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**FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II
MODULE IV**

EXAMINATION NUMBER

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Time: 1 hour

100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 pages and an Information Booklet of 2 pages (i–ii). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 2 decimal digits, unless otherwise indicated.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
18	26	14	14	8	20	/100

MATRICES AND GRAPH THEORY

QUESTION 1

1.1 It is given that matrix $A = \begin{pmatrix} -5 & 4 \\ 3 & -1 \end{pmatrix}$

Write down A^{-1} , the inverse of A , with integer elements in the matrix.

(4)

1.2 Given matrices $C = \begin{pmatrix} 2 & 6 & 3 \\ 1 & y & 4 \end{pmatrix}$ and $D = \begin{pmatrix} -2 & 4 & z \\ 1 & 1 & -2 \end{pmatrix}$

Calculate the elements x , y and z if $C - 3D = \begin{pmatrix} 8 & -6 & 12 \\ x & 0 & 10 \end{pmatrix}$

(6)

1.3 Given $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = k$, express the determinant of the following in terms of k :

(a) $\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$

(b) $\begin{pmatrix} g & h & i \\ d & e & f \\ a & b & c \end{pmatrix}$

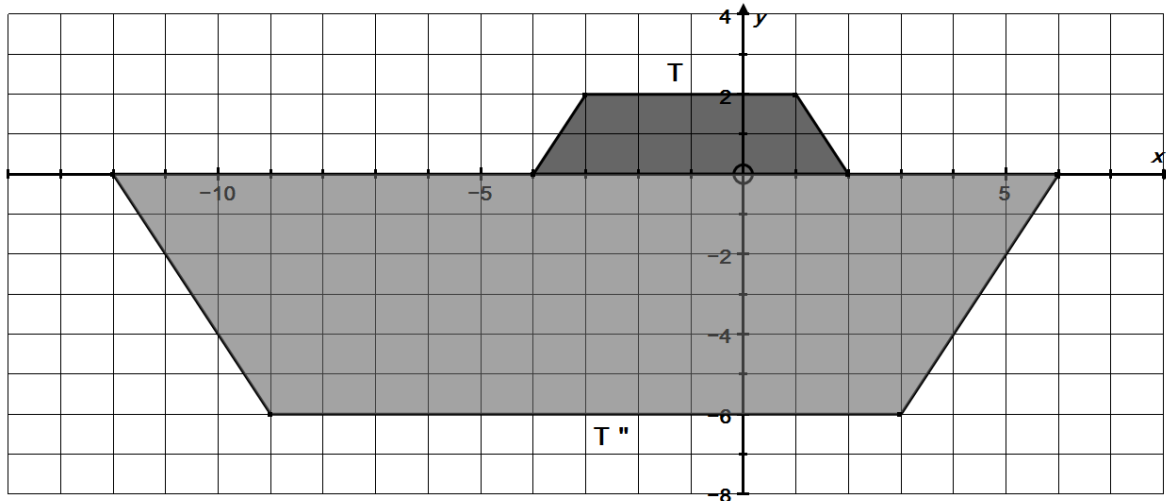
(c) $\begin{pmatrix} a & b & c \\ d & e & f \\ -3g & -3h & -3i \end{pmatrix}$

(d) $\begin{pmatrix} a-d & b-e & c-f \\ d & e & f \\ g & h & i \end{pmatrix}$

(8)
[18]

QUESTION 2

- 2.1 In the sketch below, Figure T has been translated to form Figure T' (not shown on the sketch), which has then been enlarged to create Figure T".



- (a) Describe the translation of Figure T to Figure T' in words.

(2)

- (b) Give the factor by which Figure T' was enlarged to create Figure T".

(2)

- 2.2 The point $(3; -2)$ is mapped onto $(3,232; -1,598)$ by a reflection across the line $y = mx$. Determine the inclination of the line of reflection.

(12)

2.3 A vertical line segment with endpoints $(t; v)$ and $(t; r)$ is sheared parallel to the x -axis by a factor of k .

- (a) Calculate the coordinates of the sheared endpoints in terms of the variables k , t , v , and r .

(6)

- (b) Express as an equation the relationship between m (the gradient of the line segment after it has been sheared) and k .

