



GRADE 12 EXAMINATION  
NOVEMBER 2015

**ADVANCED PROGRAMME MATHEMATICS  
ELECTIVE MODULE: MATRICES AND GRAPH THEORY**

Time: 1 hour

100 marks

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**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 6 pages, an Answer Sheet of 2 pages and an Information Booklet of 3 pages (i – iii). Please check that your question paper is complete.
  2. Detach the Answer Sheet from the middle of the question paper and hand it in together with your Answer Book.
  3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
  4. All necessary calculations must be clearly shown and writing should be legible.
  5. Diagrams have not been drawn to scale.
  6. Round off your answers to two decimal digits, unless otherwise indicated.
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## MATRICES AND GRAPH THEORY

### QUESTION 1

Consider the matrix equation  $\begin{pmatrix} 3 & 0 & -1 \\ 5 & -1 & 6 \end{pmatrix} + 2 \begin{pmatrix} 3 & 3 & 3 \\ -1 & -1 & -1 \end{pmatrix} = M$

- 1.1 Give the dimension of the matrices on the left of the equation. (2)
- 1.2 Calculate the elements of matrix M. (4)
- 1.3 This equation represents a transformation of a figure in a Cartesian plane. Describe this transformation in words. (4)

**[10]**

### QUESTION 2

Padma is to solve a set of simultaneous equations using Gaussian (row) reduction.

The equations given are:  $2x - 5y + z = -12$ ,  $4x + y + 3z = 13$ ,  $6x + 2y - z = -1$

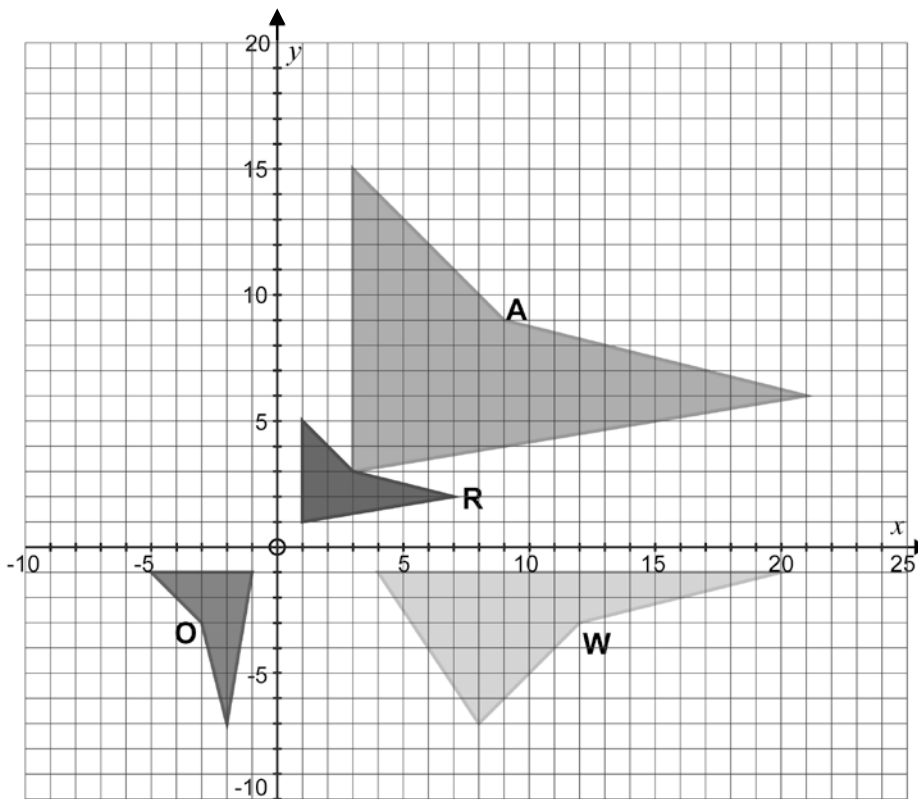
STEP 1:	2	-5	1	A	row 1
	4	1	3	B	row 2
	6	2	-1	C	row 3
STEP 2:	2	-5	1	-12	row 1
	0	11	1	37	row 2 - (2 × row 1)
	0	17	-4	35	
STEP 3:	2	-5	1	-12	
	0	11	1	37	
	0	D	0	E	

- 2.1 In Step 1, Padma writes down an augmented matrix. Give the values for A, B and C. (2)
- 2.2 In Step 2, Padma reduces the second row as indicated. Explain how she has reduced the third row in Step 2. (2)
- 2.3 In Step 3, Padma completes the reduction for the third row. Give the correct values for D and E. (3)
- 2.4 Hence, or otherwise, determine the set of simultaneous solutions to the equations. (3)

**[10]**

**QUESTION 3**

3.1 Sketched below are four figures, all transformations of each other.



Write down a single transformation matrix for each of the following:

- (a) R is mapped onto O (2)
- (b) A is mapped onto R (2)
- (c) O is mapped onto W (4)
- (d) A is mapped onto W (6)

