INTERNATIONAL SECONDARY CERTIFICATE
EXEMPLAR 2021

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II
MODULE II

MARKING GUIDELINES

Time: 1 hour 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

All alternative valid solutions will be credited accordingly.

QUESTION 1 STATISTICS

1.1 (a) $\dfrac{\binom{4}{1}\binom{7}{2}}{\binom{11}{3}} = \dfrac{28}{55} = 0,5091$ (6)

(b) $\left(\frac{4}{11}\right)\left(\frac{3}{10}\right)\left(\frac{7}{9}\right)+\left(\frac{4}{11}\right)\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)+\left(\frac{7}{11}\right)\left(\frac{4}{10}\right)\left(\frac{6}{9}\right)+\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)=\frac{7}{11}$ (7)

1.2 (a) $20(0,1)=2$ (2)

(b) $P(X \le 3) = 1-\left(\binom{5}{4}(0,3)^4(0,7)+(0,3)^5\right)$ (7)
$= 0,9692$

(c) $X \sim B(200;0,6)$
since $np > 5$ and $nq > 5$
$X \sim N\left(120;\sqrt{48}^2\right)$
$P(X > 125) \to P(X > 125,5)$
$= P\left(Z > \dfrac{125,5-120}{\sqrt{48}}\right)$
$= P(Z > 0,79)$
$= 0,5 - 0,2852$
$= 0,2148$ (7)

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QUESTION 2

2.1 (a)
$$E[X] = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{2}\right) + 3\left(\frac{2}{9}\right) + 4\left(\frac{1}{9}\right)$$
$$= 2,28$$
$$Var(X) = 1\left(\frac{1}{6}\right) + 4\left(\frac{1}{2}\right) + 9\left(\frac{2}{9}\right) + 16\left(\frac{1}{9}\right) - (2,28)^2$$
$$= 0,746$$
$$\sigma_x = 0,86 \quad (7)$$

(b) The mean would decrease and the standard deviation would increase. (2)

2.2 (a)
$$\int_0^4 \frac{k}{x+1}\,dx = 1$$
$$\left[k\ln(x+1)\right]_0^4 = 1$$
$$k(\ln 5 - \ln 1) = 1$$
$$k\ln 5 = 1$$
$$\therefore k = \frac{1}{\ln 5} \quad (6)$$

(b)
$$\frac{1}{\ln 5}\left[\ln(x+1)\right]_0^m = \frac{1}{2}$$
$$\left[\ln(m+1) - \ln 1\right] = \frac{1}{2}\ln 5$$
$$\ln(m+1) = \ln\sqrt{5}$$
$$m+1 = \sqrt{5}$$
$$\therefore m = \sqrt{5} - 1 \text{ or } (1,2361) \quad (6)$$

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QUESTION 3

3.1 (a) $P(R) = P(Z > 1,1)$
$= 0,5 - 0,3643$
$= 0,1357$ (3)

(b) $P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$
$= 0,1357 + 0,9282 - P(1,1 < Z < 1,8)$
$= 0,1357 + 0,9282 - (0,4641 - 0,3643)$
$= 0,9641$

OR

$P(R \cup Q) = P(Z > -1,8) = 0,5 + 0,4641 = 0,9641$ (6)

3.2 $X \sim N(200; 50^2)$

$$P(X > c|X > 280) = \frac{P(X > c)}{P(X > 280)} = 0,625$$

$$P(X > 280) = P\left(Z > \frac{280 - 200}{50}\right)$$
$= P(Z > 1,6)$
$= 0,5 - 0,4452$
$= 0,0548$

$$\therefore \frac{P(X > c)}{0,0548} = 0,625$$
$P(X > c) = 0,0343$

$$\therefore 1,82 = \frac{c - 200}{50}$$
$c = 291$ (8)

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QUESTION 4

4.1 (a) A 98% CI for p is:

$$\frac{1}{5} \pm 2,33\sqrt{\frac{(0,2)(0,8)}{300}}$$

(0,1462; 0,2538) (6)

(b) Since 15% is in the interval there is no evidence to suggest that the percentage of residents have approved the revised plan. (2)

4.2 (a) $H_0 : \mu_x = \mu_y$

$H_1 : \mu_x > \mu_y$

Reject H_0 if $z > 2,05$

Test Statistic:

$$z = \frac{30,06 - 29,84}{\sqrt{\frac{0,0784}{60} + \frac{0,168}{50}}} = 3,22$$

Conclusion: Since $z > 2,05$ reject H_0 and suggest sufficient evidence to support the claim that the mean volume from the first machine is greater than the mean volume of the second machine. (10)

(b) $$z = \frac{30,06 - 29,84 - 0,1}{\sqrt{\frac{0,0784}{60} + \frac{0,168}{50}}} = 1,76$$

$$P(z > 1,76) = 0,5 - 0,4608$$
$$= 0,0392$$

$\therefore \alpha > 3,9\%$ (5)

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QUESTION 5

5.1 $$\frac{9!}{3!3!} = 10080$$ (4)

5.2 An example of such an arrangement:

* C * A * L (EE) S * S * S *

6 places for other E

$$\therefore \frac{7!}{3!} \times 6 = 5040 \quad \text{or} \quad \frac{8!}{3!} - 2\left(\frac{7!}{3!}\right) = 5040$$ (6)

[10]

Total: 100 marks