GRADE 12 EXAMINATION
NOVEMBER 2020

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 1 hour 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1 $$\frac{\binom{6}{3}\checkmark+\binom{5}{3}\checkmark+\binom{3}{3}\checkmark}{\binom{14}{3}\checkmark}=\frac{31}{364}\checkmark \text{ or } (0,0852)$$ (8)

1.2 (a) $$P(X>3)=\binom{5}{4}\checkmark(0,7)^4\checkmark(0,3)\checkmark+\binom{5}{5}\checkmark(0,7)^5\checkmark(0,3)^0\checkmark=0,5282\checkmark$$ ✓binomial (8)

(b) (i) $X \sim N(42\checkmark;12,6\checkmark)$

$P(X \geq 40) \rightarrow P(X>39,5)$ ✓

$=P\left(Z>\frac{39,5-42}{\sqrt{12,6}}\right)\checkmark$

$=P(Z>-0,7)$ ✓

$=0,5\checkmark+0,2580\checkmark$

$=0,7580$ ✓ (8)

(ii) Since $np=42>5\checkmark$ and $nq=18>5\checkmark$ the Normal approximation can be used. (2)

(iii) This allows for symmetry ✓ and not for the distribution to be either positively or negatively skewed. ✓ (2)

[28]

QUESTION 2

2.1 $X \sim N(2\sigma;\sigma^2)$ ✓

$P(X>5,2)=0,9$ ✓

$\therefore -1,28\checkmark=\frac{5,2-2\sigma}{\sigma}\checkmark$

$-1,28\sigma=5,2-2\sigma\checkmark$

$0,72\sigma=5,2$ ✓

$\therefore \sigma=7,22\checkmark \Rightarrow \mu=14,44$ ✓ (8)

2.2 (a) A 94% CI for p is:

$0,2\checkmark\pm 1,88\checkmark\sqrt{\frac{(0,2)(0,8)}{100}}\checkmark$

$(0,1248\checkmark;0,2752)\checkmark$ (6)

(b) If dice was unbiased each value would have a probability of 0,1667✓ of being thrown, since 0,1667 is within the CI✓ the die would not be considered biased. (2)

[16]

QUESTION 3

3.1 (a) ✓ $E[X] = 0,67$

$\therefore 1(0,35) + 2p + 3q = 0,67$ ✓

$2p + 3q = 0,32$ ✓

$p + q + 0,85 = 1$ ✓

$\therefore p + q = 0,15$ ✓ and $2p + 3q = 0,32$

Solving simultaneously:

$p = 0,13$ ✓ and $q = 0,02$ ✓ (6)

(b) $Var(X) = (1)^2(0,35) + (2)^2(0,13) + (3)^2(0,02) - (0,67)^2$ ✓

$= 0,6011$ ✓

$\sigma_x = 0,7753$ ✓ (5)

3.2 $g(x)$ has an area ✓ greater ✓ than 1 and $h(x)$ has a negative ✓ probability. ✓ (4)

3.3 (a) x. ✓ The median is where the area is cut in half and since there is a much larger area to ✓ the right of midway of the x-values, the median will be closer to 2. ✓ (3)

(b) w, ✓ as higher and lower values more likely. ✓ (2)

(c) $P(t < 0,5) = \int_0^{0,5} \left(-\frac{1}{2}x + 1\right) dx = \frac{7}{16}$ ✓ ✓ ✓ ✓ ✓ ✓ (6)

[26]

QUESTION 4

4.1 $H_0 : \mu = 12$ ✓

$H_1 : \mu < 12$ ✓ (2)

4.2 Test Statistic: $Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$-1,96 < \frac{11,7 - 12}{\frac{0,5}{\sqrt{n}}} < -1,645$ ✓ ✓ ✓ ✓

