GRADE 12 EXAMINATION
NOVEMBER 2019

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 1 hour 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1 (a) $\dfrac{\binom{4}{1}\binom{7}{2}}{\binom{11}{3}} = \dfrac{28}{55} = 0,5091$

(b) $\left(\frac{4}{11}\right)\left(\frac{3}{10}\right)\left(\frac{7}{9}\right)+\left(\frac{4}{11}\right)\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)+\left(\frac{7}{11}\right)\left(\frac{4}{10}\right)\left(\frac{6}{9}\right)+\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)=\frac{7}{11}$

1.2 (a) $20(0,1)=2$

(b) $P(X \leq 3) = 1-\left(\binom{5}{4}(0,3)^4(0,7)+(0,3)^5\right)$
$= 0,9692$

(c) $X \sim B(200;0,6)$
since $np > 5$ and $nq > 5$
$X \sim N\left(120;\sqrt{48}^2\right)$
$P(X > 125) \rightarrow P(X > 125,5)$
$= P\left(Z > \dfrac{125,5-120}{\sqrt{48}}\right)$
$= P(Z > 0,79)$
$= 0,5 - 0,2852$
$= 0,2148$

QUESTION 2

2.1 (a)
$$E[X] = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{2}\right) + 3\left(\frac{2}{9}\right) + 4\left(\frac{1}{9}\right)$$
$$= 2,28$$
$$Var(X) = 1\left(\frac{1}{6}\right) + 4\left(\frac{1}{2}\right) + 9\left(\frac{2}{9}\right) + 16\left(\frac{1}{9}\right) - (2,28)^2$$
$$= 0,746$$
$$\sigma_x = 0,86$$

(b) The mean would decrease and the standard deviation would increase.

2.2 (a)
$$\int_0^4 \frac{k}{x+1}\,dx = 1$$
$$\left[k\,ln(x+1)\right]_0^4 = 1$$
$$k(ln\,5 - ln\,1) = 1$$
$$k\,ln\,5 = 1$$
$$\therefore k = \frac{1}{\ln 5}$$

(b)
$$\frac{1}{\ln 5}\left[\ln(x+1)\right]_0^m = \frac{1}{2}$$
$$\left[\ln(m+1) - \ln 1\right] = \frac{1}{2}\ln 5$$
$$\ln(m+1) = \ln\sqrt{5}$$
$$m+1 = \sqrt{5}$$
$$\therefore m = \sqrt{5} - 1 \text{ or } (1,2361)$$

QUESTION 3

3.1 (a) $P(R) = P(Z > 1,1)$
$= 0,5 - 0,3643$
$= 0,1357$

(b) $P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$
$= 0,1357 + 0,9282 - P(1,1 < Z < 1,8)$
$= 0,1357 + 0,9282 - (0,4641 - 0,3643)$
$= 0,9641$

OR

$P(R \cup Q) = P(Z > -1,8) = 0,5 + 0,4641 = 0,9641$

3.2 $X \sim N(200; 50^2)$

$$P(X > c|X > 280) = \frac{P(X > c)}{P(X > 280)} = 0,625$$

$$P(X > 280) = P\left(Z > \frac{280 - 200}{50}\right)$$
$= P(Z > 1,6)$
$= 0,5 - 0,4452$
$= 0,0548$

$$\therefore \frac{P(X > c)}{0,0548} = 0,625$$

$P(X > c) = 0,0343$

$$\therefore 1,82 = \frac{c - 200}{50}$$

$c = 291$

QUESTION 4

4.1 (a) A 98% CI for p is:

$$\frac{1}{5} \pm 2,33\sqrt{\frac{(0,2)(0,8)}{300}}$$

(0,1462; 0,2538)

(b) Since 15% is in the interval there is no evidence to suggest that the percentage of residents have approved the revised plan.

