$n < 16,0937$

$n = 16$

[20]

"OH, GREAT. MY INSURANCE AGENT. I WAS JUST TEXTING YOU."

QUESTION 7

(a) Patrick wants to invest some of his monthly salary. He invests a certain amount of this every month for 18 months. For each month there is a probability of 0,25 that he will buy shares in a large company, there is a probability of 0,15 that he will buy shares in a small company and there is a probability of 0,6 that he will invest in a savings account.

Determine the probability that Patrick will buy shares in a small company in exactly 3 of these 18 months. (6)

$$P(X = 3) = \binom{18}{3}(0,15)^3 (0,85)^{15}$$

$$= 0,2406$$

(b) A book club sends 6 paperback and 2 hardback books to Matthew. He chooses 4 of these books at random to take with him on holiday.

Determine the probability that he chooses exactly two paperback books.

(6)

$$P(R=2) = \frac{\binom{6}{2}\binom{2}{2}}{\binom{8}{4}}$$

$$= 0,2143$$

(c) The probability that Richard goes to the gym on any day is 0,85. On a day when he goes to the gym, the probability that he has a whey protein smoothie is 0,9. On a day when he does not go to the gym, the probability that he has a whey protein smoothie is 0,32.

Given that Richard had a whey protein smoothie, find the probability that he went to the gym that day. (4)

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{0,9 \times 0,85}{0,9 \times 0,25 + 0,1 \times 0,32}$$

$$= \frac{0,765}{0,797}$$

$$= 0,9598$$

Filling in that application form was a severe first test

[16]

QUESTION 8

Given

$P(A|B)\frac{2}{5}; P(B|A) = \frac{1}{4}$ and $P(A \cup B) = \frac{11}{20}$

(a) Determine $P(A \cap B)$ in terms of $P(A)$ (3)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \checkmark$$

$$\frac{1}{4}P(A) = P(A \cap B) \checkmark\checkmark$$

(b) Determine $P(A \cap B)$ in terms of $P(B)$ (3)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \checkmark$$

$$\frac{2}{5}P(B) = P(A \cap B) \checkmark\checkmark$$

(c) Hence or otherwise determine $P(A)$ (6)

$$\frac{1}{4}P(A) = \frac{2}{5}P(B) \checkmark$$

$$P(B) = \frac{5}{8}P(A) \checkmark$$

$$\frac{11}{20} \checkmark = P(A) + \frac{5}{8} \checkmark P(A) - \frac{1}{4} \checkmark P(A)$$

$$P(A) = \frac{2}{5} \checkmark$$

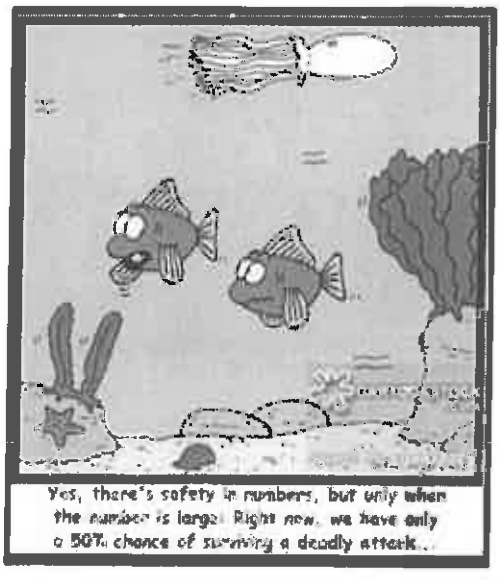
(d) Determine $P(A \cap B)$ (2)

$$P(A \cap B) = \frac{1}{4}P(A)$$

$$= \frac{1}{4} \times \frac{2}{5} \checkmark$$

$$= 0,1 \checkmark$$

[14]



[Total: 100 marks]