$-1,96 < \sqrt{n}\left(\frac{11,7 - 12}{0,5}\right) < -1,645$

$2,742 < \sqrt{n} < 3,267$ ✓ ✓

✓ $7,52 < n < 10,67$; $n \in Z$

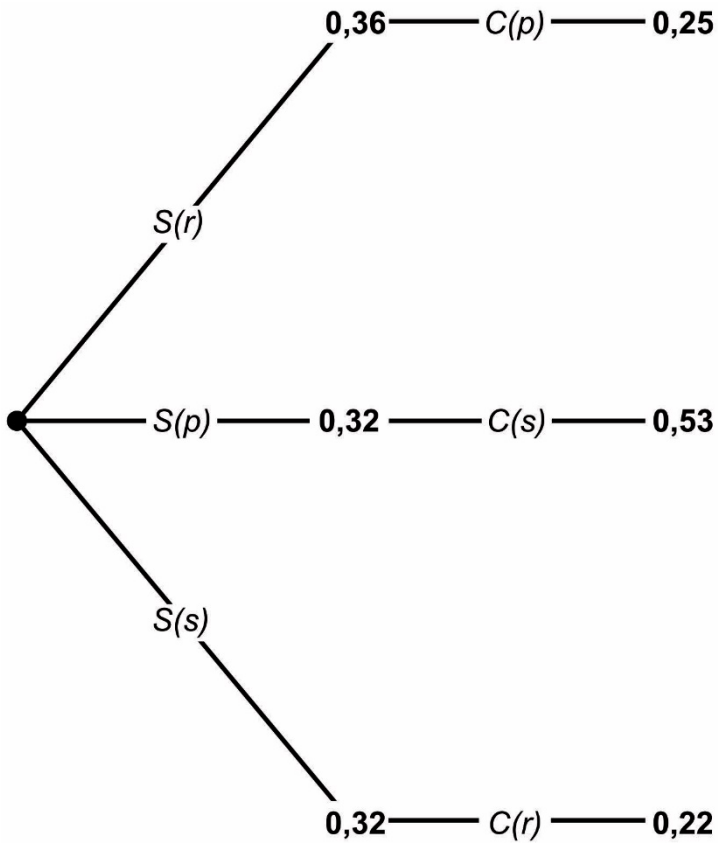
or $8 \leq n \leq 10$; $n \in Z$ ✓ (10)

[12]

QUESTION 5

5.1 $P(\text{Charlie wins}) = P(R_C)P(S_S) + P(P_C)P(R_S) + P(S_C)P(P_S)$
$= (0,22)(0,32) + (0,25)(0,36) + (0,53)(0,32)$
$= 0,33$

OR:



(8)

5.2 (a) $20 \times 19 \times 18 = 6\,840$ (5)

(b) $20^3 - 20 = 7\,980$

OR

(a) + a student winning two prizes
$= 6\,840 + 3(20 \times 19)$
$= 7\,980$ (5)

[18]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

1.1 $0,0775 = \left(1+\frac{r}{12}\right)^{12} - 1$ ✓

$r = 0,0749$ ✓

$$F_{37} = \frac{\checkmark 1\,500\left[\left(1+\frac{0,0749}{12}\right)^{\checkmark 37} - 1\right]}{\frac{0,0749}{12}} \checkmark$$

$= \text{R}62\,214,60$ or $\text{R}62\,212,31$ (exact r)

$= \text{R}62\,210$ (nearest 10) ✓ (6)

1.2 $$62\,210\left(1+\frac{\overset{\checkmark}{0,0749}}{12}\right)^{\overset{\checkmark}{48}} + \underset{\checkmark}{\frac{x\left[\left(1+\frac{\overset{\checkmark}{0,0749}}{12}\right)^{\overset{\checkmark}{48}} - 1\right]}{\frac{0,0749}{12}}} \checkmark = 206\,530,90 \checkmark$$

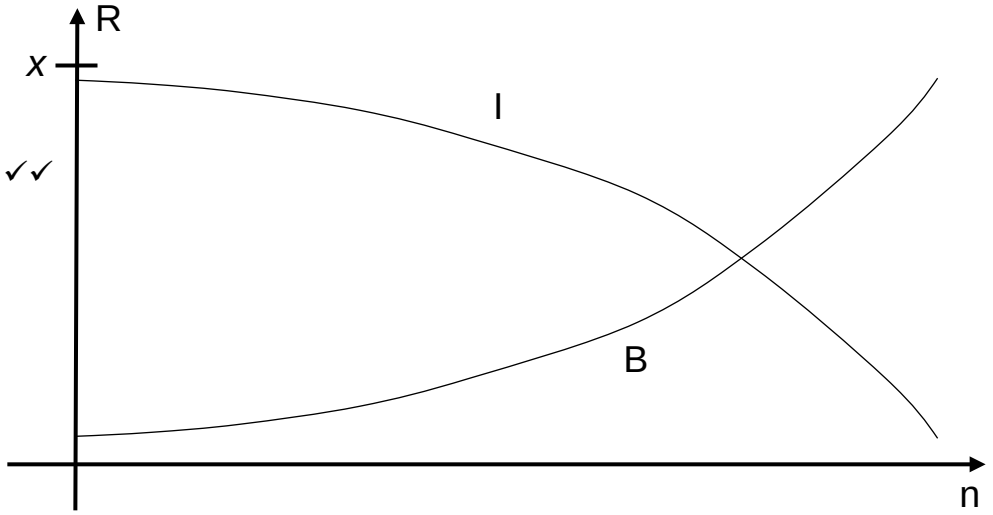
$= \text{R}2\,199,75$ or $\text{R}2\,199,99...$

