GRADE 12 EXAMINATION
NOVEMBER 2016

# ADVANCED PROGRAMME MATHEMATICS: PAPER II

## MARKING GUIDELINES

Time: 1 hour 100 marks

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**These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.**

**The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.**

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## MODULE 2 STATISTICS

## QUESTION 1

1.1 (a) $P(X=3)=\binom{10}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^7$

$=0,2013$ (5)

(b) $15\left(\frac{1}{5}\right)=3$ (2)

(c) $P(x\geq 1)>0,95$

$1-\binom{n}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^n>0,95$

$\left(\frac{4}{5}\right)^n<0,05$

$n\log\frac{4}{5}<\log 0,05$

$n>13,4251$

14 people (8)

1.2 (a) $3!\times 4!\times 8!\times 3!=34\ 836\ 480$ (5)

(b) $\frac{\binom{3}{2}\binom{4}{2}\binom{8}{2}}{\binom{15}{6}}=\frac{72}{715}$ or $0,1007$ (5)

(c) $\binom{8}{3}\binom{3}{1}\binom{4}{2}+\binom{8}{3}\binom{3}{2}\binom{4}{1}+\binom{8}{4}\binom{3}{1}\binom{4}{1}$

$=2\ 520$ (8)

**[33]**

## QUESTION 2

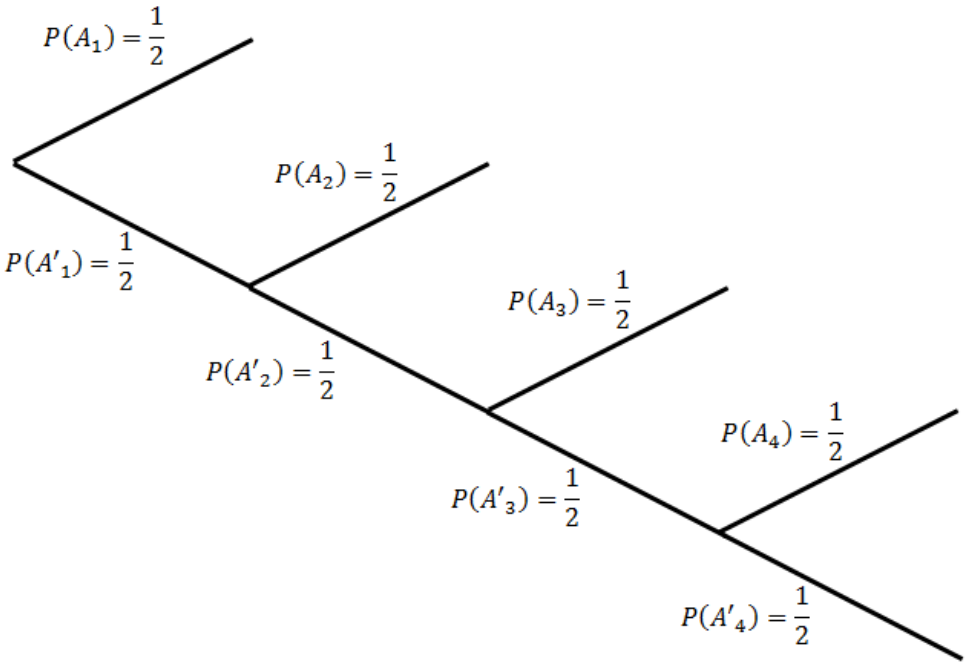
2.1 (a) $y = 10,38 - 1,54x$ (4)

(b) $r = -0,983$
Data lie close to the regression line. (2)

(c) $y = 10,38 - 1,54(35)$
$= -43,52$ (2)

(d) Reliable but caution is required as although extrapolation occurs, correlation is very strong. (2)

2.2 (a)




(4)

(b)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{16}$ |

(4)

(c) $E[X] = 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{16}\right) + 4\left(\frac{1}{16}\right) = 0,9375$ (2)

**[20]**

## QUESTION 3

3.1 (a) A 96% confidence interval for p is:

$$0,8 \pm 2,05\sqrt{\frac{(0,8)(0,2)}{250}}$$

$$(0,7481\,;0,8519)$$ (6)

(b) 96% of the time the interval will contain the population proportion. (2)