4.2 (a) $H_0 : \mu_x = \mu_y$

$H_1 : \mu_x > \mu_y$

Reject H_0 if $z > 2,05$

Test Statistic:

$$z = \frac{30,06 - 29,84}{\sqrt{\frac{0,0784}{60} + \frac{0,168}{50}}} = 3,22$$

Conclusion: Since $z > 2,05$ reject H_0 and suggest sufficient evidence to support the claim that the mean volume from the first machine is greater than the mean volume of the second machine.

(b) $$z = \frac{30,06 - 29,84 - 0,1}{\sqrt{\frac{0,0784}{60} + \frac{0,168}{50}}} = 1,76$$

$$P(z > 1,76) = 0,5 - 0,4608$$
$$= 0,0392$$
$$\therefore \alpha > 3,9\%$$

QUESTION 5

5.1 $$\frac{9!}{3!3!} = 10080$$

5.2 An example of such an arrangement:

* C * A * L (EE) S * S * S *

6 places for other E

$$\therefore \frac{7!}{3!} \times 6 = 5040 \quad \text{or} \quad \frac{8!}{3!} - 2\left(\frac{7!}{3!}\right) = 5040$$

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

1.1 B

1.2 C

1.3 A

1.4 B

QUESTION 2

2.1 $920\,000 = 1\,850\,000\,(1-i)^4$ $\therefore$ ***i* = 16,02%**

2.2 $$2\,680\,000 - 920\,000 = \frac{x\left[\left(1+\frac{0,042}{12}\right)^{46}-1\right]\left(1+\frac{0,042}{12}\right)^{3}}{\frac{0,042}{12}}$$

$\therefore$ ***x* = 34 961,87**

QUESTION 3

3.1 $x + 1\,000 = x\left(1 + \frac{0,082}{4}\right)^4$ ***x* = 11 826,46**

3.2 $2\,600(x + 0,025) + 1\,800(x) = 274$

$4\,400x = 209$

$x = 0,0475$

***x* = 4,75% + 2,5% = 7,25%**

3.3 $10\,000\left(1 + \frac{0,072}{12}\right)^n = 12\,000\left(1 + \frac{0,064}{12}\right)^n$

$$\frac{5}{6} = \left(\frac{\frac{377}{375}}{\frac{503}{500}}\right)^n = \left(\frac{1\,508}{1\,509}\right)^n$$

***n* = 275 months**

QUESTION 4

4.1 Logistics: carrying capacity present

4.2 (a) S-shaped (b) Linear

4.3 $0,65 \times 0,82 - \frac{4}{50} = 0,453$

4.4 $R_{n+1} = R_n + a.R_n\left(1 - \frac{R_n}{40000}\right) - 4\ 000 \quad with\ R_{n+1} = R_n$

$a.(18\ 000)\left(1 - \frac{18\ 000}{40\ 000}\right) = 4\ 000$ a = 0,404

4.5 $1 + 0,4 = (1 + a)^4$

a = 0,087 757 per annum

$R_{n+1} = R_n + 0,087.R_n\left(1 - \frac{R_n}{40\ 000}\right) \quad with\ R_0 = 18\ 000$