3.2 A figure in a Cartesian plane is reflected across a line through the origin, using the transformation matrix  $\begin{pmatrix} 0,342 & -0,94 \\ -0,94 & -0,342 \end{pmatrix}$ . Calculate the angle of inclination of the line of reflection. (8)

3.3 The point (2; 3) is mapped onto the point (2; 11) by the matrix  $\begin{pmatrix} p & q - 4 \\ q & p \end{pmatrix}$

- (a) Calculate the values of  $p$  and  $q$ . (8)
- (b) Hence, describe the type of transformation in words. (4)

**[34]**

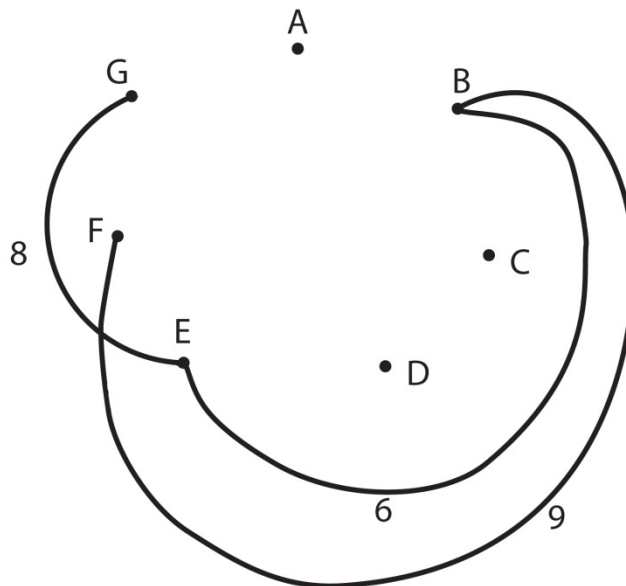
**QUESTION 4**

Answer **BOTH** parts of this question on the Answer Sheet.

A simple, connected graph is given in the form of the adjacency matrix below.

	A	B	C	D	E	F	G
A							
B	10						
C							
D	7	10	4				
E		6					
F	7	9	8	5	9		
G	8				8	5	

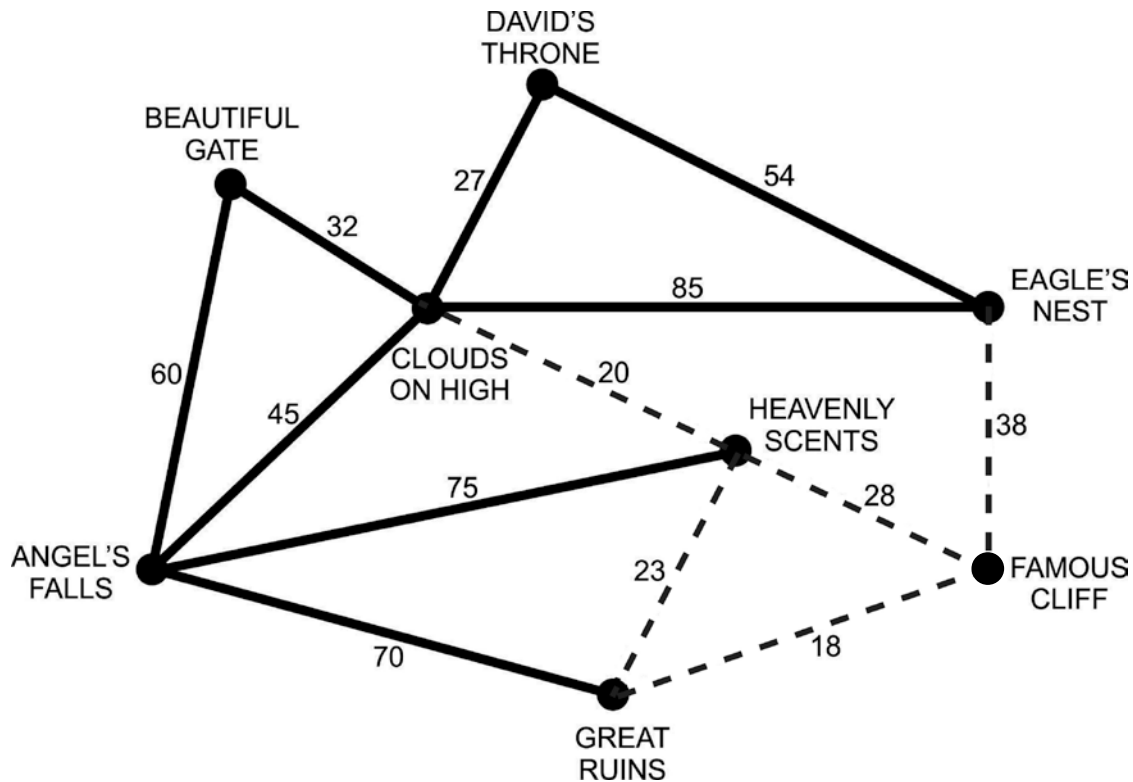
- 4.1 Complete the graph represented by the above adjacency matrix. Clearly indicate the weight of each edge in the graph. Three of the edges have already been drawn. (8)



