GRADE 12 EXAMINATION
NOVEMBER 2021

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 1 hour 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1 (a) $X \sim B\left(7; \frac{1}{7}\right)$

$$P(X>1) = 1 - \left[P(X=0) + P(X=1)\right]$$
$$= 1 - \left[\left(\frac{6}{7}\right)^7 + \binom{7}{1}\left(\frac{1}{7}\right)\left(\frac{6}{7}\right)^6\right]$$
$$= 0,2635$$

(b) $X - B\left(60; \frac{1}{7}\right)$

$np > 5$ *and* $nq > 5$

$$X \sim N\left(8,57; \sqrt{7,35}^2\right)$$
$$P(X \geq 13) \rightarrow P(X > 12,5)$$
$$= P\left(Z > \frac{12,5 - 8,57}{\sqrt{7,35}}\right)$$
$$= P(Z > 1,45)$$
$$= 0,5 - 0,4265$$
$$= 0,0735$$

1.2 (a) $\frac{\binom{5}{2}\binom{2}{1}}{\binom{7}{3}} = \frac{4}{7}$

(b)

x	1	2	3
$P(X=x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

(c) $$E[X] = 1\left(\frac{1}{7}\right) + 2\left(\frac{4}{7}\right) + 3\left(\frac{2}{7}\right)$$
$$= \frac{15}{7}$$
$$Var(X) = 1^2\left(\frac{1}{7}\right) + 2^2\left(\frac{4}{7}\right) + 3^2\left(\frac{2}{7}\right) - \left(\frac{15}{7}\right)^2$$
$$= \frac{20}{49}\ (0,408)$$

QUESTION 2

2.1 (a) $X \sim N\left(7,5\ ;\ 0,75^2\right)$

$$P(\ X < 8) = P\left(Z < \frac{8-7,5}{0,75}\right)$$
$$= P\left(Z < 0,67\right)$$
$$= 0,5 + 0,2486$$
$$= 0,7486$$

(b) $P\left(Q_1 < Z < Q_3\right) = 0,5$

$$\therefore P\left(-0,67 < z < 0,67\right) = 0,5$$

$$-0,67 = \frac{Q_1 - 7,5}{0,75} \quad \textit{and} \quad 0,67 = \frac{Q_3 - 7,5}{0,75}$$

$$\therefore Q_1 = 6,998 \qquad Q_3 = 8,003$$

(c) $200\left(0,7486\right) = 149,72$

$\therefore \approx 149$ adults

2.2 (a) $\overline{x} = \frac{5,99 + 8,01}{2} = 7$

(b) $Z\left(\frac{3,5}{\sqrt{50}}\right) = 1,01$

$$Z = 2,04$$

$$\therefore P\left(-2,04 < Z < 2,04\right) = 0,4793 \times 2$$
$$= 0,9586$$

$\therefore 96\%$

QUESTION 3

3.1 (a) C

(b) D

(c) C

(d) A

3.2 (a) $H_0 : \mu = 22$
$H_1 : \mu > 22$

(b) Reject H_0 if $Z > 1,48$

$$\therefore \frac{\bar{X} - 22}{\frac{5}{\sqrt{30}}} > 1,48$$

$$\bar{X} > 23,351$$

QUESTION 4

(a) $$\int_0^k \frac{2}{k^2} x \, dx$$

$$= \left[\frac{2x^2}{2k^2}\right]_0^k$$

$$= \frac{k^2}{k^2} - 0$$

$$= 1$$

(b) $$\left[\frac{x^2}{k^2}\right]_0^6 = \frac{1}{4}$$

$$\frac{36}{k^2} = \frac{1}{4}$$

$$k^2 = 144$$

$$k = 12$$

QUESTION 5

5.1 (a) $x^2+9x-2=50$

$x^2+9x-52=0$

$(x+13)(x-4)=0$

$x\neq -13 \cup x=4$

(b) $P(A|B')=\dfrac{P(A\cap B')}{P(B')}$

$=\dfrac{24}{29}$

(c) $P(A)=\dfrac{32}{50}=\dfrac{16}{25}$

$\therefore P(A|B')\neq P(A)$

$\therefore$ A and B are not independent

OR

$P(A\cap B)=\dfrac{4}{25}=0{,}16$

$P(A)\times P(B)=\dfrac{16}{25}\times\dfrac{21}{50}$

$=0{,}2688$

$P(A\cap B)\neq P(A)\times P(B)$

$\therefore$ A and B are not independent

5.2 $\binom{5}{3}+\binom{5}{2}+\binom{5}{4}+\binom{5}{3}=35$

(1E1N 3 others + 2E1N 2 others + 1E0N 4 others + 2E0N 3 others)