R_6 = 23 235/6

OR

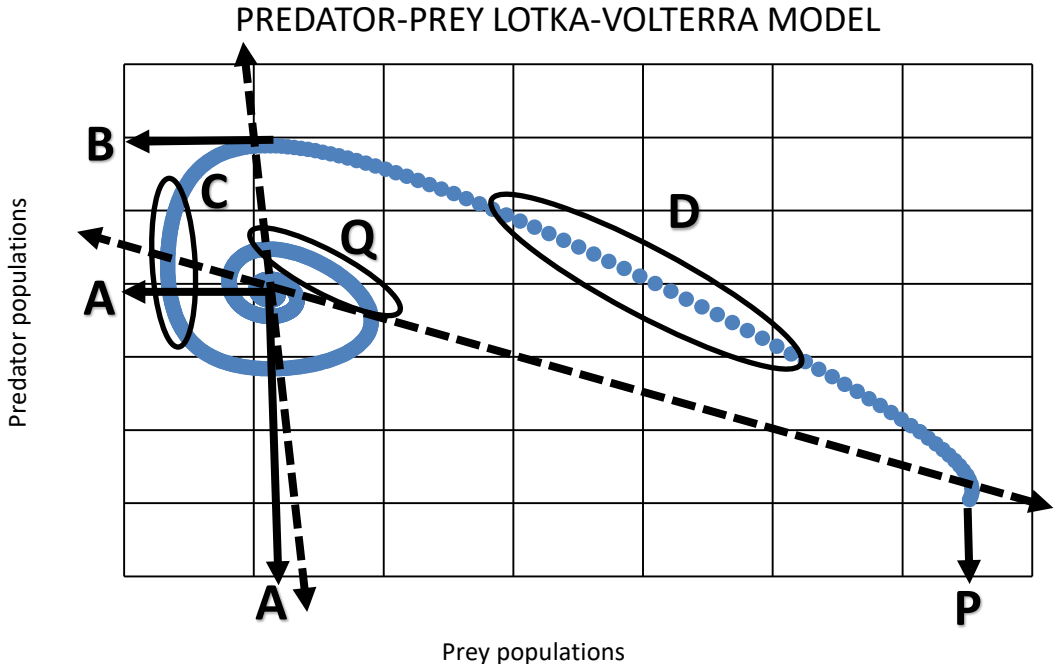
$$1 + 0,4 = \left(1 + \frac{a}{4}\right)^4$$

a = 0,351 092 four-yearly cycle, compounded per annum

$R_{n+1} = R_n + \frac{0,351092}{4}.R_n\left(1 - \frac{R_n}{40\ 000}\right) \quad with\ R_0 = 18\ 000$

R_6 = 23 235/6

QUESTION 5



5.1 (a) A on phase plot (b) B on phase plot

5.2 (a) C on phase plot (b) D on phase plot

5.3 pair of axes
accuracy (passing through equilibrium pt)
accuracy (passing through max/min values of prey)
accuracy (passing through max/min values of predator)

QUESTION 6

6.1 $T_1 = 20\ 000\left(1+\frac{0,048}{12}\right)+400 = 20\ 480$

$T_2 = 20\ 480\left(1+\frac{0,048}{12}\right)+400(1,005) = 20\ 963,92$

$T_3 = 20\ 963,92\left(1+\frac{0,048}{12}\right)+400(1,005)^2 = 21\ 451,48$

6.2 $T_n = 1,004.\ T_{n-1} + 400\ (1,005)^{n-1}$, $T_0 = 20\ 000$

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 $A^{-1} = -\frac{1}{7}\begin{pmatrix} -1 & -4 \\ -3 & -5 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 1 & 4 \\ 3 & 5 \end{pmatrix}$

1.2 $3 - 3z = 12$ $\quad z = -3$
$y - 3 = 0$ $\quad y = 3$
$1 - 3(1) = x$ $\quad x = -2$

1.3 (a) k (b) – k (c) – 3k (d) k

QUESTION 2

2.1 (a) translation 2 units right

(b) factor = – 3

2.2 $\begin{pmatrix} \cos 2A & \sin 2A \\ \sin 2A & -\cos 2A \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3,232 \\ -1,598 \end{pmatrix}$

$3\cos 2A - 2\sin 2A = 3,232$ and $3\sin 2A + 2\cos 2A = -1,598$

$\cos 2A = 0,5$ and $\sin 2A = -0,866$

$2A = 360^o - 60^o$

$A = 150^o$

2.3 (a) $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} t & t \\ v & r \end{pmatrix} = \begin{pmatrix} \boldsymbol{t + kv} & \boldsymbol{t + kr} \\ \boldsymbol{v} & \boldsymbol{r} \end{pmatrix}$

(b) $m = \frac{v - r}{(t + kv) - (t + kr)} = \frac{v - r}{k(v - r)} = \boldsymbol{\frac{1}{k}}$