3.2 (a) $\sim N(\mu\,;\sigma^2)$

$$\sigma = \frac{3}{5}\mu$$

$$P(x < 2\mu) = P\left(z < \frac{2\mu - \mu}{\frac{3}{5}\mu}\right)$$

$$= P(z < 1,67)$$

$$= 0,5 + 0,4525$$

$$= 0,9525$$ (8)

(b) $$-0,84 = \frac{\frac{1}{3}\mu - \mu}{2}$$

$$\frac{-42}{52} = -0,84 \times 2 = \frac{-2}{3}\mu$$

$$\mu = +2,52$$ (7)

**[23]**

## QUESTION 4

4.1 A one-tailed test should be used as Gareth would like to see if there has been an increase in his travel time from last year to this year. (2)

4.2 $H_0 : \mu = 45,7$

$H_1 : \mu > 45,7$

Rejection region: reject $H_0$ if $z > 1,75$

Test Statistic:

$$z = \frac{47,4 - 45,7}{\frac{3,2}{\sqrt{8}}} = 1,5026$$

Conclusion: since $z < 1,75$, we fail to reject the $H_0$ at the 4% level of significance and suggest insufficient evidence to support the claim, hence no significant increase in Gareth's journey time. (10)

**[12]**

## QUESTION 5

5.1

| | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **Arnie** | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| **Michael** | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| **Connor** | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{2}{7}$ | $\frac{3}{7}$ |

For entrance 1:

$$P(x \geq 2) = P(A \cap M \cap C') + P(A \cap M' \cap C) + P(A' \cap M \cap C) + P(A \cap M \cap C)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{6}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right)$$

$$= \frac{11}{56} \quad (6)$$

5.2 $\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{2}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{3}{7}\right)$

$$= \frac{3}{56} \quad (6)$$

**[12]**

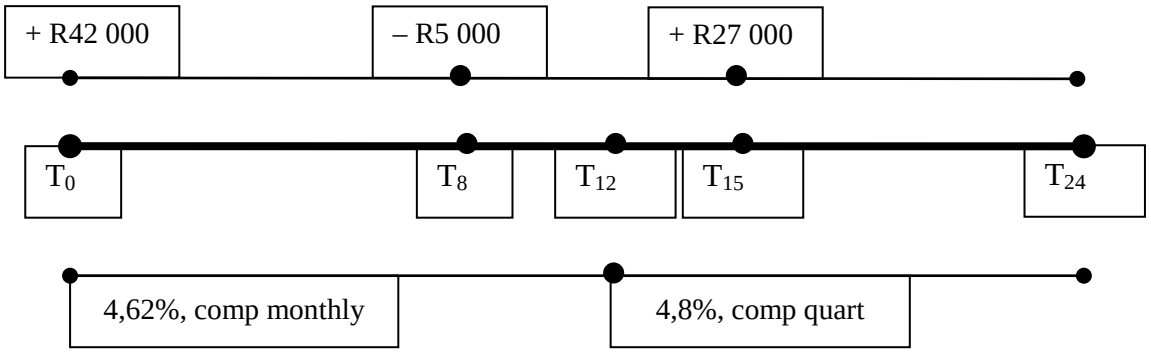
**Total for Module 2: 100 marks**

## MODULE 3 FINANCE AND MODELLING

## QUESTION 1

1.1




(6)

1.2 $42\ 000\left(1+\frac{0,0462}{12}\right)^{12}\left(1+\frac{0,048}{4}\right)^{4} = 46\ 131,4622$

$5\ 000\left(1+\frac{0,0462}{12}\right)^{4}\left(1+\frac{0,048}{4}\right)^{4} = 5\ 325,58533$

$27\ 000\left(1+\frac{0,048}{4}\right)^{3} = 27\ 983,71066$

$46\ 131,46 - 5\ 325,58 + 27\ 983,71 =$ **68 789,59**

**OR**

$42\ 000\left(1+\frac{0,0462}{12}\right)^{8} = 43\ 311,1661$

$38\ 311,1661\left(1+\frac{0,0462}{12}\right)^{4} = 38\ 904,5740$

$38\ 904,5740\left(1+\frac{0,048}{4}\right) = 39\ 371,4289$

$66\ 371,4289\left(1+\frac{0,048}{4}\right)^{3} =$ **68 789,59** (10)