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

1.1 $1\,200\,000(0,85)^8 = \text{R}326\,988,63$

1.2 $1\,200\,000(1,055)^8 = \text{R}1\,841\,623,82$

1.3 $$r_{em} = \left(1+\frac{0,1}{12}\right)^{12} - 1$$
$$= 0,1047$$

1.4 $$1\,514\,635,19 = \frac{x\left[\left(1+\frac{0,1}{12}\right)^{73} - 1\right] \cdot \left(1+\frac{0,1}{12}\right)^{24}}{\frac{0,1}{12}} - \frac{10\,000\left[(1+0,1047)^8 - 1\right]}{0,1047}$$

$$\therefore x = \text{R}13\,373,83$$

QUESTION 2

2.1 Interest = payments − reduction in outstanding balance

$$47\,131{,}31 = 12x - 36\,868{,}69$$

$$\therefore x = \text{R}7\,000$$

2.2 Outstanding balance$_{12}$ = Loan$_{12}$ − payments

$$P - 36\,868{,}69 = P\left(1+\frac{0{,}0925}{12}\right)^{12} - \frac{-7\,000\left[\left(1+\frac{0{,}0975}{12}\right)^{12}-1\right]}{\frac{0{,}0975}{12}}$$

$$\therefore P = \text{R}500\,000$$

2.3 $$500\,000 = \frac{7\,000\left[1-\left(1+\frac{0{,}0975}{12}\right)^{-n}\right]}{\frac{0{,}0975}{12}}$$

$n = 107{,}307\ldots$

$\therefore$ 108 payments = 9 years

$\therefore$ Final payment is 1 January 2030

$$\text{F.P.} = \left[500\,000\left(1+\frac{0{,}0975}{12}\right)^{107} - \frac{7\,000\left[\left(1+\frac{0{,}0975}{12}\right)^{107}-1\right]}{\frac{0{,}0975}{12}}\right].\left(1+\frac{0{,}0975}{11}\right)$$

$= \text{R}2\,159{,}18$ (12)

QUESTION 3

3.1 $T_1 = 4 + 5 = -1 = p$

$T_2 = 16 - 5(3) = 1 = q$

$T_3 = 64 - 5(9) = 19$

$19 = 7(1) + (-1)a$

$\therefore a = -12$

3.2 (a) $F_{n+1} = 0,8F_n + 1000\,;F_0 = 100\,000$

(b) $F_{n+1} = Fn$

$\therefore F_n = 0,8F_n + 1000$

$F_n = 5\,000$

(c) Equilibrium $= 30\,000$

$0,2F_n = 6\,000$

$\therefore F_{n+1} = 0,8F_n + 6\,000$

6000 fish must be added each year

QUESTION 4

4.1 Prey $= 100 \pm 5$
Predator $= 8 \pm 1$

4.2 100 prey and 50 predators

4.3 The predator population is increasing at its most rapid rate.

4.4 No effect on prey.
Increase in $K \rightarrow$ increase in predator

QUESTION 5

5.1 $a + 1 = 2,6$
$\therefore a = 1,6$
$1,6 = 0,5 \times \frac{2}{3} \times 6 \times x$
$\therefore x = 0,8$ (survival rate)

$b = 0,03$

$0,9722 = 1 - c$
$\therefore c = 0,0278$
$\therefore$ lifespan 36 cycles

$\frac{a}{K} = 0,001778$
$\therefore K = 900$

5.2 $F_{n+1} = F_n$
$0,0278F_n = 0,00005289R_nF_n$
$\therefore R_n = 526$

$R_{n+1} = R_n$
$R_n = 2,6R_n - 0,001778R_n^2 - 0,03R_nF_n$
$\therefore 0,03F_n = 1,6 - 0,001778(526)$
$\therefore F_n = 22$

QUESTION 6

6.1 $P(1+i)^{12} = P\left(1+\frac{0,12}{12}\right)^{36}\left(1+\frac{0,12}{4}\right)^{16}\left(1+\frac{0,12}{2}\right)^{10}$

$$\therefore i = \left(1+\frac{0,12}{12}\right)^{3}\left(1+\frac{0,12}{4}\right)^{\frac{4}{3}}\left(1+\frac{0.12}{2}\right)^{\frac{5}{6}} - 1$$

$$= 0,1250$$

$$= 12,5\% \text{ p.a.}$$

6.2 Increase $= 1\%$ per day

$= 365\%$ per year, compounding daily

$r_{eNR} = \left(1+\frac{3,65}{365}\right)^{365} - 1$ or $(1.01)^{365} - 1 = 36.783434$ from the growth rate

$$= 36,783\ldots$$

$$= 3\,678,34\% \text{ p.a.}$$

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 $\tan J = \sqrt{3}, \varphi = 60$

$$\begin{pmatrix} cos\,2(60) & sin\,2(60) \\ sin\,2(60) & -cos\,2(60) \end{pmatrix} \begin{pmatrix} 1 & 4 & -1 & 3 \\ 5 & 2 & -1 & -2 \end{pmatrix} \quad \text{(order)}$$

$$= \begin{pmatrix} 3.83 & -0.27 & -0.37 & -3.23 \\ 3.37 & 4.46 & -1.37 & 1.6 \end{pmatrix} \text{ (signs, values)}$$