- 4.2 Use Kruskal's algorithm to create a spanning tree of minimum length. Indicate the order in which edges are chosen, and give the length of the spanning tree. (8)  
[16]

**QUESTION 5**

Dylan promotes eco-tourism in the rural communities of Limpopo. The accompanying graph represents routes between various sights, where the weight of the edges are distances measured in kilometres. The edges printed in bold are tarred roads, while the dashed edges are gravel roads.

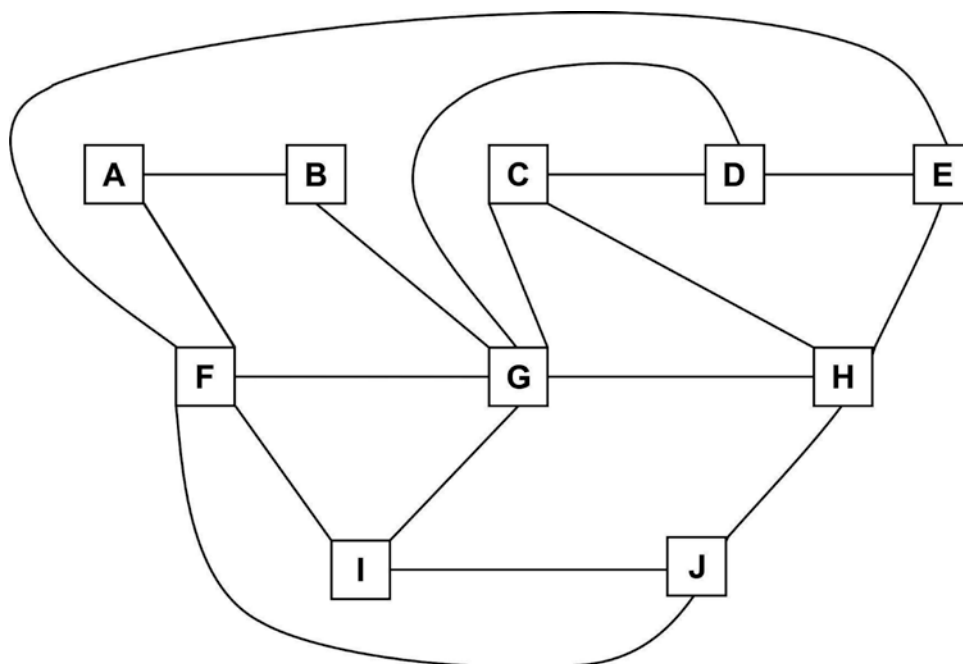


- 5.1 Dylan wants to scout out the area and so needs to drive a circuit using all the roads. Which roads should he use twice in order to minimise his travelling distance? (4)
- 5.2 On a particular day, Dylan decided to leave out the road between 'Clouds on High' and 'Eagle's Nest'. He still drove a circuit using all the other roads.
- Design a possible circuit for him to travel starting at 'Angel's Falls' that would keep his travelling distance to a minimum. State the circuit, clearly indicating which roads need to be doubled, also state the length of this circuit. (8)
- 5.3 On another day, Dylan needs to get from 'Famous Cliff' to 'Angel's Falls'. He has to pass through at least 'Great Ruins' and 'Heavenly Scents'. Travelling on a gravel road costs him 1,3 times what it costs to travel on a tar road. Find, by inspection, the cheapest route for him to travel. Show calculations to validate your answer. (6)
- [18]**

**QUESTION 6**

**Answer all FOUR parts of this question on the Answer Sheet.**

The graph below shows the interaction between ten chemicals, labelled A to J, each stored in its own bottle. Edges directly connecting the vertices A to J indicate the chemicals that react with each other. As the laboratory assistant, you need to store the ten bottles in as few containers as possible. None of the chemicals in each container may react with each other.



Follow the steps of Welsh and Powell's algorithm given below to obtain an estimate of the minimum number of containers needed to store the chemicals.

- 6.1 STEP 1:
- (a) Mark the vertex with the highest degree with a circle. (1)
  - (b) Now mark those vertices that are not directly connected to this starting vertex, or to each other, also with a circle. (3)
- 6.2 STEP 2:
- (a) Mark the vertex of highest degree that has not yet been marked in STEP 1 with a square. (1)
  - (b) Consider all vertices not yet marked with a circle: mark **as many as possible** with a square that are not directly connected to this second starting vertex, or to each other. (3)
- 6.3 STEP 3: Continue this process until all vertices are marked. Use a triangle for this step. (2)
- 6.4 What is the minimum number of containers needed? (2)
- [12]**

**Total: 100 marks**