**[16]**

## QUESTION 2

$$1\ 000\ 000 - 30\ 000 = \frac{30\ 000\ \left[1-\left(1+\frac{0,064}{4}\right)^{-n}\right]}{\frac{0,064}{4}}$$

$$0,482\ 666 = 1,016^{-n}$$

n = 45,890 quarters = **11 years, 5 or 6 months**

**OR**

$$1\ 000\ 000 = \frac{30\ 000\ \left(1+\frac{0,064}{4}\right)\ \left[1-\left(1+\frac{0,064}{4}\right)^{-(n+1)}\right]}{\frac{0,064}{4}}$$

$$0,475\ 066 = 1,016^{-(n+1)}$$

$\therefore$ n = 45,890 quarters = **11 years, 5 or 6 months**

**[12]**

## QUESTION 3

3.1 33 000 = 30 400(1 + r) ***r* = 8,55%**
30 400 = 28 000(1 + r) ***r* = 8,57%** (6)

3.2 $\frac{270\ \left[\left(1+\frac{0,052}{12}\right)^{18\times12+1}-1\right]}{\frac{0,052}{12}}$ = **96 928,20** (6)

3.3 96 928,20 – 270 × (18 × 12 + 1) = **38 338,20** (4)

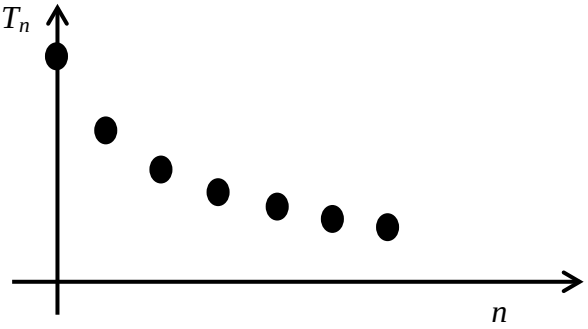
3.4 $33\ 000 + 30\ 400\left(1+\frac{0,052}{12}\right)^{12} + 28000\left(1+\frac{0,052}{12}\right)^{24}$
= **96 080,86**
**Yes**, it will cover the costs (8)

**[24]**

## QUESTION 4

4.1 $T_3 = 4(2) + 3(-2) - 4(-1)^{2-1} = 6$
$T_4 = 4(-2) + 3(2) - 4(-1)^{3-1} = -6$
$T_5 = 4(2) + 3(-2) - 4(-1)^{4-1} = 6$ (5)

4.2




decreasing
discrete
exponential
*y*-int, and no *x*-int

(4)

4.3 $P_{n+1} = \overbrace{54 + r \cdot 54}\left(1 - \frac{54}{120}\right) = 54 + 29,7r$

$P_{n+2} = P_{n+1} + r \cdot P_{n+1}(1 - \frac{P_{n+1}}{120})$

$70 = (54 + 29,7r) + r(54 + 29,7r)(1 - \frac{54 + 29,7r}{120})$

$r = 0,27$ ('solve' function) (9)

**[18]**

## QUESTION 5

5.1 Single data points, not joined by a curve
Measurements taken at intervals; measurements between intervals not considered (2)

5.2 1 800 < prey < 3 800 (2)

5.3 35–40 (2)

5.4 rate of decrease : steeper gradient (2)

5.5 (a) b – rate of deadly interactions between predator and prey (3)

(b) 1/c = life span of predator; hence predators live for shorter periods
**OR** cF = how many predators die in each cycle; hence more predators die in each cycle.
Fewer predators implies increase in prey populations or slower decline. (3)

5.6 C prey initially increasing and predator initially decreasing with tendency to equilibrium. (4)

**[18]**

## QUESTION 6

6.1 G (2)

6.2 31 (2)

6.3 $T_{n+1} = 2,\cdot T_n + 1\ ,\ T_1 = 1$ (4)

6.4 $1\ 000\ 000 = 2^n - 1$ $\quad n < 19,93 = 19$ (4)

**[12]**

**Total for Module 3: 100 marks**

## MODULE 4 MATRICES AND GRAPH THEORY

### QUESTION 1

1.1 $2 \times 4 + 4 \times (-1) = z$ **$z = 4$**

$x + 2y = 4$ and $4x - y = -11$ **$x = -2,\ y = 3$** (8)

1.2 (a) false (2)

(b) false (2)

(c) true (2)

(d) true (2)