QUESTION 3

3.1 More zeroes, hence easier multiplications.

3.2 $det = -(-1).\begin{vmatrix} 2 & 1 & 0 \\ 9 & 3 & 1 \\ -1 & 5 & 7 \end{vmatrix} + 0 - 3.\begin{vmatrix} 2 & 2 & 0 \\ 4 & 9 & 1 \\ 0 & -1 & 7 \end{vmatrix} + 0$ **= –248**

OR

$Det = -0 + 0 - 1.\begin{vmatrix} 2 & 2 & 1 \\ -1 & 0 & 3 \\ 0 & -1 & 5 \end{vmatrix} + 7.\begin{vmatrix} 2 & 2 & 1 \\ -1 & 0 & 3 \\ 4 & 9 & 3 \end{vmatrix}$ **= –248**

3.3 $\frac{1}{-248}\begin{pmatrix} -192 & 32 & 42 & -6 \\ 100 & 4 & -49 & 7 \\ -64 & -72 & 14 & -2 \\ 60 & 52 & -17 & -33 \end{pmatrix}$

QUESTION 4

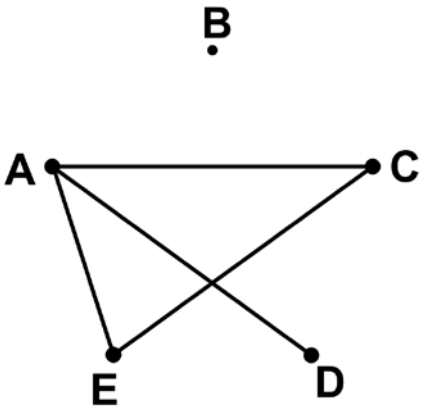
4.1 (a) *n – 1*

(b) *n/2(n – 1)*

(c) *n(n – 1)*

4.2 (a) A, B, C, D, E, B, D or its reverse or many other options
Start at A or D, end at D or A, use all edges once only.

(b)



5 vertices, 4 edges, connectivity

QUESTION 5

5.1 all vertices do not have the same degrees

5.2 no graphs have HCs

5.3 A, C, D

QUESTION 6

6.1

DF	10
HC	9
HA, DE	8
JD, GF	7
AJ	6
AB	5

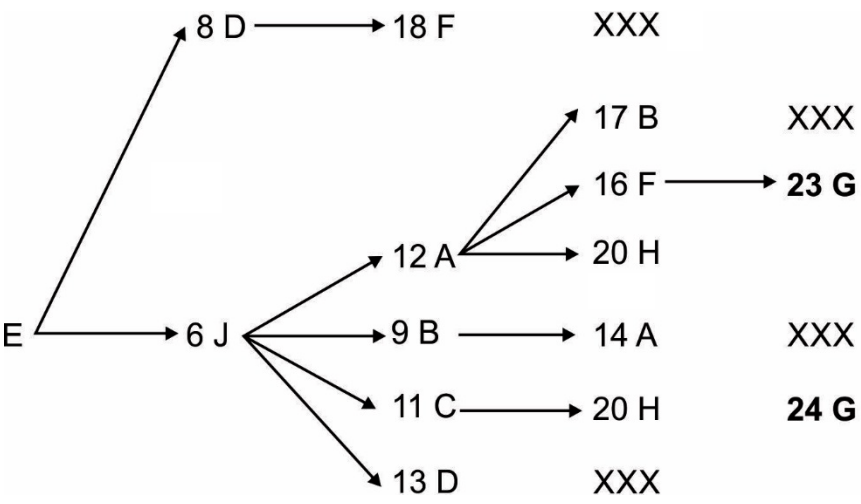
max spanning tree = 60

6.2

	A	B	C	D	E	F	G	H	J	
E				8E					6E	
J	12J	9J	11J	13 J						
D						18D				
B	14B									
C								20C		
A		17A				16A		20A		
F							**23F**			
H							**24F**			

E J A F G = 23

OR



E J A F G = 23

Total for Module 4: 100 marks