1.2 (a) $\begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$ Sign, Shear matrix, order $\begin{pmatrix} 1 & 4 & -1 & 3 \\ 5 & 2 & -1 & -2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 4 & -1 & 3 \\ 2 & -10 & 2 & -11 \end{pmatrix}$$

(b) The area of A = Area A' factor 1.

QUESTION 2

2.1 Tr(M) = Tr(N)
9 = x

2.2 $$\begin{pmatrix} 7-x & 3 & 1 \\ 0 & 2+x & 3 \\ 2 & 8 & -x \end{pmatrix}$$

2.3 $(7-x)\left[(2+x)(-x)-24\right]+2\left[9-(2+x)\right] = 0$ sign determinant = 0

$(7-x)\left(-x^2-2x-22\right)=0$

$x = 7$ and non-real roots

QUESTION 3

3.1 360/60 = 6°

3.2 (a) 6 × 12 = 72°,

order matrix sign $\begin{pmatrix} cos(-72) & -sin(-72) \\ sin(-72) & cos(-72) \end{pmatrix}\begin{pmatrix} 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 5,71 \\ 1,85 \end{pmatrix}$

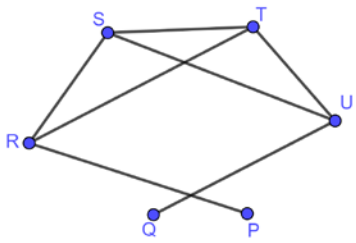
(b) 360/12 ÷ 5 = 6° – this is for a full hour, we want 1/5 of a hour

$$\begin{pmatrix} cos(-6) & -sin(-6) \\ sin(-6) & cos(-6) \end{pmatrix}\begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0,42 \\ 3,98 \end{pmatrix}$$

QUESTION 4

4.1 R; S; T; P

4.2



(correct from every vertex)

4.3 Every vertex must have an even degree.

4.4 Q R U P T Q S P Q (last tick for returning to Q)

QUESTION 5

5.1 Minimum spanning tree

5.2 Kruskal

5.3 AB = 1
AD = 4
DE = 2
EF = 2
FM = 3
MJ = 1
MH = 2
HG = 1

AC = 5
JK or MK = 6
KL = 3
GI = 8 or HI = 8 total: 38

5.4 GI /HI JK/MK (candidate identifies uniqueness comes from having unique edges)

QUESTION 6

6.1 EH – 1
CE – 2
GH – 4
BD – 4
CD – 4
AB – 8
+ FG – 1
+CF – 2
= 26

6.2 A-F-G-H-E-C-D-B = 22 + 8(AB) = 30 (return path to A)

6.3 Any path between $26 \leq x \leq 30$ (starts and ends at A, weighting is less than= 30, greater than =26

QUESTION 7

7.1 $P^{-1} = \left[\frac{1}{-11}\begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix}\right]$ (1/det) (matrix)

7.2 $A = PDP^{-1}$

$= -\frac{1}{11}\begin{pmatrix} 10 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix}$ Order

$= -\frac{1}{11}\begin{bmatrix} -77 & -110 \\ -11 & 22 \end{bmatrix}$ Multiplying

= A $\therefore LHS = RHS$ simplifying

7.3 $A^5 = A \cdot A \cdot A \cdot A \cdot A$

$= \left(PDP^{-1}\right)\left(PDP^{-1}\right)\left(PDP^{-1}\right)\left(PDP^{-1}\right)\left(PDP^{-1}\right)$

$= P D^5 P^{-1}$ (identify the pattern)

$= \frac{-1}{11}\begin{pmatrix} 10 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 8^5 & 0 \\ 0 & -3^5 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix}$

$= \frac{-1}{11}\begin{pmatrix} 327680 & -243 \\ 32768 & 243 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix}$ or $\frac{-1}{11}\begin{pmatrix} 10 \cdot 8^5 & -3^5 \\ 8^5 & 3^5 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix}$

$= \begin{pmatrix} 29767 & 30010 \\ 3001 & 2758 \end{pmatrix}$

Total for Module 4: 100 marks