$\therefore x = \text{R}2\,200$ (nearest rand) ✓✓ (9)

[15]

QUESTION 2

2.1



✓✓

The amount that goes to interest, *I*, is a percentage of the outstanding balance and therefore this follows the general shape of *OB*. The value of *B* = *x* – *I*. Therefore, the graph of *B* is *I* reflected in the *x*-axis and shifted up by *x*. ✓✓ (4)

2.2 (a) $$317\ 279{,}95 \overset{\checkmark}{=} \frac{x\left[1-\left(1+\frac{0{,}1125}{12}\right)^{\overset{\checkmark}{-132}}\right]}{\frac{0{,}1125}{12}} \checkmark$$

$$\therefore x = \text{R}4\ 200 \checkmark$$ (5)

(b) $$P\left(1+\frac{0{,}1125}{12}\right)^{\overset{\checkmark\checkmark}{7}} = \frac{\overset{\checkmark}{4\ 200}\left[1-\left(1+\frac{0{,}1125}{12}\right)^{\overset{\checkmark}{-173}}\right]}{\frac{0{,}1125}{12}} \checkmark$$

$$\therefore P = \text{R}358\ 836{,}34 \checkmark\checkmark$$ (7)

(c) Reduction in outstanding balance
$= 358\ 836{,}34 - 317\ 279{,}95$ ✓
$= \text{R}41\,556{,}39$ ✓

Amount paid
$= 41 \times 4\ 200$ ✓
$= \text{R}172\ 200$ ✓

Interest paid
$= 172\ 200 - 41\,556{,}39$ ✓
$= \text{R}130\ 644$ ✓ (6)

(d) $317\ 279{,}95\left(1+\frac{0{,}1125}{12}\right)^{2} = \frac{x\left[1-\left(1+\frac{0{,}1125}{12}\right)^{-130}\right]}{\frac{0{,}1125}{12}}$ ✓✓

$= \text{R}4\ 312{,}59$ ✓✓ (8)
[30]

QUESTION 3

3.1 *b* is the regular payment ✓
F_0 is the initial amount in the account ✓ (2)

3.2 26 200 = 15 000 *a* + *b* ..... (1) ✓
38 296 = 26 200 *a* + *b* ..... (2) ✓
(2) – (1): 12 096 = 11 200a ✓✓
✓ *a* = 1,08 and *b* = 10 000 ✓ (6)

3.3 *r* = 0,08 = 8% ✓✓ (2)
[10]

QUESTION 4

4.1 Logistical growth ✓ (1)

4.2 0 = –0,0005S + 0,35 ✓✓
s = 700 ✓ (3)

4.3 *r* = 0,35 ✓✓ (2)

4.4 $S_{n+1} = S_n + 0{,}35S_n\left(1-\frac{S_n}{700}\right)$ ✓✓ (4)

4.5 8 ✓✓ (2)
[12]

QUESTION 5

5.1 (a) This term represents the reduction in the zebra population caused by attacks by lions. ✓✓ (2)

(b) For equilibrium of lions:

✓ $L_E = L_E + f.b.Z_E.L_E - cL_E$ ✓

$\therefore 0 = L_E\left(f.b.Z_E - c\right)$ ✓

$\therefore Z_E = \frac{c}{f.b}$ ✓

For equilibrium of zebra:

✓ $Z_E = Z_E + aZ_E\left(1-\frac{Z_E}{K}\right) - b.Z_E.L_E$ ✓

$\therefore 0 = Z_E\left[a\left(1-\frac{Z_E}{K}\right) - bL_E\right]$ ✓

$\therefore bL_E = a\left(1-\frac{Z_E}{K}\right)$ ✓

substituting from above: ✓

$\therefore L_E = \frac{a}{b}\left(1-\frac{c}{K.f.b}\right)$ ✓ (10)

(c) ✓ $8 = \frac{\overset{\checkmark}{0,8}}{\underset{\checkmark}{0,05}}\left(1-\frac{\overset{\checkmark}{1\ 000}}{K}\right)$

$K = 2\ 000$ ✓ (5)

5.2 (a) Increase in '*a*' implies an increase in equilibrium of predator: B ✓✓ (2)

(b) Increase in '*f*' implies an increase in predator and a decrease in prey: C ✓✓ (2)

[21]

QUESTION 6

6.1 $x^2 - 8x + 12 = 0$ ✓

$(x-2)(x-6) = 0$

$x = 2 \quad or \quad x = 6$ ✓✓

$\therefore u_n = A.2^n + B.6^n$ ✓

$-1 = A + B$

$6 = 2A + 6B$ ✓

$3 = A + 3B$

$\therefore B = 2$ and $A = -3$ ✓✓ (7)

6.2 $(x-2)(x-3) = 0$ ✓

$\therefore x^2 - 5x + 6 = 0$ ✓

$u_n = 5u_{n-1} + 6u_{n-2}; \quad u_0 = 8; \quad u_1 = 21$

✓✓ ✓ (5)

[12]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 $8+8y+3x=38$
$8y+3x=30$ ✓✓ ①

$20+4y+x=46$ ✓✓ ②
$4y+x=26$

① – 2×② ✓

x = –22 ✓ y = 12 ✓ (7) R

1.2 $\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

$2R_3+3R_2$ $\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 2 & 3 & 9 \\ 0 & 0 & 11 & 55 \end{array}\right)$ ✓✓

$R_3 / 11$ $\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 2 & 3 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right)$ ✓

R_2-3R_3 $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 2 & 0 & -6 \\ 0 & 0 & 1 & 5 \end{array}\right)$ ✓✓
R_1+4R_3

x = 10 ✓ y = –3 ✓ z = 5 ✓ (8) R
[15]

QUESTION 2

2.1 (a) $\begin{pmatrix} 2 & 2 & 6 \\ 4 & 2 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 5 \\ 1 & -1 & -1 \end{pmatrix}$ ✓✓ ✓ (3) R

(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ✓✓ $\begin{pmatrix} 2 & 2 & 6 \\ 4 & 2 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -4 & -2 & -2 \\ -2 & -2 & -6 \end{pmatrix}$ ✓ ✓ (4) R

2.2 $\begin{pmatrix} \cos 225 & -\sin 225 \\ \sin 225 & \cos 225 \end{pmatrix}$ ✓✓ $\begin{pmatrix} \frac{2}{\sqrt{2}} & 0 \\ 0 & \frac{2}{\sqrt{2}} \end{pmatrix}$ ✓✓ $\Rightarrow$ $\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$ ✓✓ ✓ (8) K/C

2.3 $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ ✓ $\begin{pmatrix} 3 & -1 \\ 4 & 0 \end{pmatrix}$ ✓✓ $= \begin{pmatrix} \frac{-3+4\sqrt{3}}{2} & \frac{4+3\sqrt{3}}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ ✓ ✓

$3\cos 2\theta + 4\sin 2\theta = \frac{-3+4\sqrt{3}}{2}$ ✓✓ and $-\cos 2\theta = \frac{1}{2}$ ✓✓

$\theta = 60°$ ✓ $\therefore\ m = \sqrt{3}$ ✓✓ (12) C

[27]

QUESTION 3

3.1 (a) True ✓✓ (2) K

(b) False, change sign ✓✓ (2) K

(c) True ✓✓ (2) K

3.2 (a) Expanding by a row/column that has the most zeros ✓✓ (2) K

(b) p = –2 ✓ q = 2 ✓ (2) K

(c) $-2\begin{vmatrix} 4 & 1 & 2 \\ 3 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & 0 & 1 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$

$= -2[4(2-3) - (3-9) + 2(9-18)]$ ✓✓ $+ 2[3(1-4) - 0 + (8-3)]$ ✓✓

$= 24$ (4) R

[14]