**[16]**

### QUESTION 2

2.1 (a) translation 3 units right and 1 unit down (3)

(b) yes (1)

(c) $M = \begin{pmatrix} 4 & 2 & 5 \\ 3 & 2 & -1 \end{pmatrix}$ (2)

2.2 (a) shear of factor 3, y-axis invariant (3)

(b) no (1)

(c) determinant (1)

(d) det = 1 area of figure and image unchanged (3)

2.3 $\tan A = 3$ $A = 71{,}565^{o}$

$$\begin{pmatrix} \cos 143{,}13 & \sin 143{,}13 \\ \sin 143{,}13 & -\cos 143{,}13 \end{pmatrix} \begin{pmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{pmatrix} = \begin{pmatrix} -0{,}39 & 0{,}92 \\ 0{,}92 & 0{,}39 \end{pmatrix}$$ (8)

**[22]**

## QUESTION 3

3.1 $\begin{pmatrix} 6 & 2 & -3 \\ 6 & 3 & 1 \\ 9 & 3 & -2 \end{pmatrix}^{-1} = \frac{1}{15}\begin{pmatrix} -9 & -5 & 11 \\ 21 & 15 & -24 \\ -9 & 0 & 6 \end{pmatrix}$ **OR** $\begin{pmatrix} -0,6 & -0,333 & 0,733 \\ 1,4 & 1 & -1,6 \\ -0,6 & 0 & 0,4 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{15}\begin{pmatrix} -9 & -5 & 11 \\ 21 & 15 & -24 \\ -9 & 0 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ 15 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{15}\begin{pmatrix} -5 \\ 75 \\ 30 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ 5 \\ 2 \end{pmatrix}$$ (8)

3.2 $6(3t - 3) - 2(6t - 9) - 3\,(18 - 27) = 0$
$6t + 27 = 0$ **$t = -4,5$** (6)

3.3 scale factor of 1,5: **$w = 3$** (2)
**[16]**

## QUESTION 4

4.1 VW (2)

4.2 triangular inequality does not hold (2)

4.3 W V R Q U P S T W or W V R U Q P S T W (4)

4.4 PSTW = 15
$2 + 3 + x + 3 + 2 < 15$ $x < 5$ so **$x = 4$** (6)
**[14]**

## QUESTION 5

5.1 English, Swahili (2)

5.2 E → X (48) → Z (45) → S (72) → F (64) → P (58) → A (60) → E (85)
**= 432 min** (8)

5.3 E → P (65) → A (60) → F (62) → S (64) → X (62) → Z (45) → E (50)
**= 408 min**

**OR**

E → Z (50) → X (45) → S (62) → F (64) → A (62) → P (60) → E (65)
**= 408 min**

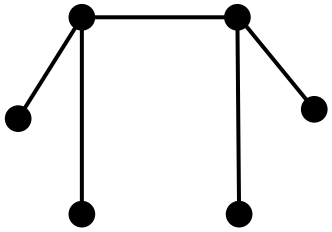
**OR** any other acceptable **circuit < 412 min** (10)
**[20]**

## QUESTION 6

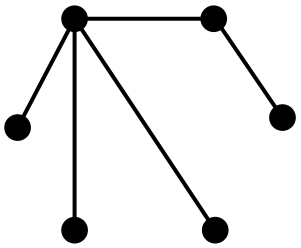
6.1 5 edges (2)

6.2 $2 \times 5 = 10$ (2)

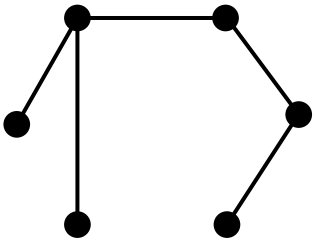
6.3 (8)



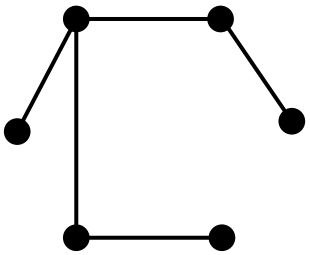
Vertices 1, 1, 1, 1, 3, 3



Vertices 1, 1, 1, 1, 2, 4



Vertices 1, 1, 1, 2, 2, 3



Vertices 1, 1, 1, 2, 2, 3

**[12]**

**Total for Module 4: 100 marks**

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