(4)

[26]

QUESTION 3

Consider the matrices $M = \begin{pmatrix} 2 & 2 & 1 & 0 \\ -1 & 0 & 3 & 0 \\ 4 & 9 & 3 & 1 \\ 0 & -1 & 5 & 7 \end{pmatrix}$ and $N = \begin{pmatrix} -192 & -100 & -64 & -60 \\ -32 & 4 & 72 & 52 \\ 42 & 49 & 14 & 17 \\ 6 & 7 & 2 & -33 \end{pmatrix}$

- 3.1 Explain in words why we should use the second row or the fourth column of M , rather than the first row, to calculate the determinant of M .

(2)

- 3.2 Hence, or otherwise, show through calculation that the determinant of M is -248 .

(6)

- 3.3 It is now given that N is the matrix of minors of M . Hence write down the inverse of M .

(6)
[14]

QUESTION 4

4.1 A complete graph has n vertices. State in terms of n :

- (a) the number of edges present in a spanning tree of the graph.

(2)

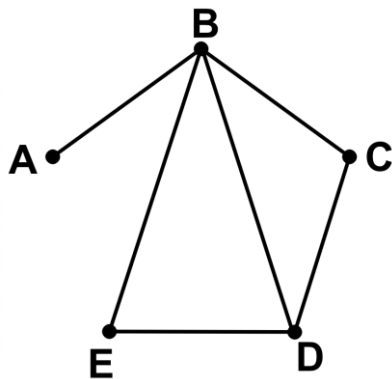
- (b) the number of edges present in the graph.

(2)

- (c) the sum of the degrees of the vertices of the graph.

(2)

4.2 Answer the questions with reference to the sketch.



- (a) Design an Eulerian path on the graph. Clearly state the order of the edges chosen, as well as the start and end vertices.

(4)

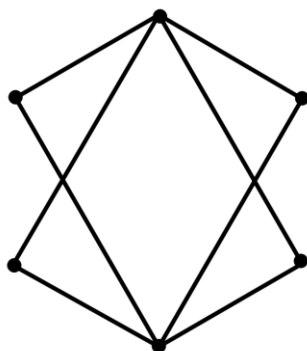
- (b) Draw the complement of the graph. Label the vertices carefully.

(4)

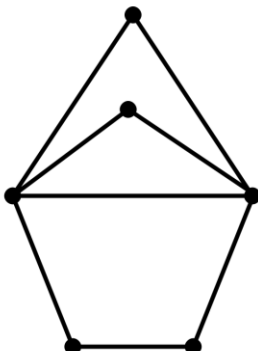
[14]

QUESTION 5

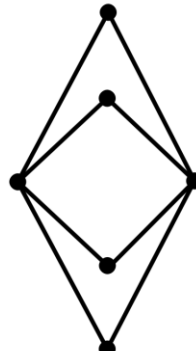
Four graphs with the same order and size are given below.



GRAPH A



GRAPH B



GRAPH C



GRAPH D

5.1 None of these graphs are regular. Briefly explain why.

(2)

5.2 State which graphs (if any) have Hamiltonian circuits.

(2)

5.3 List the graphs that are isomorphic to each other.

(4)

[8]

QUESTION 6

A graph is represented by the following adjacency matrix:

	A	B	C	D	E	F	G	H	J
A		5				4		8	6
B	5								3
C								9	5
D					8	10			7
E				8					6
F	4			10			7		
G						7		4	
H	8		9				4		
J	6	3	5	7	6				

- 6.1 Use the principles of Kruskal's Algorithm to find the spanning tree of **maximum** length. Clearly state the order in which you choose the edges, as well as the length of the tree.

(10)

- 6.2 Use Dijkstra's Algorithm to find the **shortest** route from vertex E to vertex G.
Show clear evidence of your working, e.g. a tree diagram, table record, in-out box, etc. Be sure to state your final route, as well as its length.

	A	B	C	D	E	F	G	H	J
A		5				4		8	6
B	5								3
C								9	5
D					8	10			7
E				8					6
F	4			10			7		
G						7		4	
H	8		9				4		
J	6	3	5	7	6				

(10)
[20]

Total: 100 marks