QUESTION 4

4.1 (a) $\frac{n(n+1)}{2}$ ✓✓ (2) K

4.2 Yes, ✓ every vertex has equal degree ✓ (2) K

4.3 C ✓✓ (2) K

4.4 Fig 1 & Fig 2 ✓✓ (2) K
[8]

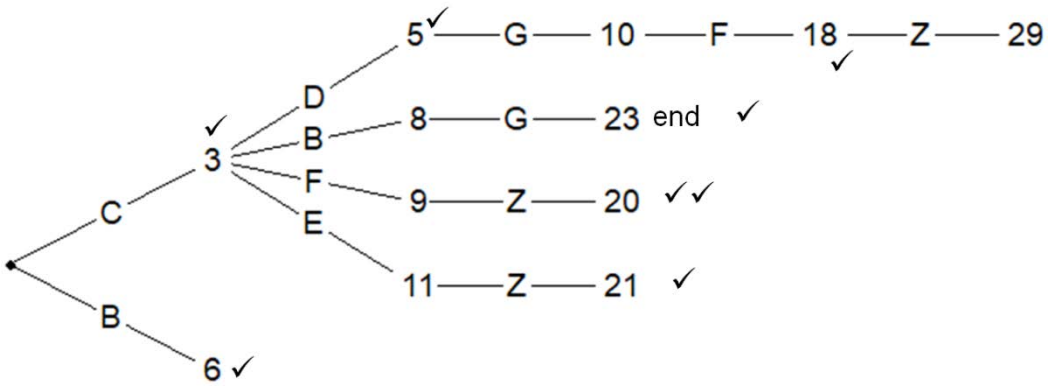
QUESTION 5

5.1 Remove vertex D ✓

AC = 3
EF = 3 ✓
CB = 5 ✓
CF = 6 ✓
FG = 8 ✓
EZ = 10 ✓
CD = 2

Total = 37 ✓ (7) R

5.2



C → F → Z = 20 minutes (8)

5.3 $B \to F \quad 2 < BF \leq 4 \;;\; BF \in \mathbb{Z}$
$\therefore$ BF = 3
BF = 4

$A \xrightarrow{6} B \xrightarrow{3} F \xrightarrow{11} Z \quad \therefore$ 20 ✓
$A \xrightarrow{6} B \xrightarrow{4} F \xrightarrow{11} Z \quad \therefore$ 21 ✓
No difference ✓ (3) C
[18]

QUESTION 6

6.1

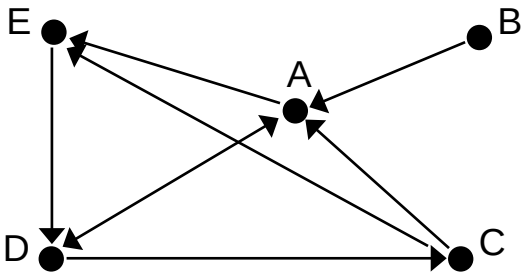
	A	B	C	D	E	
A	0	1	1	0	0	2
B	0	0	1	1	1	3 ✓
C	0	1	0	1	0	2 ✓
D	0	1	0	0	1	2 ✓
E	1	1	1	0	0	3 ✓

(4) C

6.2 B and E ✓✓ (2) C

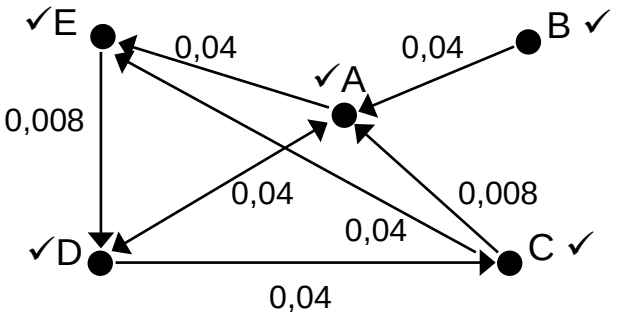
6.3 7 edges ✓✓

A_1 ; B_1 ; C_2 ; D_2 ; E_1 ✓✓



(4) C

6.4 (a)



✓

(6) P

(b) C and A ✓
E and D ✓

(2) P
[18]

Total for Module 4: 100 marks